LINEAR EQUATIONS IN TWO VARIABLES AND THEIR GRAPHS (§2.1)

Let $a, b, c \in \mathbb{R}$ with a and b not both zero,

$$\boxed{ax + by = c} \text{ — general form of line}$$

is the equation of a line.

$a = 0 \Rightarrow$ horizontal line $by = c \Rightarrow y = c/b$

$b = 0 \Rightarrow$ vertical line $ax = c \Rightarrow x = c/a$

We can easily see if $c \neq 0$ then $y = c/b$ has no x-intercept. Likewise, if $c \neq 0$ then $x = c/a$ has no y-intercept. However in most cases we have both x and y intercepts. In such a case with $x_o, y_o \neq 0$ we

$$\boxed{\frac{x}{x_o} + \frac{y}{y_o} = 1} \text{ — intercept-intercept form of a line}$$

You can check, $x = 0 \Rightarrow \underbrace{y = y_o}_{\text{y-intercept}}$ & $y = 0 \Rightarrow \underbrace{x = x_o}_{\text{x-intercept}}$.

The most important form of a line is

$$\boxed{y = mx + b} \text{ — line with slope } m \text{ and y-intercept } b$$

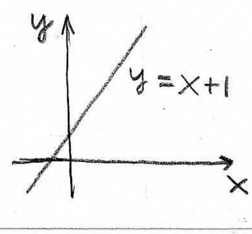
The slope can be calculated from two points on the line $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{"rise"}}{\text{"run"}}$

E97 Find line through $(1,2)$ and $(3,4)$

$m = \dfrac{4-2}{3-1} = \dfrac{2}{2} = 1$ the slope of our line

Now let's use algebra to find the "b" in $y = mx + b = x + b$.

$(1,2)$ is on the line thus $2 = 1 + b \Rightarrow b = 1 \quad \therefore \boxed{y = x + 1}$

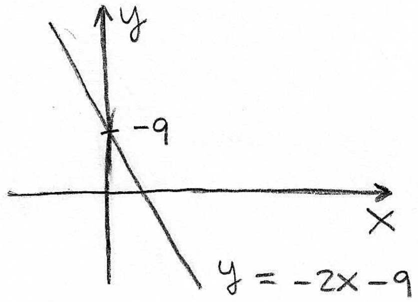


E98 Given $m = -2$ and the point $(0,-9)$ is on a line find the equation of the line and graph it.

$$y = mx + b = -2x + b$$

$$(0,-9): \quad -9 = -2(0) + b \Rightarrow b = -9$$

$$\therefore \boxed{y = -2x - 9}$$



Def^n/ Two lines $y = m_1 x + b_1$ and $y = m_2 x + b_2$ are said to be

i.) parallel if $m_1 = m_2$

ii.) perpendicular if $m_1 = \frac{-1}{m_2}$

(I'm ignoring the special cases of vertical and horizontal lines)

E99 Find the line perpendicular to $3x + 4y = 7$ that passes through $(-2/3, 7/8)$.

$$3x + 4y = 7 \Rightarrow y = \frac{7}{4} - \frac{3}{4}x$$

$$\Rightarrow y_\perp = b + \frac{4}{3}x \quad \text{(the } \perp \text{ line)}$$

The point $(-2/3, 7/8)$ is on the $\perp$ line so we have

$$\frac{7}{8} = b + \frac{4}{3}\left(\frac{-2}{3}\right) = b - \frac{8}{9}$$

$$b = \frac{7}{8} + \frac{8}{9} = \frac{63+64}{72} = \frac{127}{72}$$

$$\boxed{y_\perp = \frac{127}{72} + \frac{4}{3}x}$$