NAME ____

MATH 121:	MISSION 3: INVERSE FUNCTIONS, EXPONENTIALS AND LOGS

Please print this assignment single-sided and write your solutions neatly in the provided white-space (if you write solutions on your own paper and do not print this there is a 30pt penalty). $\boxed{\textit{Box}}$ *your answers for full credit and show work. Use interval notation to express solution set for inequality questions. At least 100pts to earn here. Note, the last third of this is literally taken from an old Final Exam.*

Problem 73: Let $f(x) = x^2 + 4x + 11$. Is this function one-to-one ? Use either an algebraic or a graphical argument to decide the question.

Problem 74: Let $f(x) = 3x + 1$. Is this function one-to-one ? Use either an algebraic or a graphical argument to decide the question.

Problem 75: Let $f(x) = \sqrt{x-4}$. Is this function one-to-one ? Use either an algebraic or a graphical argument to decide the question.

Problem 76: Let $f(x) = \sqrt{x-4}$. Calculate $f^{-1}(x)$ and graph both $y = f(x)$ and $y = f^{-1}(x)$.

Problem 77: Given the function $f(x) = \dfrac{2x+7}{3-x}$ calculate the formula for $f^{-1}(y)$. Find the range of f.

Problem 78: Given the function $f(x) = 3 + \dfrac{7}{x+2}$ calculate the formula for $f^{-1}(y)$.

Problem 79: Solve $2^{3x+1} = 1$.

Problem 80: Solve $4^x = 2^{-3x+1}$

Problem 81: Solve $10^{x^2-7} = 100$

Problem 82: Solve $\sqrt{e^x} = e^{1-x}$

Problem 83: Simplify the following (you should not need a calculator for these)

(a.) $2^{\log_2(3)}$

(b.) $3^{2\log_3(5)}$

(c.) $\log_2(8)$

(d.) $\log_3\left(\frac{1}{27}\right)$

(e.) $\log(5000) - \log(5)$

(f.) $\log_{15}(25) + \log_{15}(9)$

(g.) $\ln(e^3\sqrt{e})$

Problem 84: Collapse the following expression into an expression using a single logarithm:

$$\ln(A) - \ln(B) + 2\ln(C) =$$

Problem 85: Collapse the following expression into an expression using a single logarithm:

$$2\ln(x) + \frac{1}{2}\ln(y+1) - \ln(z+1) =$$

Problem 86: Collapse the following expression into an expression using a single logarithm:

$$3\log(x) + \frac{1}{3}\log(y+1) - 3 =$$

Problem 87: Expand the expression and rewrite it in terms of $A = \log x$, $B = \log y$ and $C = \log z$:

$$\log(100xy^2z^3) =$$

Problem 88: Expand the expression and simplify if possible

$$\ln\left(\frac{x^2e^{-x}}{\sqrt{x+3}}\right) =$$

Problem 89: Solve $2\log_3(A-B) = \log_3 C$ for A.

Problem 90: Solve $\log_2(x-10)=5$.

Problem 91: Solve $\ln(3x+2)=0$

Problem 92: Solve $\log(x+2)=2$

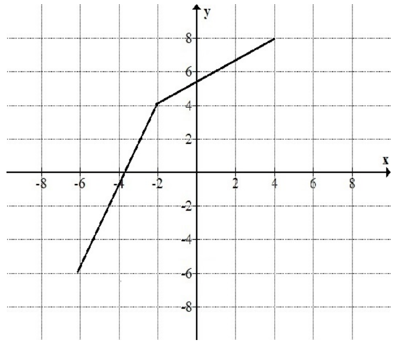
Problem 93: Let $f(x)=\log(x-2)$. Graph $y=f(x)$ and find the domain and range of f:

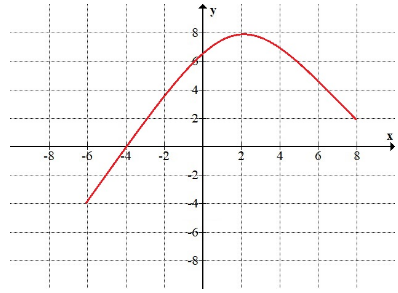
Problem 94: Let $f(x) = 2^{-x} - 3$. Graph $y = f(x)$ and find the domain and range of f:

Problem 95: Given the function $f(x) = 3 + 2\log_3(x - 1)$ calculate the formula for $f^{-1}(y)$.

Problem 96: Given the function $f(x) = 3 + 2^{3x-7}$ calculate the formula for $f^{-1}(y)$.

Problem 97: If possible, graph the inverse function for each function graph below. If the function does not have an inverse explain why.





Problem 98: Solve $\log_x(3) = 2$

Problem 99: Solve $2^x = 5$ and write the approximate answer to two decimal places.

Problem 100: (2pts) Solve $3^x = 2^{x-1}$.

