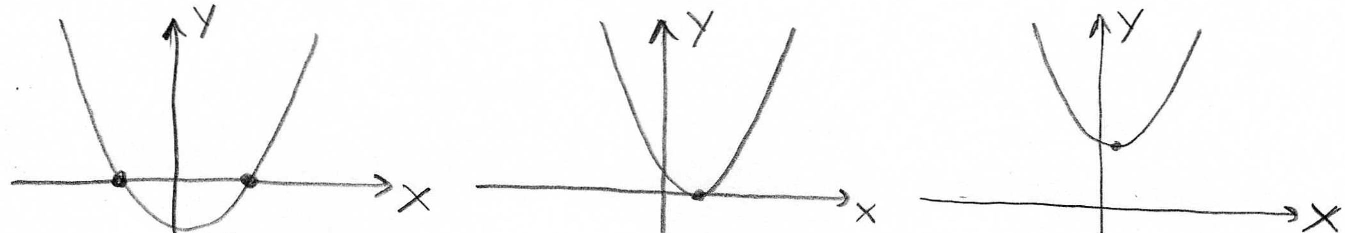
GRAPHING QUADRATIC EQUATIONS

The graph of $ax^2+bx+c = y$ is a parabola.
The Quadratic Eq$^{\underline{n}}$ had 3 cases which correspond to 3 distinct graphical possibilities (assume $a>0$)

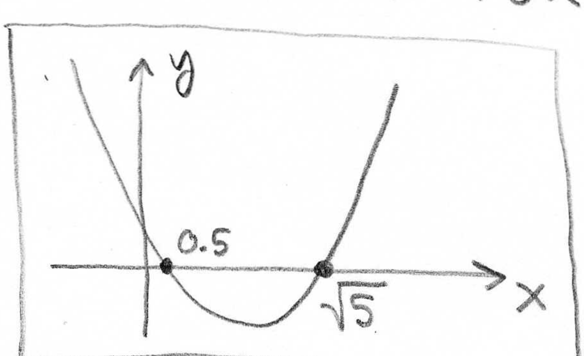


$b^2-4ac>0$ | $b^2-4ac=0$ | $b^2-4ac<0$

- The y-intercept of $y=ax^2+bx+c$ is the constant c.
- The x-intercepts are given by the quadratic eq$^{\underline{n}}$ in cases I or II.
- The vertex is at $x = -b/2a$ in all cases.

Observation: if we have a way to graph the eq$^{\underline{n}}$ $y=ax^2+bx+c$ then we can read sol$^{\underline{n}}$'s to the eq$^{\underline{n}}$ $ax^2+bx+c=0$ from the locations of the x-intercepts. Warning: can't rely on this alone because I don't allow graphing calculator on test or homework. (You can use graphing calculator for guidance and insight but you should figure out the algebra w/o technology in the end)

$\boxed{\text{EG7}}$ Given the graph of $y=ax^2+bx+c$ below find sol$^{\underline{n}}$'s to $ax^2+bx+c=0$



(Given)

$x = 0.5$ and $x=\sqrt{5}$ are the sol$^{\underline{n}}$'s

Applications of Quadratic Equations

E68 The position eq^n for a ball thrown straight up with velocity V_0 at height S_0 is given by the quadratic eq^n

$$\boxed{S = -16t^2 + V_0 t + S_0}$$

where S is in (ft) and t and V_0 are in S and $\frac{ft}{s}$.

Suppose $S_0 = 3ft$ and $V_0 = 20ft/s$. How high does the ball travel? When does it hit the ground $(s=0)$?

$$S = -16t^2 + 20t + 3$$

We have $a=-16$, $b=20$ and $c=3$. We'll find $s=0$ when $-16t^2+20t+3=0$, use quad. eq^n,

$$t = \frac{-20 \pm \sqrt{400-4(-16)(3)}}{-32}$$

$$= \frac{-20 \pm \sqrt{592}}{-32} \longrightarrow \underbrace{t_1 = \frac{20+\sqrt{592}}{32}}_{\text{time when ball hits ground.}} \text{ or } \underbrace{t_2 = \frac{20-\sqrt{592}}{32}}_{\text{negative, ignore for physical reasons.}}$$

The eq^n $s = -16t^2+20t+3$ has a graph which is a parabola that opens down. The vertex will give the maximum s.

$$t_{max} = \frac{-b}{2a} = \frac{-20}{2(-16)} = \frac{10}{16} = \frac{5}{8}$$

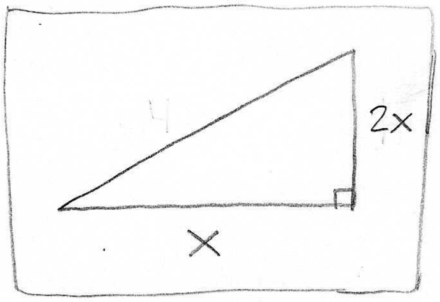
$$S_{max} = -16\left(\frac{5}{8}\right)^2 + 20\left(\frac{5}{8}\right) + 3$$

$$= \frac{-16(25) + 20(5)(8) + 3(64)}{64}$$

$$= \frac{-400+800+192}{64}$$

$$= \frac{592}{64} = \frac{16(37)}{64} = \frac{37}{4} = \boxed{9.25\ ft}$$

E 69 The triangle below is a right triangle. Find the width x if the area is known to be 100 and the height is $2x$.



