NAME(1PT)

MATH 121 MISSION 2: RATIONAL ALGEBRA & FUNCTIONS

Please print this assignment single-sided and write your solutions neatly in the provided white-space (if you write solutions on your own paper and do not print this there is a 30pt penalty). Box your answers for full credit and show work. Use interval notation to express solution set for inequality questions. At least 100pts to earn here. Thanks!

Problem 33: Add, subtract, multiply or divide and simplify the expression into lowest terms:

(a.) 
$$\frac{x}{3} + \frac{x-1}{2} + 3$$

(b.) 
$$\frac{3}{x} - \frac{x+1}{x^2}$$

(c.) 
$$\frac{x}{x+3} - \frac{x+1}{x+2}$$

(d.) 
$$\frac{x^2-1}{x+3} \cdot \frac{x^2-9}{x^2+2x+1}$$

(e.) 
$$\frac{1+\frac{1}{x+1}}{2+\frac{1}{x+1}}$$

(f.) 
$$\frac{x}{x+3} - \frac{3x}{x^2+6x+9}$$

(g.) 
$$(x^2 + 3x + 2)\left(\frac{1}{x+1} - \frac{1}{x+2}\right)$$

**Problem 34:** Solve  $\frac{x}{x-3} = 4$ .

**Problem 35:** Solve  $\frac{x+2}{x+3} = \frac{4}{x}$ .

**Problem 36:** Solve  $\frac{4}{x} - \frac{3}{x^2} = 1$ .

**Problem 37:** Solve  $\frac{x-11}{x+3} \le 2$ .

**Problem 38:** Solve 
$$\frac{(x+3)(x-4)}{(x-2)(x-6)} \ge 0.$$

**Problem 39:** Solve 
$$\frac{(x+3)^2(x+1)}{(x^2-9)x^3} \le 0.$$

**Problem 40:** Solve 
$$\frac{x^2 + 4x + 5}{x^2 - 13x + 42} \ge 0.$$

**Problem 41:** Solve  $\frac{x}{x+3} \ge \frac{2}{x+2}$ .

**Problem 42:** Given x, y > 0, find values of C, A and B for which:

(a.) 
$$Cx^A y^B = \frac{\sqrt{25xy^3}}{\sqrt[3]{8x^2y^5}}$$

(**b.**) 
$$Cx^A y^B = \frac{39x\sqrt{y}}{13x^2\sqrt[3]{y}}$$

(c.) 
$$Cx^{A}y^{B} = \frac{1}{x} \cdot (x^{2}\sqrt{y}) \cdot \frac{y^{3/2}}{x}$$

**Problem 43:** Solve  $x + \sqrt{x-3} = 5$ 

**Problem 44:** Solve  $\sqrt{x+1} + \sqrt{x-4} = 5$ 

**Problem 45:** Solve 
$$\frac{\sqrt{x+2}}{\sqrt{4x+1}} = \frac{2}{3}$$

**Problem 46:** Solve  $x = \sqrt[3]{3x^2 - 2x}$ 

**Problem 47:** Solve  $\frac{x}{\sqrt[3]{x}} = \pi$  (leave answer in terms of  $\pi$ )

**Problem 48:** Solve  $\frac{1}{x} - \frac{3}{\sqrt{x}} + 2 = 0$ 

**Problem 49:** For each formula, sketch the graph of y = f(x) and find the domain and range:

(a.) 
$$f(x) = \sqrt{3-x}$$

**(b.)** 
$$f(x) = \frac{1}{x-2}$$

(c.) 
$$f(x) = 1 + \frac{1}{x^2}$$

(d.) 
$$f(x) = 3 + 2(x-2)^2$$

(e.) 
$$f(x) = \sqrt[3]{x-1}$$

(f.) 
$$f(x) = 2 - |x - 3|$$

**Problem 50:** Let  $f(x) = \begin{cases} x^2 + 3 & \text{if } x < 1 \\ x^3 & \text{if } x \ge 1 \end{cases}$ . Graph y = f(x) and evaluate f(2) and f(0). What is the range of f?

**Problem 51:** Rewrite the formula for  $f(x) = \frac{x-1}{\sqrt{(x-1)^2}}$  as a casewise-defined function. Also, find the domain and range of the function.

