Show your work.

Problem 1 Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$.

a. compute $A - 2B^T$ $= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1-2 & 2 & 3 \\ 4 & 5 & 6-2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 4 \end{bmatrix}}.$$

b. compute AB

$$\begin{matrix} (2\times 3) & (3\times 2) \end{matrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}}.$$

Problem 2 Solve $x_1+x_2+x_3+x_4 = 1$, $3x_1+6x_2+4x_3+7x_4 = 5$ and $2x_2+2x_3+x_4 = 1$ simultaneously. You are given that $\text{rref}\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 3 & 6 & 4 & 7 & 5 \\ 0 & 2 & 2 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 7/4 & 3/4 \\ 0 & 0 & 1 & -5/4 & -1/4 \end{array}\right]$. (you do not need to do any Gaussian elimination, I did it for you by telling you this!)

$$\boxed{\begin{aligned} x_1 &= \tfrac{1}{2} - \tfrac{1}{2}x_4 \\ x_2 &= \tfrac{3}{4} - \tfrac{7}{4}x_4 \\ x_3 &= -\tfrac{1}{4} + \tfrac{5}{4}x_4 \quad \text{where } x_4 \text{ is } \underline{\text{free}}. \end{aligned}}$$

$(x_4 \in \mathbb{R}$, no restriction$)$

Problem 3 Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$. Use row operations on A to calculate rref(A). Show steps and denote row-operations explicitly with the notation we used in lecture.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow[r_3 - 2r_1]{r_2 - 3r_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[r_3 - r_2]{r_1 - r_2} \boxed{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}.$$

(of course, you could find rref(A) by a different sequence of steps.)