Polynomials

A polynomial in x of degree $n \geq 0$ $(n \in \mathbb{N})$ has the form

$$\boxed{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0} \leftarrow \underline{\text{standard form}}$$

where $a_n \neq 0$ and $a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are the coefficients.
A polynomial with real coefficients has $a_0, a_1, a_2, \ldots, a_n \in \mathbb{R}$
and we say <u>a_n is the leading coefficient</u> while
a_0 is the <u>constant term</u>.

<u>Polynomial Multiplication</u>: If we multiply a polynomial with M-summands with another polynomial with N-summands the result is a polynomial with MN-summands. However, usually there is a simplification from collecting terms of the same degree. (Read this again once we've done a bunch of examples)

$\boxed{E19}$ $14x - \frac{1}{2}x^5 = \underbrace{\frac{-1}{2}x^5 + 14x}$

standard form, $n = 5$
leading coefficient is $\frac{-1}{2} = a_5$

$\boxed{E20}$ $\sqrt{y^2 - y^4}$ is <u>not</u> a polynomial

☺^{2} + ☺ + 1 is a polynomial in ☺ with leading coefficient $a_2 = 2$. This is a <u>quadratic</u> polynomial

$\boxed{E21}$ $3x(x^2 - 2x) = 3x(x^2) - 3x(2x)$
$= 6x^3 - 6x^2$

we multiplied the given expression and wrote the answer in standard form.

<u>Write expressions in standard form after performing algebraic steps</u>

$\boxed{E22}$ $6x + 5 - (x-3) = 6x + 5 - x - (-3)$

$= \underline{5x + 8}.$

$\boxed{E23}$ $2y\left(4 - \frac{1}{4}y^3\right) = 8y - \frac{1}{2}y^4 = \underline{\frac{-1}{2}y^4 + 8y}.$

$\boxed{E24}$ $(3x-5)(2x+1) = 6x^2 + 3x - 10x - 5$

$= \underline{6x^2 - 7x - 5}.$

$\boxed{E25}$ $(x+10)(x-10) = x^2 - 10x + 10x - 100$

$= \underline{x^2 - 100}.$

<u>Remark:</u> $(x+a)(x-a) = x^2 - a^2$ is a <u>special form</u> known as the "difference of perfect squares".

$\boxed{E26}$ $(2x+3)^2 = (2x+3)(2x+3)$

$= 4x^2 + 6x + 6x + 9$

$= \underline{4x^2 + 12x + 9}.$

$\boxed{E27}$ $5x(x+1) - 3x(x+1) = (5x - 3x)(x+1)$

$= 2x(x+1)$

$= \underline{2x^2 + 2x}.$

$\boxed{E28}$ $x(x+1)(x^2+3x-7) = (x^2+x)(x^2+3x-7)$

$= x^4 + 3x^3 - 7x^2 + x^3 + 3x^2 - 7x$

$= \underline{x^4 + 4x^3 - 4x^2 - 7x}.$

E29 $(x+1)^3 = (x+1)(x+1)(x+1)$

$= (x+1)(x^2+2x+1)$

$= x^3 + 2x^2 + x + x^2 + 2x + 1$

$= \underline{x^3 + 3x^2 + 3x + 1}.$

Binomial Th^m

$$(a+b)^n = a^n + na^{n-1}b + \cdots + nab^{n-1} + b^n$$

the ... is the tricky part
but pascal's triangle will
give the necessary coefficients.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Its a neat trick
for finding the coefficients
in the binomial expansion.
(this is just for fun)

$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$(x+2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 90x + 32$

E30 $(x+4)(3+x)(x^2-2) = (x+4)(3x^2 - 6 + x^3 - 2x)$

$= 3x^3 - 6x + x^4 - 2x^2 + 12x^2 - 24 + 4x^3 - 8x$

$= \underline{x^4 + 7x^3 + 10x^2 - 14x - 24}.$