

(allow 8 minutes)

Name:

MATH 221 BOSS FIGHT 1: BONUS QUIZ ON BASIS FOR COL(A) AND NULL(A) (+20PTS)

A basis is a set of vectors. Write your answers as appropriate sets of vectors. Box your answers.

Problem 1: You are given that:

$$A = \begin{bmatrix} 1 & -3 & 1 & 3 \\ 3 & -9 & 1 & 5 \\ 2 & -6 & 1 & 4 \end{bmatrix} \quad \text{has} \quad \text{rref}(A) = \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a.) (2pts) Find the basis for Col(A),

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(b.) (8pts) Find the basis for Null(A).

$$x_1 = 3x_2 - x_4$$

$$x_3 = -2x_4$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3x_2 - x_4 \\ x_2 \\ -2x_4 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \quad \therefore \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$$

Problem 2: You are given that:

$$A = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -2 & 2 & 2 & 2 \end{bmatrix} \quad \text{has} \quad \text{rref}(A) = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a.) (2pts) Find the basis for Col(A),

$$\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$$

(b.) (8pts) Find the basis for Null(A).

$$x_1 = x_2 + x_3 + x_4$$

$$\begin{aligned} x &= (x_2 + x_3 + x_4, x_2, x_3, x_4) \\ &= x_2(1, 1, 0, 0) + x_3(1, 0, 1, 0) + x_4(1, 0, 0, 1) \end{aligned}$$

$$\left\{ (1, 1, 0, 0), (1, 0, 1, 0), (1, 0, 0, 1) \right\} \quad \text{a.k.a.} \quad \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$