Number Systems

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ = natural numbers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ = integers

$\mathbb{Q} = \{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ with } q \neq 0 \}$ = rational numbers

$\mathbb{R} = (-\infty, \infty)$ = real numbers

$\mathbb{C} = \{ a + ib \mid a, b \in \mathbb{R} \text{ with } i = \sqrt{-1} \}$ = complex numbers

$\boxed{\text{E1}}$ Examples of numbers,

$32 \in \mathbb{N}$ ("$\in$" means "element of")

$234{,}216{,}247 \in \mathbb{N}$

$-7 \in \mathbb{Z}$ means -7 is an integer.

$0 \notin \mathbb{N}$ means zero is not a natural number

$\frac{3}{4} \in \mathbb{Q}$ means $\frac{3}{4}$ is a rational number

$1, \frac{1}{2}, \frac{1}{3} \in \mathbb{Q}$ reads 1, $\frac{1}{2}$ and $\frac{1}{3}$ are all rational numbers.

$\pi \in \mathbb{R}$ means π is a real number

$\pi \notin \mathbb{Q}$ means π is <u>not</u> a rational number.
the decimal expansion of π is

$$\pi = 3.1415\ldots$$

the pattern does not repeat, this shows us evidence that π is <u>irrational</u> (not rational)

Decimal Expansions of Rational Numbers

It can be shown that $a + ar + ar^2 + ar^3 + \cdots = \dfrac{a}{1-r}$ for $|r| < 1$. This is the geometric series result.

(we prove this in math 132)

$\boxed{E2}$ Consider that $0.\overline{111} = 0.1 + 0.01 + 0.001 + 0.0001 + \cdots$ identify that $a = 0.1$ and $r = 0.1$ thus,

$$0.\overline{111} = \frac{0.1}{1-0.1} = \frac{0.1}{0.9} = \boxed{\frac{1}{9} = 0.111\ldots}$$

$\boxed{E3}$ Find fraction with decimal expansion $0.137137137\ldots$

Notice
$$0.\overline{137} = 0.137 + 0.000137 + 0.000000137 + \cdots$$
$$= 0.137 + \frac{1}{1000}(0.137) + \left(\frac{1}{1000}\right)^2(0.137) + \cdots$$

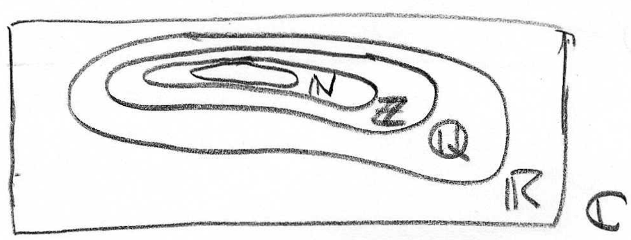
Identify this is geometric series with $a = 0.137$ and $r = \frac{1}{1000}$. Thus

$$0.\overline{137} = \frac{0.137}{1 - \frac{1}{1000}} = \frac{0.137}{\frac{999}{1000}} = \boxed{\frac{137}{999} = 0.\overline{137}}$$

Remark: these examples should help you do §8.3# 93, 94

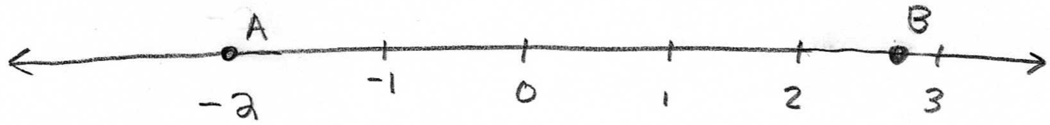
What are Real Numbers?

We can think of real numbers as all the possible decimal expansions. Some of these are repeating decimals, those numbers are in $\mathbb{Q}$. Other numbers are irrational like $\pi = 3.1415\ldots$ or $e = 2.71\ldots$.



Number Line and Inequalities

The real numbers correspond to a line,



the point $A = -2$ while the point $B = e = 2.71\ldots$ of course you could just as well say $B = 2.7$ if you were just given the number line above.

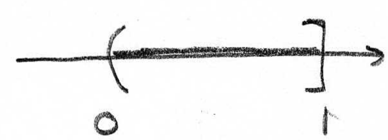
X is a number with $-2 < X < 3$

$\boxed{E4}$ Inequalities, number lines and set notation. Three ways to express the same idea

i.) $X \in [0,1] \iff 0 \leq X \leq 1 \iff$

0 and 1 are <u>included</u>

ii.) $X \in (0,1] \iff 0 < X \leq 1 \iff$



0 is <u>excluded</u>
1 is <u>included</u>

iii.) $X \in [0,\infty) \iff 0 \leq X \iff$

0 is <u>included</u>
<u>∞ is not a real number</u>

Cases i & ii are examples of <u>bounded</u> sets. Example iii. is an <u>unbounded</u> set.

E5 Give an inequality for z if $z \in (20, 23]$

$$20 < z \leq 23$$

<u>Observation</u>: The distance between $a, b \in \mathbb{R}$ is given by the distance formula $d(a,b) \equiv |a-b|$ where $|\cdot|$ denotes <u>absolute value</u>.

$$\boxed{|x| \equiv \begin{cases} x & : x \geq 0 \\ -x & : x < 0 \end{cases}}$$

definition of absolute value.

E6

$$d(-2,1) = |-2-1| = |-3| = 3.$$
$$d(-2,3) = |-2-3| = |-5| = 5.$$
$$d(3,1) = |3-1| = 2.$$

<u>Request</u>: please read pgs. 7-8 of your text to refresh your memory on how arithmetic works.

E7 Which value for x makes the expression below undefined?

$$\boxed{\frac{x}{x-1}} \leftarrow$$ <u>what must we avoid?</u>

E8 The coefficients are the numbers next to the variables in a polynomial expression,

$$x^4 - x^2 + 3x + 2 = x^4 + 0 \cdot x^3 - 1x^2 + 3x + 2x^0$$

<u>Coefficients</u>	1	0	-1	3	2

variable is x.