"Other" types of Equations (§1.6)

In this section we combine techniques or extend techniques we discussed in previous examples. We learn a few new tricks about eliminating radicals.

E72 $x^4 - 16 = 0$ let $u = x^2$ then $x^4 = u^2$

$$u^2 - 16 = 0$$
$$(u+4)(u-4) = 0$$
$$(x^2+4)(x^2-4) = 0$$

either $x^2 + 4 = 0$ or $x^2 - 4 = 0$

$x^2 = -4$ $\quad$ $x^2 = 4$

$x = \pm 2i$ $\quad$ $x = \pm 2$

We find two <u>real</u> <u>number</u> sol^{n}'s $\boxed{x = \pm 2}$

(there are complex sol^{n}'s $x = \pm 2i$, but I'll usually just ask you to find all the real sol^{n}'s)

E73 $x^4 + 7x^3 + 12x^2 = 0$

$$x^2(x^2 + 7x + 12) = 0$$
$$x(x+3)(x+4) = 0$$

we find sol^{n}'s $\boxed{x = 0,\ x = -3 \text{ and } x = -4}$

E74 $2x + 9\sqrt{x} = 5$ let $u = \sqrt{x}$ then $u^2 = x$
substitute to find

$$2u^2 + 9u = 5$$
$$2u^2 + 9u - 5 = 0$$
$$u = \frac{-9 \pm \sqrt{81 - 4(2)(-5)}}{4}$$
$$u = \frac{-9 \pm \sqrt{121}}{4} = \frac{-9 \pm 11}{4}$$

We find $u = 2/4$ or $u = -20/4 = -5$. We cannot keep the $u = -5$ sol^{n} since $u = \sqrt{x} \geq 0$. Thus $1/2 = \sqrt{x}$ yields the sol^{n} $\boxed{x = 1/4}$

$\boxed{E75}$ $x^3 = x$ divide by x

$x^2 = 1$

$x = \pm\sqrt{1} = \pm 1$

what did I miss? Why do we generally avoid division when doing algebra? (this is not universally the case, sometimes we divide w/o much care for detail, but here we want to find <u>all</u> sol^{n}'s so division got us into trouble)

$\boxed{E76}$ $F = \dfrac{GmM}{R^2}$ solve for G.

$$G = \frac{R^2 F}{mM}$$

(here division by m, M is ok since $m, M \neq 0$)

$\boxed{E77}$ Solve: $\sqrt[3]{2x+5} + 3 = 0$ (looks like §1.6 #34)

$\sqrt[3]{2x+5} = -3$ now cube both sides.

$2x+5 = (-3)^3 = -27.$

$2x = -32$

$\boxed{x = -16}$

$\boxed{E78}$ Solve $\sqrt{x} - \sqrt{x-5} = 1$

$\sqrt{x} = 1 + \sqrt{x-5}$: square both sides

$x = (1+\sqrt{x-5})^2 = 1 + 2\sqrt{x-5} + x - 5$

$\Rightarrow 4 = 2\sqrt{x-5}$

$\Rightarrow 2 = \sqrt{x-5}$: square both sides.

$\Rightarrow 4 = x - 5 \quad \therefore \boxed{x = 9}$

$\boxed{E79}$ Solve $\sqrt{x-6} = \sqrt{x+6}$ if possible.

$$x-6 = x+6$$
$$0 = 12 \Rightarrow \underline{\text{no sol}^{\text{n}}\text{'s exist}}.$$

Remark: graphically this means $y=\sqrt{x-6}$ and $y=\sqrt{x+6}$ have no intersection.

$\boxed{E80}$ $(x-5)^{3/2} = 125$

$$\left[(x-5)^{3/2}\right]^{2/3} = (125)^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$$

$$\Rightarrow x-5 = 25 \therefore \boxed{x = 30}$$

$\boxed{E81}$ $\boxed{\dfrac{1}{x} - \dfrac{1}{x+1} = \dfrac{1}{2}}$: find sol$^{\text{n}}$'s if possible.

$$\frac{x+1-x}{x(x+1)} = \frac{1}{2} \quad : \text{ made common denominator.}$$

$$\frac{1}{x(x+1)} = \frac{1}{2}$$

flipped eq$^{\text{n}}$ over.

$$x(x+1) = 2$$
$$x^2+x-2 = 0$$
$$(x+2)(x-1) = 0$$
$$\boxed{x = -2 \text{ or } x = 1}$$

you can check these make sense in given eq$^{\text{n}}$

$\boxed{E82}$ Solve $\dfrac{20-x}{x} = x$

$$20-x = x^2$$
$$x^2+x-20 = 0$$
$$(x+5)(x-4) = 0$$
$$\boxed{x = -5 \text{ or } x = 4}$$

E83 $|2x-1| = 5$. There are two eq^{n}'s to solve here

1.) $2x - 1 = 5 \Rightarrow 2x = 6 \Rightarrow \boxed{x = 3}$

2.) $-(2x-1) = 5 \Rightarrow -2x = 4 \Rightarrow \boxed{x = -2}$

E84 Solve $|2x-1| = 5$ another way.

$$|2x-1| = \sqrt{(2x-1)^2} = 5$$

: $\underbrace{|u| = \sqrt{u^2}}$ way to convert cases to algebraic statements.

$$(2x-1)^2 = 25$$
$$4x^2 - 4x + 1 = 25$$
$$4x^2 - 4x - 24 = 0$$
$$x^2 - 4x - 6 = 0$$
$$(x+2)(x-3) = 0$$
$$\boxed{x = -2 \quad \text{or} \quad x = 3}$$

I guess E83 is easier way.

E85 $|x^2+1| = -2$. No sol^{n}'s possible since $|x^2+1| > 0$.

E86 $|x+1| = x^2 - 5$

$$\pm(x+1) = x^2 - 5$$

$\nearrow$
$$x + 1 = x^2 - 5$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \quad \text{or} \quad x = -2$$

$\searrow$
$$-x - 1 = x^2 - 5$$
$$x^2 + x - 4 = 0$$
$$x = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$
$$x = \frac{-1+\sqrt{17}}{2} \quad \text{or} \quad x = \frac{-1-\sqrt{17}}{2}$$

Notice $|x+1| \geq 0$ thus our sol^{n}'s are only genuine if $x^2 - 5 \geq 0$. You can check that $x = -2$ and $x = \frac{-1+\sqrt{17}}{2}$ have $x^2 < 5$ thus $\boxed{x = 3 \text{ and } x = \frac{-1-\sqrt{17}}{2} \text{ are the sol}^{n}\text{'s.}}$