FACTORING POLYNOMIALS

The examples we just completed $\boxed{E21} \to \boxed{E30}$ went the reverse direction. We took polynomials that were nice and factored and multiplied them out. Why? Well, I hope that those make what we do here a little less mysterious. Ultimately we can tell if we factored correctly by multiplying out our factored form to see if we get back where we started. I usually do that in my head when I factor, I check my answer to be safe.

$\boxed{E31}$ $$x^2 + 5x + 6 = \underline{(x+3)(x+2)}.$$

You can check, $(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6.$

$\boxed{E32}$
$$\begin{aligned} \tfrac{1}{2}x^2 + 3x + \tfrac{9}{2} &= \tfrac{1}{2}(x^2 + 6x + 9) \\ &= \tfrac{1}{2}(x+3)(x+3) \\ &= \underline{\tfrac{1}{2}(x+3)^2}. \end{aligned}$$

Notice that factoring out the $\frac{1}{2}$ made it easier to see how to factor the polynomial

$\boxed{E33}$
$$\begin{aligned} x^2 - 5 &= x^2 - (\sqrt{5})^2 \\ &= \underline{(x+\sqrt{5})(x-\sqrt{5})}. \end{aligned}$$

This is the difference of perfect squares pattern. It's good to know this one. I don't expect you know the special forms for trinomials. (on pg. 34) We'll approach the problem of factoring a cubic by a more brute-force tactic.

$\boxed{E34}$ $\frac{2}{3}x(x-3) - 4(x-3) = \left[\frac{2}{3}x - 4\right](x-3)$

$$= \frac{2}{3}\left[x - \left(\frac{3}{2}\right)4\right](x-3)$$

$$= \underline{\frac{2}{3}(x-6)(x-3)}.$$

The <u>common factor</u> of $(x-3)$ stood out to begin with. Then to make things pretty I brought the $\frac{2}{3}$ out front. Both of the factors $(x-6)$ and $(x-3)$ are called <u>monic</u> because the <u>leading coefficient is one</u>.

$\boxed{E35}$ Factor the following poly. by grouping,

$$x^5 + 2x^3 + x^2 + 2 = x^3(x^2+2) + x^2 + 2$$

$$= \underline{(x^3+1)(x^2+2)}.$$

to go further I need to discuss more about zeroes of cubics. Your text would use a special form to break down x^3+1 into pieces.

$\boxed{E36}$ Again look for grouping,

$$x^3 + 5x^2 - 5x - 25 = x^2(x+5) - 5(x+5)$$

$$= (x^2-5)(x+5)$$

$$= \underline{(x+\sqrt{5})(x-\sqrt{5})(x+5)}.$$

$\boxed{E37}$ $(x^2+8)^2 - 36x^2 = u^2 - v^2$: $\underline{u = x^2+8,\ v = 6x}$ substituted.

$= (u+v)(u-v)$: diff. of perfect squares.

$= (x^2+8+6x)(x^2+8-6x)$

$= (x^2+6x+8)(x^2-6x+8)$

$= \underline{(x+4)(x+2)(x-4)(x-2)}.$

Hmm, this sure does seem similar to § P.4 # 100.

$\boxed{E38}$ $7(3x+2)^2(1-x)^2 + (3x+2)(1-x)^3 =$ look for greatest common factor.

$= (3x+2)(1-x)^2\left[7(3x+2) + 1 - x\right]$

$= (3x+2)(x-1)^2\left[21x + 14 + 1 - x\right]$

$= \underline{(3x+2)(x-1)^2(20x+15)}.$

Problems § P.4 # 106 & 108 would seem to require similar thinking.

$\boxed{E39}$ $x^2+1 = x^2+1$: cannot factor over $\mathbb{R}$.

$(x^2+1)^2 = (x^2+1)^2$: again this is irreducible over $\mathbb{R}$.

$(x^2+1)^4 = (x^2+1)^4$: can't break down further over $\mathbb{R}$.

__Remark:__ over $\mathbb{C}$ we have $x^2+1 = (x+i)(x-i)$ but for now we're just factoring over $\mathbb{R}$.