Please print this assignment single-sided and write your solutions neatly in the provided white-space (if you write solutions on your own paper and do not print this there is a 30pt penalty). Box your answers for full credit and show work. Use interval notation to express solution set for inequality questions. At least 100pts to earn here. Thanks!

Problem 33: Add, subtract, multiply or divide and simplify the expression into lowest terms:
(a.) $\frac{x}{3}+\frac{x-1}{2}+3$
(b.) $\frac{3}{x}-\frac{x+1}{x^{2}}$
(c.) $\frac{x}{x+3}-\frac{x+1}{x+2}$
(d.) $\frac{x^{2}-1}{x+3} \cdot \frac{x^{2}-9}{x^{2}+2 x+1}$
(e.) $\frac{1+\frac{1}{x+1}}{2+\frac{1}{x+1}}$
(f.) $\frac{x}{x+3}-\frac{3 x}{x^{2}+6 x+9}$
(g.) $\left(x^{2}+3 x+2\right)\left(\frac{1}{x+1}-\frac{1}{x+2}\right)$

Problem 34: Solve $\frac{x}{x-3}=4$.

Problem 35: Solve $\frac{x+2}{x+3}=\frac{4}{x}$.

Problem 36: Solve $\frac{4}{x}-\frac{3}{x^{2}}=1$.

Problem 37: Solve $\frac{x-11}{x+3} \leq 2$.

Problem 38: Solve $\frac{(x+3)(x-4)}{(x-2)(x-6)} \geq 0$.

Problem 39: Solve $\frac{(x+3)^{2}(x+1)}{\left(x^{2}-9\right) x^{3}} \leq 0$.

Problem 40: Solve $\frac{x^{2}+4 x+5}{x^{2}-13 x+42} \geq 0$.

Problem 41: Solve $\frac{x}{x+3} \geq \frac{2}{x+2}$.

Problem 42: Given $x, y>0$, find values of $C, A$ and $B$ for which:
(a.) $C x^{A} y^{B}=\frac{\sqrt{25 x y^{3}}}{\sqrt[3]{8 x^{2} y^{5}}}$
(b.) $C x^{A} y^{B}=\frac{39 x \sqrt{y}}{13 x^{2} \sqrt[3]{y}}$
(c.) $C x^{A} y^{B}=\frac{1}{x} \cdot\left(x^{2} \sqrt{y}\right) \cdot \frac{y^{3 / 2}}{x}$

Problem 43: Solve $x+\sqrt{x-3}=5$

Problem 44: Solve $\sqrt{x+1}+\sqrt{x-4}=5$

Problem 45: Solve $\frac{\sqrt{x+2}}{\sqrt{4 x+1}}=\frac{2}{3}$

Problem 46: Solve $x=\sqrt[3]{3 x^{2}-2 x}$

Problem 47: Solve $\frac{x}{\sqrt[3]{x}}=\pi$ (leave answer in terms of $\pi$ )

Problem 48: Solve $\frac{1}{x}-\frac{3}{\sqrt{x}}+2=0$

Problem 49: For each formula, sketch the graph of $y=f(x)$ and find the domain and range:
(a.) $f(x)=\sqrt{3-x}$
(b.) $f(x)=\frac{1}{x-2}$
(c.) $f(x)=1+\frac{1}{x^{2}}$
(d.) $f(x)=3+2(x-2)^{2}$
(e.) $f(x)=\sqrt[3]{x-1}$
(f.) $f(x)=2-|x-3|$

Problem 50: Let $f(x)=\left\{\begin{array}{ll}x^{2}+3 & \text { if } x<1 \\ x^{3} & \text { if } x \geq 1\end{array}\right.$. Graph $y=f(x)$ and evaluate $f(2)$ and $f(0)$. What is the range of $f$ ?

Problem 51: Rewrite the formula for $f(x)=\frac{x-1}{\sqrt{(x-1)^{2}}}$ as a casewise-defined function. Also, find the domain and range of the function.