**Problem 52:** Determine if the equations below define y as a single function of x by solving the equations below for y:

(a.)  $x^2 + (y-1)^2 = 4$ 

**(b.)** 
$$y^3 + 2x^2 - 1 = 0$$

(c.) 
$$\frac{3x-y}{2y-x} = 1$$

Problem 53: Equations are generally much more exotic than we can reasonably test in-class in Math 121. Let's use computer graphing system to explore a bit. For each formula below, sketch the graph you see in Desmos and determine if the equation defines a function in view of the vertical line test:

(a.) 
$$2x^2 + xy + 2y^2 = 7$$

**(b.)** 
$$(x^2 + y^2 - 9)(x^8 - y^4) = 0$$

Problem 54: Determine the domain of the functions given below:

(a.) 
$$f(x) = \frac{1}{\sqrt{x^2 + 6x + 20}}$$

**(b.)** 
$$f(x) = \sqrt{9 - x^2}$$

**Problem 55:** Suppose f and g are functions for which f(1) = 13 and g(1) = 3 whereas f(-2) = 7 and g(13) = 8. Given this data, calculate the following:

(a.) (gf)(1)
(b.) (f + g)(1)
(c.) (3f)(-2)
(d.) (g ∘ f)(1)

**Problem 56:** Suppose f(x) = 3x + 1 and  $g(x) = 1 + \frac{1}{x^2}$ . Find the (unsimplified) formulas for:

**(a.)** (fg)(x)

**(b.)** 
$$(f-g)(x)$$

(c.)  $(f \circ g)(x)$ 

(d.)  $(g \circ f)(x)$ 

**Problem 57:** Solve  $K = \frac{1}{2}mv^2$  for v.

**Problem 58:** Suppose  $U = \frac{-Gm_1m_2}{R+h}$ . Solve for h.

**Problem 59:** Let  $f(x) = 5x^5 + 4x^4 + 3x^3 + 2x^2 + 1$  and  $g(x) = x^3 + 2x - 3$ . Use polynomial long division to find p(x) and r(x) for which  $\frac{f(x)}{g(x)} = p(x) + \frac{r(x)}{g(x)}$  where  $\deg(r(x)) < \deg(g(x))$ :

**Problem 60:** Let  $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$ . Factor f(x) completely over  $\mathbb{R}$ . Thou shall begin by dividing f(x) by  $x^2 - 3x + 2$  via polynomial long division.

**Problem 61:** Let  $f(x) = x^5 - 4x^4 + 5x^3 - 8x^2 + 32x - 40$ . Factor f(x) completely over  $\mathbb{R}$ . Hint: f(2+i) = 0 so you know a particular quadratic polynomial factors f(x).

**Problem 62:** Let  $f(x) = x^4 - 6x^3 + 14x^2 - 21x + 35$ .

- (a) Use long-division to divide f(x) by  $g(x) = x^2 6x + 10$ , is g(x) a factor of f(x)?
- (b) Calculate the value of f(3+i)

**Problem 63:** Find the formula for the real polynomial f(x) of least degree which has f(2i) = 0, f(-3) = 0 and f(1) = 40.

**Problem 64:** Find the formula for the real polynomial f(x) of least degree which has a factor (x + 2) with multiplicity 3 and f(-1) = 0 given that f(3) = 200.

**Problem 65:** Given f(0) = 32, find the polynomial f(x) of least degree with the graph below:



**Problem 66:** Consider  $f(x) = x^4 + x^3 - 13x^2 - x + 12$ . Evaluate f(1) and f(-1) and factor f(x) completely over  $\mathbb{R}$ .

**Problem 67:** Sketch the graph of  $y = (2x - 20)(2x - 1)(x + 4)^2(x - 5)^2$ .

**Problem 68:** Consider  $f(x) = \frac{x-2}{x+3}$ . Sketch the graph y = f(x) and label any x-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

**Problem 69:** Consider  $f(x) = \frac{x}{x^2 - 9}$ . Sketch the graph y = f(x) and label any x-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

**Problem 70:** Consider  $f(x) = \frac{x+4}{x^2-16}$ . Sketch the graph y = f(x) and label any x-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

**Problem 71:** Consider  $f(x) = \frac{x^2 - 6x + 9}{2x^2 - 3x - 9}$ . Sketch the graph y = f(x) and label any *x*-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.

**Problem 72:** Consider  $f(x) = \frac{12}{x+4} - \frac{3}{x-3} - 2$ . Sketch the graph y = f(x) and label any x-intercepts, vertical asymptotes, horizontal asymptotes, and any holes in the graph.