Rational Expressions

Factoring and the laws of algebra are our chief weapons.

E40 $\dfrac{y^2-16}{y+4} = \dfrac{\cancel{(y+4)}(y-4)}{\cancel{y+4}} = \underline{y-4}$.

E41 Hint for # 26, $x^3+x^2-9x-9 = (x+3)(x^2-2x-3)$

$$\begin{array}{r l} & x^2-2x-3 \\ x+3 \,\big) & x^3+x^2-9x-9 \\ & \underline{x^3+3x^2} \\ & \quad -2x^2-9x-9 \\ & \quad \underline{-2x^2-6x} \\ & \qquad -3x-9 \end{array}$$

We'll cover this soon, for now just use this fact to help simplify # 26 of § P.5.

E42 $\dfrac{9x^2+9x}{2x+2} = \dfrac{9(x^2+x)}{2(x+1)} = \dfrac{9x\cancel{(x+1)}}{2\cancel{(x+1)}} = \boxed{\dfrac{9x}{2}}$.

E43 $\left(\dfrac{5}{x-1}\right)\left(\dfrac{x-1}{25(x-2)}\right) = \dfrac{5\cancel{(x-1)}}{25\cancel{(x-1)}(x-2)} = \boxed{\dfrac{1}{5(x-2)}}$.

E44 $\dfrac{x^2-36}{x} \div \dfrac{x^3-6x^2}{x^2+x} = \dfrac{\left(\dfrac{x^2-36}{x}\right)}{\left(\dfrac{x^3-6x^2}{x^2+x}\right)}$ ← prefered notation.

this notation ought not be used in algebra. We stop this notation in advanced math. (I'm following the text here)

$$= \frac{(x^2-36)(x^2+x)}{x(x^3-6x^2)}$$

$$= \frac{(x+6)\cancel{(x-6)}\cancel{x}(x+1)}{\cancel{x}\cancel{(x-6)}x^2}$$

$$= \boxed{\frac{(x+6)(x+1)}{x^2}}$$

Common Denominator Examples

E45

$$\frac{5}{x-1}+\frac{2}{x+3}=\frac{5(x+3)}{(x-1)(x+3)}+\frac{2(x-1)}{(x+3)(x-1)}$$: multiplied by one twice

$$=\frac{5(x+3)+2(x-1)}{(x-1)(x+3)}$$

$$=\frac{5x+15+2x-2}{(x-1)(x+3)}$$

$$=\boxed{\frac{7x+13}{(x-1)(x+3)}}$$

E46

$$\frac{-1}{x}+\frac{2}{x^2+1}+\frac{1}{x^3+x}=\frac{-(x^2+1)}{x(x^2+1)}+\frac{2x}{(x^2+1)x}+\frac{1}{x(x^2+1)}$$

$$=\frac{-x^2-1+2x+1}{x(x^2+1)}$$

$$=\frac{-x^2+2x}{x(x^2+1)}$$

$$=\frac{x(2-x)}{x(x^2+1)}$$

$$=\boxed{\frac{2-x}{x^2+1}}.$$

Remark: I hope these help with § P.5 # 49 & 52.

E47 $x^6 - x^{-2} = x^{8-2} - x^{-2}$

$= x^8 x^{-2} - x^{-2}$

$= (x^8 - 1)x^{-2}$

$= \boxed{\dfrac{x^8 - 1}{x^2}}$

E48 $3x^2(x-2)^{1/2} - 7(x-2)^{-1/2} = \dfrac{3x^2(x-2)}{(x-2)^{1/2}} - \dfrac{7}{(x-2)^{1/2}}$

$= \dfrac{3x^2(x-2) - 7}{\sqrt{x-2}}$

$= \boxed{\dfrac{3x^3 - 6x^2 - 7}{\sqrt{x-2}}}$

__technically__: this is an __algebraic__ expression, this problem (and those like it §P.5 # 65, 66, 67, 73, 74 etc...) ought not be in the rational expressions section. Just saying, Same here?

E49 Simplify the following "difference quotient" by rationalizing the numerator.

$$\frac{\sqrt{x+2} - \sqrt{x}}{2} = \frac{\sqrt{x+2} - \sqrt{x}}{2}\left(\frac{\sqrt{x+2} + \sqrt{x}}{\sqrt{x+2} + \sqrt{x}}\right)$$

$$= \frac{(\sqrt{x+2})^2 + \cancel{\sqrt{x}\sqrt{x+2}} - \cancel{\sqrt{x}\sqrt{x+2}} - (\sqrt{x})^2}{2(\sqrt{x+2} + \sqrt{x})}$$

$$= \frac{x + 2 - x}{2(\sqrt{x+2} + \sqrt{x})}$$

$$= \boxed{\frac{1}{\sqrt{x+2} + \sqrt{x}}}$$

I'm not so convinced this is "simpler" but this is what I'd like you to try for §P.5 # 74.

More on simplifying Rational Expressions

E50 $$\frac{16-5x-x^2}{x} = \frac{16}{x} - \frac{5x}{x} - \frac{x^2}{x} = \boxed{\frac{16}{x} - 5 - x}$$

If we have a sum in the numerator then we can rewrite the given fraction as a sum of fractions.

In contrast: $$\frac{x}{16-5x-x^2} \neq \frac{x}{16} - \frac{x}{5x} - \frac{x}{x^2}$$

(a common and unfortunate error.)

E51 $$\frac{2(3x-1)^{1/3} - (2x+1)(\cancel{\tfrac{1}{3}})(3x-1)^{-2/3}(\cancel{3})}{(3x-1)^{2/3}} =$$

multiply by $\frac{(3x-1)^{2/3}}{(3x-1)^{2/3}}$

$$= \frac{2(3x-1)^{1/3}(3x-1)^{2/3} - (2x+1)\cancel{(3x-1)^{-2/3}}\,\cancel{(3x-1)^{2/3}}}{(3x-1)^{2/3}(3x-1)^{2/3}}$$

$$= \frac{2(3x-1) - (2x+1)}{(3x-1)^{4/3}}$$

$$= \frac{6x-2-2x-1}{(3x-1)^{4/3}}$$

$$= \boxed{\frac{4x-3}{(3x-1)^{4/3}}}$$

Remark: if I were to ask such a question on a test I would probably give some guidance. I might ask what are A, B, C, D for

$$\frac{2(3x-1)^{1/3} - (2x+1)(\tfrac{1}{3})(3x-1)^{-2/3}}{(3x-1)^{2/3}} = \frac{Ax+B}{(Cx-1)^D}$$

[you'd find $A=4$, $B=-3$, $C=3$, $D=4/3$]