Inequality Properties for Real Variables

1.) $a < b$ and $b < c \Rightarrow a < c$: transitive property

2.) $a < b$ and $c < d \Rightarrow a + c < b + d$: addition of inequalities.

3.) $a < b \Rightarrow a + c < b + c$: addition of constant

4.) $c > 0$ and $a < b \Rightarrow ac < cb$

5.) $c < 0$ and $a < b \Rightarrow ac > cb$

Multiplication by negative number <u>reverses</u> the inequality.

$\boxed{E87}$ Solve $x + 7 \leq 12$ and sketch the sol^{n} on the number line.

$x \leq 12 - 7$

$\underline{x \leq 5}$.

$\boxed{E88}$ Again solve and sketch sol^{n} on number line.

$4 - 2x < 3(3 - x)$

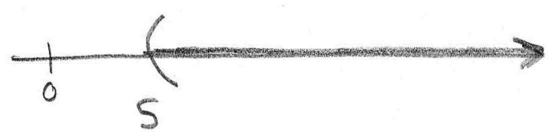
$4 - 2x < 9 - 3x$

$4 < 9 - 3x + 2x$

$4 - 9 < -x$

$-5 < -x$

$\Rightarrow \underline{5 > x}$.

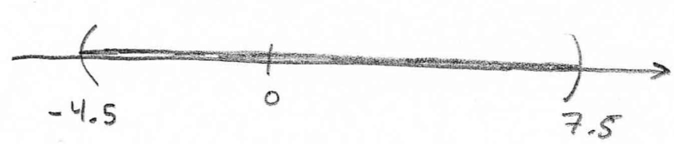


$\boxed{E89}$ $-4 < \frac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

$\underline{-4.5 < x < 7.5}$.



Absolute Values and Inequalities : Let $a > 0$

$$|x| < a \iff -a < x < a$$

$$|x| > a \iff x > a \quad \text{or} \quad x < -a$$

Remember $|x|$ gives distance of x from zero on the number line. This means $|x| < a$ says the distance of x from zero is less than a. Whereas $|x| > a$ says x has distance larger than a from the origin. This makes those inequalities easier to think about.

$\boxed{E90}$ Describe sol^{n} set for $\left|\frac{x}{2}\right| > 1$

$$\Rightarrow 2\left|\frac{x}{2}\right| > 2$$

$$\Rightarrow |x| > 2$$

$$\Rightarrow \boxed{x > 2 \quad \text{or} \quad x < -2}$$

$x \in (-\infty, -2) \cup (2, \infty)$

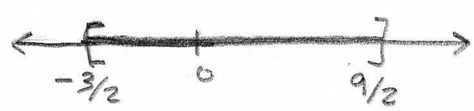
$\boxed{E91}$ Describe sol^{n} set for $|2x - 3| \le 6$

$$-6 \le 2x - 3 \le 6$$

$$-3 \le x - \frac{3}{2} \le 3$$

$$-3 + \frac{3}{2} \le x \le 3 + \frac{3}{2}$$

$$\boxed{-\frac{3}{2} \le x \le \frac{9}{2}}$$



$x \in [-3/2, 9/2]$

$\boxed{E92}$ Describe sol^{n} set for $|x - 2| < 0$.

The sol^{n} set is empty since $|x - 2| \ge 0$ by definition of absolute value.

Polynomial Inequalities (§1.8)

> **Def$^{\underline{n}}$/** A critical number for an algebraic expression is a value for which there is division by zero or the expression has a zero.

E93 Find critical numbers of $\dfrac{x^2+5x+6}{x(x-7)}$.

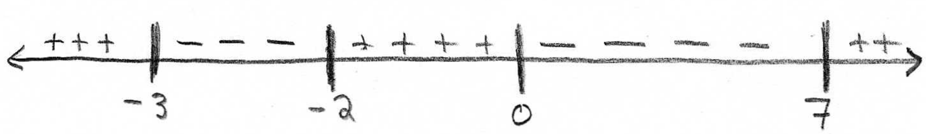
Notice we have division by zero for $x=0$ or $x=7$

Also $x^2+5x+6 = (x+3)(x+2)$ thus the numerator will be zero if $x=-3$ or $x=-2$. This shows the critical #'s are $\underline{x=-3,-2,0,7 \text{ for this expression}}$.

