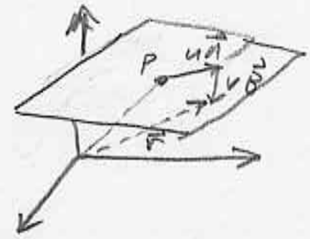
Problem 1 [3pt] Find a parametrization of a plane which contains the point $P = (1,2,3)$ and the vectors $\vec{A} = \langle 1,0,1\rangle$ and $\vec{B} = \langle 0,1,-2\rangle$ (these vectors are tangent to the plane).

$$\vec{r}(u,v) = P + u\vec{A} + v\vec{B}$$

$$\boxed{\vec{r}(u,v) = \langle 1+u,\ 2+v,\ 3+u-2v\rangle}$$



Problem 2 [5pts] Let $A = [v_1|v_2|v_3|v_4|v_5]$ and suppose $\text{rref}(A) = \begin{bmatrix} 1 & -2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Given this data, answer the following questions:

(a.) is $S = \{v_1, v_2\}$ a linearly independent set? If not then provide a linear dependence of the vectors in S

By CCP $V_2 = -2V_1$ hence <u>not</u> LI.

(b.) is $S = \{v_1, v_3, v_5\}$ a linearly independent set? If not then provide a linear dependence of the vectors in S

By CCP $V_5 = V_1 + V_3$ hence <u>not</u> LI.

(c.) state the basis of the column space of A and find the $\dim(Col(A))$.

The pivot columns of A are V_1, V_3 $\therefore$ $Col(A) = \text{span}\{V_1, V_3\}$
a nice basis is $\boxed{\{V_1, V_3\}}$ thus $\dim(Col(A)) = \boxed{2}$

(d.) derive a basis for the solution set of $Ax = 0$ where $x = (x_1, x_2, x_3, x_4, x_5)$ and find the $\dim(Null(A))$.

$$Ax = 0 \Rightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{bmatrix} 2x_2 - 2x_4 - x_5 \\ x_2 \\ -3x_4 - x_5 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \underbrace{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{W_1} + x_4 \underbrace{\begin{bmatrix} -2 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}}_{W_2} + x_5 \underbrace{\begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{W_3}$$

Thus $\boxed{\{W_1, W_2, W_3\}}$ is basis for Null(A) and $\boxed{\nu = 3}$

(e.) is $[1,1,1,1,1]$ in $Row(A)$? (provide a short calculation to support your claim)

$$a[1,-2,0,2,1] + b[0,0,1,3,1] = [1,1,1,1,1]$$

$$\left.\begin{aligned} a &= 1 \\ -2a &= 1 \end{aligned}\right\} \Rightarrow 1 = \tfrac{-1}{2} \quad \rightarrow\leftarrow$$

$\vdots$

$\therefore$ no such $a, b \in \mathbb{R}$ exist hence $[1,1,1,1,1] \notin Row(A)$.

Problem 3 [2pts] Suppose $x = s + 7t$ and $y = 2s + 3t$ and $z = 4s - t$ parametrize some space in $\mathbb{R}^3$.

(a.) what type of space is this (a point, line, plane, volume etc...) ?

this is a plane (containing $\langle 1,2,4\rangle$ & $\langle 7,3,-1\rangle$

$$\vec{r}(s,t) = s\langle 1,2,4\rangle + t\langle 7,3,-1\rangle$$

(b.) find the cartesian equation(s) of the space. How many equations in x, y, z should we expect are required?

Solve $x = s + 7t$ (I) for x, y, z.

$y = 2s + 3t$ (II)

$z = 4s - t$ (III)

take (II) $-$ 2(I) to eliminate s

$$y - 2x = 3t - 14t = -11t \quad \text{(IV)}$$

Likewise (III) $-$ 2(II) to eliminate s,

$$z - 2y = -t - 6t = -7t \quad \text{(V)}$$

Solve both (IV) and (V) for t,

$$t = \boxed{\frac{-1}{11}(y - 2x) = \frac{-1}{7}(z - 2y)}$$

(Can clean up)