Problem 52: Determine if the equations below define $y$ as a single function of $x$ by solving the equations below for $y$ :
(a.) $x^{2}+(y-1)^{2}=4$
(b.) $y^{3}+2 x^{2}-1=0$
(c.) $\frac{3 x-y}{2 y-x}=1$

Problem 53: Equations are generally much more exotic than we can reasonably test in-class in Math 121. Let's use computer graphing system to explore a bit. For each formula below, sketch the graph you see in Desmos and determine if the equation defines a function in view of the vertical line test:
(a.) $2 x^{2}+x y+2 y^{2}=7$
(b.) $\left(x^{2}+y^{2}-9\right)\left(x^{8}-y^{4}\right)=0$

Problem 54: Determine the domain of the functions given below:
(a.) $f(x)=\frac{1}{\sqrt{x^{2}+6 x+20}}$
(b.) $f(x)=\sqrt{9-x^{2}}$

Problem 55: Suppose $f$ and $g$ are functions for which $f(1)=13$ and $g(1)=3$ whereas $f(-2)=7$ and $g(13)=8$. Given this data, calculate the following:
(a.) $(g f)(1)$
(b.) $(f+g)(1)$
(c.) $(3 f)(-2)$
(d.) $(g \circ f)(1)$

Problem 56: Suppose $f(x)=3 x+1$ and $g(x)=1+\frac{1}{x^{2}}$. Find the (unsimplified) formulas for:
(a.) $(f g)(x)$
(b.) $(f-g)(x)$
(c.) $(f \circ g)(x)$
(d.) $(g \circ f)(x)$

Problem 57: Solve $K=\frac{1}{2} m v^{2}$ for $v$.

Problem 58: Suppose $U=\frac{-G m_{1} m_{2}}{R+h}$. Solve for $h$.

Problem 59: Let $f(x)=5 x^{5}+4 x^{4}+3 x^{3}+2 x^{2}+1$ and $g(x)=x^{3}+2 x-3$. Use polynomial long division to find $p(x)$ and $r(x)$ for which $\frac{f(x)}{g(x)}=p(x)+\frac{r(x)}{g(x)}$ where $\operatorname{deg}(r(x))<\operatorname{deg}(g(x))$ :

Problem 60: Let $f(x)=x^{4}-10 x^{3}+35 x^{2}-50 x+24$. Factor $f(x)$ completely over $\mathbb{R}$. Thou shall begin by dividing $f(x)$ by $x^{2}-3 x+2$ via polynomial long division.

Problem 61: Let $f(x)=x^{5}-4 x^{4}+5 x^{3}-8 x^{2}+32 x-40$. Factor $f(x)$ completely over $\mathbb{R}$. Hint: $f(2+i)=0$ so you know a particular quadratic polynomial factors $f(x)$.

Problem 62: Let $f(x)=x^{4}-6 x^{3}+14 x^{2}-21 x+35$.
(a) Use long-division to divide $f(x)$ by $g(x)=x^{2}-6 x+10$, is $g(x)$ a factor of $f(x)$ ?
(b) Calculate the value of $f(3+i)$

Problem 63: Find the formula for the real polynomial $f(x)$ of least degree which has $f(2 i)=0$, $f(-3)=0$ and $f(1)=40$.

Problem 64: Find the formula for the real polynomial $f(x)$ of least degree which has a factor $(x+2)$ with multiplicity 3 and $f(-1)=0$ given that $f(3)=200$.

Problem 65: Given $f(0)=32$, find the polynomial $f(x)$ of least degree with the graph below:


Problem 66: Consider $f(x)=x^{4}+x^{3}-13 x^{2}-x+12$. Evaluate $f(1)$ and $f(-1)$ and factor $f(x)$ completely over $\mathbb{R}$.

Problem 67: Sketch the graph of $y=(2 x-20)(2 x-1)(x+4)^{2}(x-5)^{2}$.

Problem 68: Consider $f(x)=\frac{x-2}{x+3}$. Sketch the graph $y=f(x)$ and label any $x$-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

Problem 69: Consider $f(x)=\frac{x}{x^{2}-9}$. Sketch the graph $y=f(x)$ and label any $x$-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

Problem 70: Consider $f(x)=\frac{x+4}{x^{2}-16}$. Sketch the graph $y=f(x)$ and label any $x$-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

Problem 71: Consider $f(x)=\frac{x^{2}-6 x+9}{2 x^{2}-3 x-9}$. Sketch the graph $y=f(x)$ and label any $x$-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

Problem 72: Consider $f(x)=\frac{12}{x+4}-\frac{3}{x-3}-2$. Sketch the graph $y=f(x)$ and label any $x$-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