> <u>BIG IDEA</u>: algebraic expressions can only change signs at critical numbers. (the intermediate value theorem of calculus [math 131] proves this assertion, we'll just use it in our course)

E94 Solve $\dfrac{x^2+5x+6}{x(x-7)} < 0$ via critical #'s and a "sign-chart"

Here's the plan. We draw the number line and plot all the critical #'s. Then we need only test a point in each subinterval to see the answer.



← sign chart for $\dfrac{x^2+5x+6}{x(x-7)}$

Evaluate $\dfrac{x^2+5x+6}{x(x-7)} = \dfrac{(x+3)(x+2)}{x(x-7)}$ at $x=-4,-2.5,-1,1$ to supply the pictured plus (+) and minus (−) on the line

Our sol$^{\underline{n}}$ to $\dfrac{x^2+5x+6}{x(x-7)} < 0$ is $\boxed{-3<x<2 \text{ or } 0<x<7}$

$\boxed{E95}$ Find sol^{n} set for $\frac{1}{x+2} + \frac{1}{x-3} > 0$. To begin we find critical numbers. Clearly $x = -2$ and $x = 3$ cause division by zero. Next make common denominator,

$$\frac{1}{x+2} + \frac{1}{x-3} = \frac{x+2+x-3}{(x+2)(x-3)} = \frac{2x-1}{(x+2)(x-3)}$$

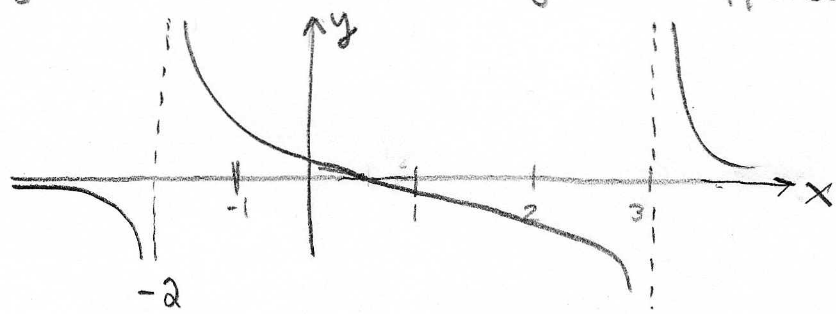
this is zero when $2x - 1 = 0$, $x = 1/2$ is also a critical #.

$$\text{---}\underset{-2}{|}\text{+++++}\underset{1/2}{|}\text{-----}\underset{3}{|}\text{++++}\rightarrow \quad \frac{2x-1}{(x+2)(x-3)}$$

You can test $x = -3, 0, 1$ and 4 to get the signs.

Thus $\boxed{\frac{1}{x+2} + \frac{1}{x-3} > 0 \text{ for } -2 < x < 1/2 \text{ and } x > 3}$

Remark: later we'll study graphs like $y = \frac{1}{x+2} + \frac{1}{x-3}$ and the sign chart will have great application there



Critical numbers turn out to be where there is either a vertical asymptote, hole in the graph or a zero.

$\boxed{E96}$ What domain of x will make $\sqrt{16-x^2}$ real-valued?

Need $16 - x^2 \geq 0$ ($16 - x^2$ is the "radicand")

Notice $16 - x^2 = (4-x)(4+x)$ has critical #s $x = \pm 4$ only.

$$\text{--}\underset{-4}{|}\text{+++++}\underset{4}{|}\text{--}\rightarrow \quad 16 - x^2 \qquad \therefore \quad \underline{16 - x^2 \geq 0 \text{ for } x \in [-4, 4]}$$

$$\boxed{\text{dom}\left(\sqrt{16-x^2}\right) = [-4, 4] = \{x \mid -4 \leq x \leq 4\}}$$