NAME(1PT) _____

MATH 121:	MISSION 1: POLYNOMIAL ALGEBRA & FRIENDS

Please print this assignment single-sided and write your solutions neatly in the provided white-space (if you write solutions on your own paper and do not print this there is a 30pt penalty). $\boxed{\textit{Box}}$ *your answers for full credit and show work. Use interval notation to express solution set for inequality questions. At least 100pts to earn here. Thanks!*

Problem 1: Solve the equations

(a.) $3(x+8) = 2x - 7$,

(b.) $3x + 13 - x = 2(x+5)$,

(c.) $\dfrac{x}{3} - \dfrac{x+4}{2} = \dfrac{1}{7}$.

Problem 2: Find the equation for the line described below:

(a.) the line parallel to $2x - y = 3$ with y-intercept 7,

(b.) the line perpendicular to $y = -3x + 1$,

(c.) the line through $(-3, 2)$ and $(6, 7)$,

(d.) the line perpendicular to $y = 4$ through the point $(3, 7)$

Problem 3: Find the point on $y = 3x + 2$ which is closest to $(4, 5)$. Be sure your solution includes a picture to help explain your logic.

Problem 4: Solve $3x + 1 < 9 - x$

Problem 5: Solve $3x + 1 < 4x + 7 \leq 9 - x$

Problem 6: Solve $2|x - 3| = 20$.

Problem 7: Solve $|3x + 17| < -1$.

Problem 8: For best results, notice the interplay of the following tasks.

(a.) graph $y = |x - 2|$,

(b.) solve $|x - 2| = 1$,

(c.) solve $|x - 2| < 1$,

(d.) solve $|x - 2| > 1$.

Problem 9: Solve $|2x + 3| > 8$.

Problem 10: Solve the system and write the solution set for:

(a.) $y = 3x - 2$ and $y = 4 - x$.

(b.) $y = 3x - 2$ and $y - 3x = 1$.

(c.) $x + y = 3$ and $x - 3 = y$.

Problem 11: Graph the lines and check that your graphs are logically consistent with your answers from the previous problem.

(a.) $y = 3x - 2$ and $y = 4 - x$.

(b.) $y = 3x - 2$ and $y - 3x = 1$.

(c.) $x + y = 3$ and $x - 3 = y$.

Problem 12: Solve the system of equations $x + y = 6 - z$, $y - z + 1 = 0$ and $3z = 9$.

Problem 13: Solve the following system of equations.

$$\begin{aligned} 3x + 4y + 5z &= 22 \\ x + 2y - z &= 6 \\ x - 2y + z &= 0 \end{aligned}$$

Problem 14: Solve the following system of equations.

$$\begin{aligned} x + 2y + z &= 4 \\ 3x + y - z &= 0 \\ 2x + 3y + z &= 1 \end{aligned}$$

Problem 15: Complete the square and factor over $\mathbb{R}$ if possible, if not possible then write **prime**

(a.) $x^2 - 10x + 20$

(b.) $x^2 + 3x - 1$

(c.) $x^2 - 16x + 64$

Problem 16: Complete the square and factor over $\mathbb{C}$

(a.) $x^2 + 4x - 7$

(b.) $2x^2 + 3x + 4$

(c.) $3x^2 + 6x + 3$

Problem 17: Factor the quadratic polynomials given below either by direct factoring or completing the square. If the polynomial does not factor over $\mathbb{R}$ then write **prime**:

(a.) $-x^2 + 12x - 11$

(b.) $x^2 - 6x + 8$

(c.) $6x^2 - x - 40$

(d.) $x^2 - 11$

(e.) $6x^2 - 7$

(f.) $6x^2 - 19x - 7$

(g.) $x^2 + 6x + 13$

(h.) $x^2 - 26x + 169$

(i.) $\frac{1}{2}x^2 + x$

(j.) $(2x - 3)^2 - (x + 1)^2$

Problem 18: Solve the quadratic equations over $\mathbb{C}$:

(a.) $(2x+3)^2 = 11$

(b.) $(x+2)^2 + 3 = 0$

(c.) $11x^2 + 3x - 2 = 0$

Problem 19: Graph each parabola. Find the vertex, x-intercepts and sketch the graph:

(a.) $y = -2(x-3)^2 + 5$

(b.) $y = x^2 + 6x - 7$

(c.) $y = 2x^2 - 4x + 2$

Problem 20: A cat is fired from a circus cannon and is observed to follow a trajectory given by

$$y = 2 + 10t - 5t^2$$

where t is in seconds and y is in meters. Find the time and height which the cat reaches the top of the flight. Also, if $y = 0$ denotes ground level (where there is lava) then at what time does the cat hit the lava ?

