

CURVE FITTING

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We often find ourselves faced with the problem of solving several linear equations simultaneously in the context of fitting a curve.

E218 Find a parabola through $(0,0)$, $(2,-2)$, $(4,0)$.

Ignore for a moment our wealth of knowledge concerning zeros and factoring. Attack this brute-force.

$$Y = f(x) = ax^2 + bx + c$$

$$(0,0) : 0 = c \quad : R1$$

$$(2,-2) : -2 = 4a + 2b + c \quad : R2$$

$$(4,0) : 0 = 16a + 4b + c \quad : R3$$

This time our system has variables a, b, c .

Clearly $R1$ says $c = 0$ then

$$\begin{array}{l} 4(R2) : (-8 = 16a + 8b) \\ R3 : - (0 = 16a + 4b) \end{array}$$

$$-8 = 4b$$

$$\therefore b = -2$$

$$\text{Then } R2 : -2 = 4a + 2(-2)$$

$$-2 = 4a - 4$$

$$2 = 4a$$

$$\therefore a = \frac{1}{2}$$

Thus the parabola

$$Y = \frac{1}{2}x^2 - 2x$$

goes through the given points.

Remark: We were given zeros! We could say $f(x) = Ax(x-4)$ from the beginning then $f(2) = -2$

$$\text{Says } -2 = 2A(-2) \Rightarrow A = -\frac{1}{2} \Rightarrow Y = -\frac{1}{2}x(x-4)$$

(this remark makes §6.3#45 easier)

E219 Find the circle that goes through the points $(0,0)$, $(2,2)$ and $(4,0)$. We are given that the general form of the circle eqⁿ is

$$x^2 + y^2 + Dx + Ey + F = 0$$

Our goal: find D, E, F that fit the given data.

$$(0,0): 0^2 + 0^2 + D(0) + E(0) + F = 0 \quad \therefore \underline{F = 0}$$

nice.

$$(2,2): 4 + 4 + 2D + 2E = 0: \textcircled{2}$$

$$(4,0): 16 + 0 + 4D + 0E = 0 \quad \therefore 16 + 4D = 0$$

$$\Rightarrow D = \frac{-16}{4} = -4$$

② reveals that

$$8 + 2D + 2E = 0$$

$$8 + 2(-4) + 2E = 0$$

$$\Rightarrow \underline{E = 0}$$

Thus the circle is $\boxed{x^2 + y^2 - 4x = 0}$

This is same as $\underline{(x-2)^2 + y^2 = 4}$ its a circle of radius 2 centered at $(2,0)$.

Its easy to check $(0,0)$, $(2,2)$ and $(4,0)$ are on the circle as requested.

Remark: Solving systems can be tedious, but it is one of the more useful skills you can gain in this course. The matrix methods we cover in chapter 7 are even easier if you're allowed a good calculator (not on my tests, I don't require it so it's not allowed)