Laws of Exponents & Radicals

Assume $a, b \neq 0$ and denominators $\neq 0$

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

$$a^0 = 1$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$|a| = \sqrt{a^2}$$

$$|a|^2 = a^2$$

$\boxed{E9}$

$$\frac{x^2}{x^4} = x^{2-4} = x^{-2} = \frac{1}{x^2}$$

$$\frac{x^3 x^{-1}}{(x^2+7)^0} = \frac{x^{3-1}}{1} = x^2$$

$$(5x)^4 = 5^4 x^4 = (25)(25)x^4 = 625x^4$$

$$\frac{(6x)^2}{\left(\frac{x}{y}\right)^{-2}} = \frac{6^2 x^2}{\left(\frac{y}{x}\right)^2} = \frac{x^2}{y^2} \cdot 6^2 x^2 = \frac{36x^4}{y^2}$$

E10 Simplify the expression.

$$\frac{y}{\frac{1}{y^3}} = \frac{y}{y^{-3}} = y^{1-(-3)} = y^4.$$

In words, the denominator of the denominator is the numerator of the overall expression.

$$\frac{1}{\frac{1}{a}} = a$$

E11 Simplify the expression, write as one fraction,

$$4^{-1} + 5^{-1} = \frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \boxed{\frac{9}{20}}$$

we made a <u>common</u> <u>denominator</u>.

E12 Simplify. Write as a single fraction,

$$\left((-3)^2\right)^{-2} + 1 = \left((-3)(-3)\right)^{-2} + 1$$

$$= 9^{-2} + 1$$

$$= \frac{1}{81} + \frac{81}{81}$$

$$= \boxed{\frac{82}{81}}$$

E13 "Evaluation" simply means to substitute the value for x into the expression which is given. For example, Evaluate $x^2 + 1$ for $x = 3$ means

$$\left(x^2 + 1\right|_{x=3} = 9 + 1 = 10.$$

(the evaluation bar notation)

Def^n/ Let $a, b \in \mathbb{R}$ and $n \in \mathbb{N}$ with $n \geq 2$. If $a = b^n$ then b is the n^{th} root of a. If b has the same <u>sign</u> as a then $b = \sqrt[n]{a}$ is called the principal n^{th} root, n = index of radical

$\boxed{E14}$ $25 = b^2 \Rightarrow b = 5$ or -5, here $\sqrt{25} = 5$ is the principle root.

<u>Properties of Radicals : let $n \in \mathbb{N}$</u>

$$\sqrt[n]{a} \equiv a^{1/n} \quad , \quad \sqrt{a} \equiv \sqrt[2]{a} = a^{1/2}.$$

$$\left(\sqrt[n]{a}\right)^m = \left(a^{1/n}\right)^m = a^{m/n}$$

no mention of "n" implicitly indicates we are using the <u>square root</u>.

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\left(\sqrt[n]{a}\right)^n = \left(a^{1/n}\right)^n = a$$

<u>Remark</u>: if n is even then $\sqrt[n]{a}$ is a real number only if $a \geq 0$.

$\boxed{E15}$ $\sqrt{x^2y^2} = \sqrt{x^2}\sqrt{y^2} = xy$

$$\sqrt{25x} = \sqrt{25}\sqrt{x} = 5\sqrt{x}$$

$$x\frac{1}{\sqrt[3]{x^2}} = x\frac{1}{x^{2/3}} = x^{1-\frac{2}{3}} = x^{1/3} = \sqrt[3]{x}.$$

$$\sqrt{3x^2} + 2x = \sqrt{3}\sqrt{x^2} + 2x = \sqrt{3}\,|x| + 2x$$

E16 $x^2 = \frac{y^3}{27} \Rightarrow \sqrt[3]{x^2} = \sqrt[3]{\frac{y^3}{27}} = \frac{\sqrt[3]{y^3}}{\sqrt[3]{27}} = \frac{y}{3}$

$\Rightarrow (x^2)^{1/3} = \frac{y}{3} \Rightarrow \boxed{y = 3x^{2/3}}$

E17

$$\sqrt[3]{16x^5} = \sqrt[3]{16}\sqrt[3]{x^5} = \sqrt[3]{2^4}\sqrt[3]{x^5} = 2^{4/3}x^{5/3}$$

OR

$$\sqrt[3]{16x^5} = \sqrt[3]{8}\sqrt[3]{x^3}\sqrt[3]{2x^2} = 2x\sqrt[3]{2x^2}$$

there is certainly ambiguity in what constitutes a "simplified" answer. There are many ways to write a given expression.

E18

$$\frac{\sqrt{x(x+3)}}{x+3} + \frac{\sqrt{3(x+3)}}{x+3} = \frac{\sqrt{x}\sqrt{x+3}}{x+3} + \frac{\sqrt{3}\sqrt{x+3}}{x+3}$$

$$= \left(\sqrt{x} + \sqrt{3}\right)\left(\frac{\sqrt{x+3}}{x+3}\right)$$

$$= \boxed{\frac{\sqrt{x} + \sqrt{3}}{\sqrt{x+3}}}$$

Notice: we <u>cannot</u> simplify $\sqrt{x} + \sqrt{3}$. You might like to say $\sqrt{x} + \sqrt{3} = \sqrt{x+3}$ but this is <u>FALSE</u>. When in doubt we can evaluate and see if we're correct

$x = 1$: $\sqrt{1} + \sqrt{3} \cong 1 + 1.7 = 2.7$ <u>BUT</u> $\sqrt{1+4} = 2$.

Clearly $\sqrt{1} + \sqrt{3} \neq \sqrt{1+3}$.

(In fact, $\sqrt{x} + \sqrt{3} = \sqrt{x+3}$ only for $x = 0$)