Logarithmic Functions (§5.2)

Since $a^x = a^y \Rightarrow x = y$ we have $f(x) = a^x$ (for $a > 0$) is a one-one function. This means there is an inverse function for $f(x) = a^x$. We define,

Def^n/ Let $a > 0$ with $a \neq 1$ if $f(x) = a^x$ then we define $f^{-1}(x) \equiv \log_a(x)$. In the special cases
i.) $a = e$ we write $f(x) = e^x$ and $f^{-1}(x) = \ln(x)$
ii.) $a = 10$ we write $f(x) = 10^x$ and $f^{-1}(x) = \log(x)$

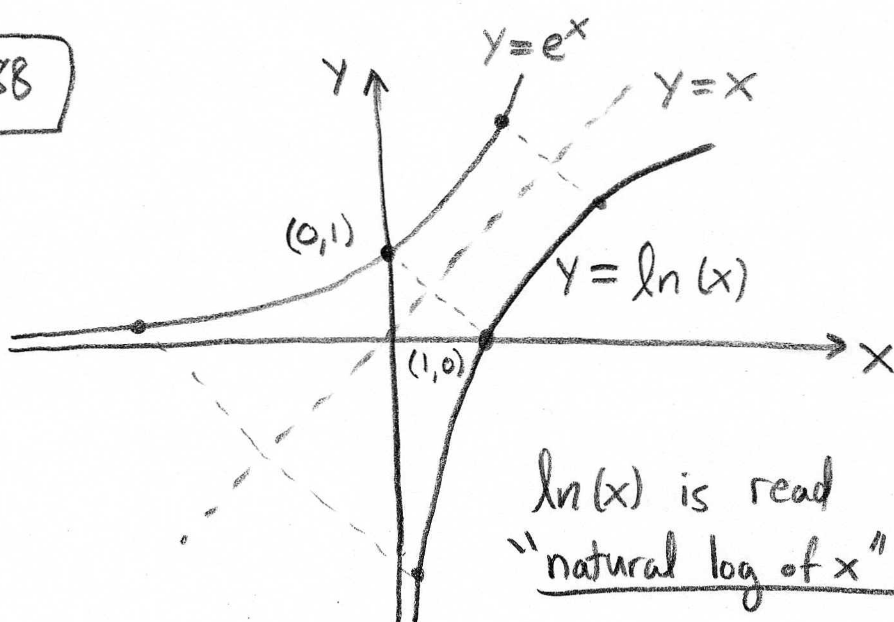
Remark: If $a > 0$ and $a \neq 1$ then $f(x) = a^x$ has $f : \mathbb{R} \longrightarrow (0, \infty)$ thus $f^{-1} : (0, \infty) \longrightarrow \mathbb{R}$ with

$$f(f^{-1}(x)) = x \quad \text{for } x \in (0, \infty)$$
$$\boxed{a^{\log_a(x)} = x \quad \text{for } x > 0}$$

$$f^{-1}(f(x)) = x \quad \text{for } x \in \mathbb{R}$$
$$\boxed{\log_a(a^x) = x \quad \text{for } x \in \mathbb{R}}$$

these conditions define the way $\log_a(x)$ really works.

E188



$\ln(x)$ is read "natural log of x"

the graph of a logarthimic function can be obtained by reflecting an exponential function across $y = x$.

$\boxed{E189}$ Let $f(x) = \log_3(x)$ calculate $f(0)$, $f(1)$, $f(3)$, $f(1/3)$ and $f(9)$ if possible.

$\underline{f(0)}$ not possible $0 \notin \text{dom}(\log_3(x))$

$\underline{f(1)}$ $y = \log_3(1)$

$$3^y = 3^{\log_3(1)} = 1 = 3^0 \Rightarrow y = 0 \Rightarrow \boxed{\log_3(1) = 0}$$

$\underline{f(3)}$ $y = \log_3(3) = \log_3(3^1) = \boxed{1}$

$\underline{f(1/3)}$ $y = \log_3\left(\frac{1}{3}\right) = \log_3(3^{-1}) = \boxed{-1}$

$\underline{f(9)}$ $y = \log_3(9) = \log_3(3^2) = \boxed{2}$

If I had asked for $f(2) = \log_3(2)$ then we'd need a calculator. Probably we'd need the change of base formula as well since $\log_3$ is not a built in function for many calculators.

BASIC LOG. PROPERTIES

1.) $\log_a(1) = 0$
2.) $\log_a(a) = 1$
3.) $\log_a(a^x) = x$ & $a^{\log_a(x)} = x$
4.) $\log_a(x) = \log_a(y) \Rightarrow x = y$

We saw 1.) and 2.) explained for $a = 3$ in $\boxed{E189}$

E190 We can use logs to simplify exponential eq$^{\underline{n}}$'s.

$$e^{x^2} = e^x \Rightarrow \ln(e^{x^2}) = \ln(e^x)$$
$$\Rightarrow x^2 = x$$
$$\Rightarrow x^2 - x = x(x-1) = 0 \quad \therefore \boxed{x = 0 \text{ or } x = 1}$$

Or, we could use exponentials to simplify log. eq$^{\underline{n}}$'s

$$\ln(3x-2) = \ln(x^2)$$
$$\Rightarrow e^{\ln(3x-2)} = e^{\ln(x^2)}$$
$$3x - 2 = x^2 \Rightarrow x^2 - 3x + 2 = 0$$
$$\Rightarrow (x-1)(x-2) = 0$$
$$\Rightarrow \boxed{x = 1 \text{ or } x = 2}$$

<u>Remark</u>: we could also just refer to the one-one properties of exp. and ln to accomplish these calculations.

E191 Suppose $f(x) = 10^x$. Graph $\log(f)$ verses x.

$\log(f(x)) = \log(10^x) = x$. The logarithm straightens out the exponential function. This sort of plot is useful for certain applications.