(given)

The area $= \frac{1}{2}(\text{base})(\text{height})$ thus

$$100 = \frac{1}{2}(x)(2x)$$

$$100 = x^2$$

$$x = \pm 10$$

But, x is a length which are positive so $\boxed{x = 10}$

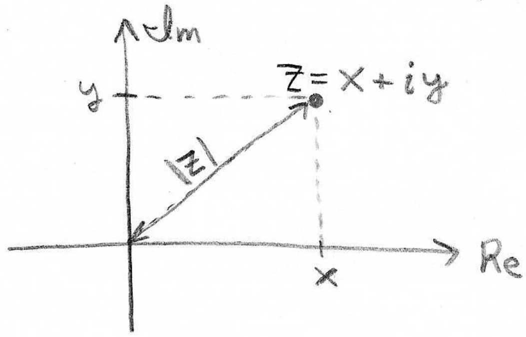
Complex Numbers

A complex number has the form $a+ib$ where $a, b \in \mathbb{R}$ and $i^2 = -1$. We add, subtract, multiply and divide complex numbers just like real numbers, the new thing is $i = \sqrt{-1}$. We say $z \in \mathbb{C}$ if $z = x + iy$ for $x, y \in \mathbb{R}$ and

$$\text{Re}\{x+iy\} = x \quad : \text{the real part.}$$

$$\text{Im}\{x+iy\} = y \quad : \text{the imaginary part.}$$

The conjugate of $z = x+iy$ is denoted $z^* \equiv x - iy$.
The modulus of z is $|z| = \sqrt{zz^*} = \sqrt{x^2+y^2}$ this gives the distance of $z = x+iy$ from the origin in the complex plane,



$\boxed{\text{E70}}$ A few sample complex number arithmatic problems

$$(5+16i)(2-i) = 10-5i+32i-16i^2$$
$$= 10-16(-1)+27i$$
$$= \underline{26+27i}.$$

Rewrite the expression in the $a+ib$ form,

$$\frac{3}{i}+4 = \frac{3i}{i^2}+4 = \underline{-3i+4}.$$

$$\frac{1}{3+2i} = \frac{1}{3+2i}\left(\frac{3-2i}{3-2i}\right) = \frac{3-2i}{9+6i-6i-4i^2} = \frac{3-2i}{13}$$
$$= \underline{\frac{3}{13}-\frac{2}{13}i}.$$

$$\frac{1+i}{i}-\frac{3}{4-i} = \frac{i+i^2}{i^2}-\frac{3(4+i)}{(4-i)(4+i)}$$
$$= -i+1-\frac{12+3i}{16+1}$$
$$= 1-\frac{12}{17}-\left(1+\frac{3}{17}\right)i$$
$$= \underline{\frac{15}{17}-\frac{20}{17}i}.$$

$$\frac{5i}{(2+3i)^2} = \frac{5i}{4+12i-9}$$
$$= \left(\frac{5i}{12i-5}\right)\left(\frac{-12i-5}{-12i-5}\right)$$
$$= \frac{-60i^2-25i}{-144i^2+25}$$
$$= \frac{60-25i}{169}$$
$$= \underline{\frac{60}{169}-\frac{25}{169}i}.$$

$$\sqrt{-144} = \sqrt{144}\sqrt{-1} = \underline{12i}.$$

E71 Application of complex numbers to Impedance

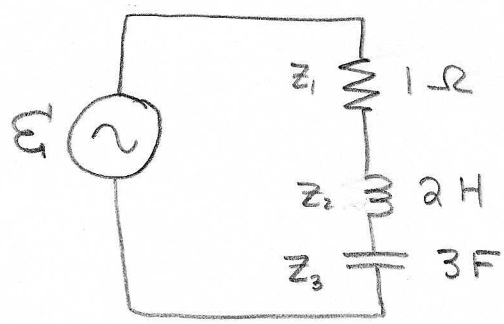
Impedance is complex resistance. (Assuming the frequency for the voltage source is $\omega = 1$)

$Z = R$ for resistor

$Z = Li$ for inductor

$Z = -Ci$ for capacitor

The "effective" or Thevenin impedance is found for the series circuit below as follows:



(Ohms) $Z_1 = 1$

(Henries) $Z_2 = 2i$

(Farads) $Z_3 = -3i$

$$Z_{net} = 1 + 2i - 3i = 1 - i$$

If the voltage source $\mathcal{E}$ has 10 V peak then "Ohm's Law" says $10 = I(1-i)$ thus

$$I = \frac{10}{1-i} = \frac{10(1+i)}{(1-i)(1+i)} = \frac{10+i}{2} = 5 + \frac{1}{2}i$$

Then $I_{peak} = |I| = \sqrt{25 + \frac{1}{4}} = \sqrt{\frac{101}{4}}$. The peak current will be $\sqrt{\frac{101}{4}}$ Amperes.

Remark: this is known as the Phasor Method. It's neat because it allows us to analyze an AC-circuit circuit as if it is a DC-circuit. I have hidden the frequency dependence. I'd be happy to point you to further things to read about this if you wish. (Also see § 1.5 # 83)