Problem 21: If Santa's cost of manufacturing fully-automatic BB-guns is given by

$$C(x) = 30(x - 4)^2 + 100$$

where x is the hourly wage of elves in bit-coin and $C(x)$ is in American Dollars, then what is the minimum cost to assemble the BB-guns ?

Problem 22: Use the binomial theorem to expand the polynomials below into standard form:

(a.) $(x - 3)^5$

(b.) $(2x + 1)^4$

Problem 23: Multiply the polynomials to check each assertion. Circle True or False as appropriate:

(a.) TRUE FALSE $x^2 + 4x + 5 = (x + 1)(x + 5)$

(b.) TRUE FALSE $x^3 + 3x^2 + 3x + 1 = (x + 1)^3$

(c.) TRUE FALSE $x^2(x - 6)(x + 6) = x^4 - 36x^2$

(d.) TRUE FALSE $(a + b + c)(a^2 + b^2) = a^3 + a^2b + a^2c + b^3 + b^2c$

(e.) TRUE FALSE $x^4 + 4x^3 - 6x^2 - 36x - 27 = (x + 3)^2(x^2 - 2x - 3)$

Problem 24: Find real values for B and C for which $x^2 + Bx + C = 0$ has the solution

(a.) $x = 3$ and $x = -2$

(b.) $x = 2 - 3i$

(c.) $x = 2 - \sqrt{3}$ and $x = 2 + \sqrt{3}$

Problem 25: Factor the following polynomial $f(x)$ completely over $\mathbb{R}$ given that $f(c) = 0$ where:

(a.) $c = -3$ and $f(x) = x^3 - 12x^2 - 79x - 102$

(b.) $c = -2 + i\sqrt{3}$ and $f(x) = x^3 + 5x^2 + 11x + 7$

(c.) $c = 1/2$, $c = 1/3$, $c = 1/5$ and $f(x) = 30x^3 - 31x^2 + 10x - 1$

Problem 26: Solve the following polynomial equations over $\mathbb{R}$:

(a.) $(x^2 + 1)(x^2 - 9)(x + 5) = 0$

(b.) $x^4 + 3x^2 = -2$

(c.) $(x+2)(x^2 - 10x + 1) = 0$

(d.) $x(x^2 + 3x - 4) = 8(4 - 3x - x^2)$

(e.) $16x^4 = 81$

(f.) $x^3 = 8$

Problem 27: Factor the following polynomials completely over $\mathbb{R}$

(a.) $27x^3 + 125$

(b.) $(x^2 - 13x + 42)^2$

(c.) $x^7 - 100x^3$

(d.) $x^4 + 2x^2 + 1$

(e.) $x^5 - 4x^3 - 5x$

(f.) $(x^2 + 4x + 5)(x^2 + 4x - 5)$

(g.) $(x^2 + 6x + 1)(x^2 - 25)$

Problem 28: Solve the following inequalities:

(a.) $x^2 < 2x + 3$

(b.) $(x+4)(x^2-9) \geq 0$

(c.) $x^5 - 81x > 0$

(d.) $(x-1)(x-2)(x-3)(x-4) < 0$

Problem 29: Sketch the graph of each equation below and decide whether the curve is a circle, ellipse or a hyperbola:

(a.) $(x+2)^2 + (y-3)^2 = 16$

(b.) $9x^2 + 4y^2 = 36$

(c.) $(x-3)^2 - (y-1)^2 = 1$

Problem 30: Solve $x^2 + y^2 = 7$ and $x^2 - y^2 = 1$. Does your answer make sense graphically ?

Problem 31: Complete the square for both x and y and decide whether the given equation describes a circle, ellipse, hyperbola, point or nothing:

(a.) $x^2 + 2x - y^2 + 4y = 0$

(b.) $x^2 - 2x + y^2 - 4y + 10 = 0$

(c.) $x^2 - 3x + y^2 + 8y = 0$

(d.) $2x^2 + 4x + 9y^2 - 18y = 0$

Problem 32: Solve $x^2 + y^2 = 4$ and $x + y = 2$. Does your answer make sense graphically ?