Problem 101: Newton's Law of Cooling describes how the temperature of an object changes as it either gains or loses energy to assume the same temperature as the ambient surrounding. A hot cup of coffee loses energy as it cools to room temperature. A cold can of soda gains energy as it warms to room temperature. The formula for the temperature of an object Y at time t is given by

$$Y(t) = R + Ae^{-kt}$$

where R, A, k are constants and $k > 0$ and R is the ambient or room temperature. Suppose we have a cup of coffee initially heated to $150^o\ F$ then after 10 minutes the coffee cools to $120^o\ F$. If the room temperature is $70^o\ F$ then find the time at which the coffee cools to $80^o\ F$.

Problem 102: Suppose $A(t) = 5 + 2e^{-0.3t}$. Find the time t for which $A(t) = 6$. Please express your answer as a decimal number and round the answer to nearest thousandth.

Problem 103: Find a constant k for which $2^x = 3^{kx}$ for every x:

Problem 104: Let $P(t) = 10e^{-2t}$ where t is in seconds. If $P(t) = 10\left(\frac{1}{2}\right)^{t/\tau}$ then what is the value of τ ? Also, calculate $P(\tau)$ and $P(2\tau)$ and comment on the meaning of τ.

Problem 105: Find the domain of $f(x) = \ln(16 - x^2)$

Problem 106: Find the domain of $f(x) = \ln(x-2) + \ln(x-10)$

Problem 107: Solve $4^x + 2^x - 12 = 0$

Problem 108: Solve $e^{-x} + e^x = 3$

Problem 109: Solve $\ln(x-1)+\ln(x+1)=\ln(3)$

Problem 110: Solve $\log(x+4)+\log(x+2)=1$

Problem 111: Solve $\ln(x^2 - 6x + 8) = 0$

Problem 112: Match by filling in the blanks with the appropriate choice of A, B, C, or N (where N indicates the expression is not a simplification of A., B. or C.). Note: the choices may be repeated.

(A.) $\log x + \log y^2 - \log z$

(B.) $3 + \log x$

(C.) $\ln \sqrt{\dfrac{e^x y^2}{z^4}}$

(i.) ____________ simplifies to $\log(x + y^2 - z)$

(ii.) ____________ simplifies to $x/2 + \ln y - 2\ln z$

(iii.) ____________ simplifies to $\log(1000x)$.

(iv.) ____________ simplifies to $\log\left(\frac{xy^2}{z}\right)$

(v.) ____________ simplifies to $\dfrac{\ln(1000x)}{\ln(10)}$

Problem 113: Factor the polynomials below completely over $\mathbb{R}$. If the polynomial is prime then write "prime" as your answer.:

(a.) $x^2 - 7$

(b.) $x^2 - 6x + 8$

(c.) $x^2 + 6x + 1$

(d.) $6x^2 + 19x - 7$

(e.) $x^2 + 6x + 10$

(f.) $x^4 + 14x^3 + 49x^2$

Problem 114: Solve the following inequalities. Give answer in interval notation.

(a.) $3x + 1 \leq 7 - 2x$

(b.) $|x - 2| + 1 < 10$

(c.) $\dfrac{x+4}{(2x-3)(x-8)} \geq 0$

Problem 115: Find the solutions (real or complex) of the following equations

(a.) $x^3 - 4x^2 + 3x = 0$

(b.) $(2x - 3)^2 = -11$

Problem 116: Find the solution set for each of the following:

(a.) $\dfrac{6x}{x-6} = 9 + \dfrac{x^2}{x-6}$

(b.) $x = 3 + \sqrt{2x - 3}$

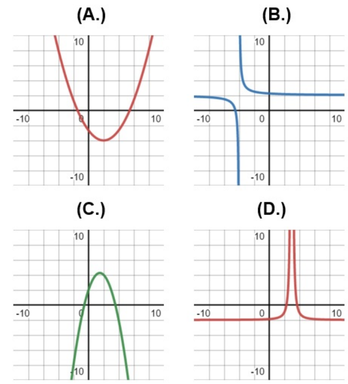
Problem 117: Match by filling in the blanks with the appropriate choice of A, B, C, or D.

(i.) ____________ is the graph $y = 2 + \dfrac{1}{x+4}$

(ii.) ____________ is the graph $y = \dfrac{1}{3}(x-2)^2 - 4$

(iii.) ____________ is the graph $y = \dfrac{1}{(x-3)^2} - 2.$

(iv.) ____________ is the graph $y = -x^2 + 3x + 2$



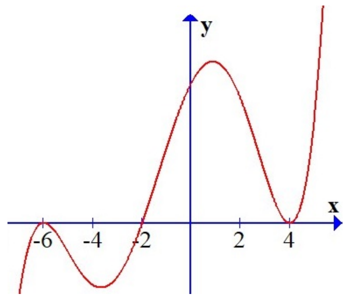
Problem 118: Find the coordinates of the vertex of the parabola $y = x^2 - 4x + 5$.

Problem 119: Fill-in-the blanks below with for $y = \dfrac{x^2 - 4x}{x^2 - 16}$. If there is a hole in the graph, please find its coordinates.

x-intercept is at: ____________ vertical asymptote is at: ____________ hole in graph at: ____________

Problem 120: Find the center and the radius of the circle with equation $x^2 + 18x + y^2 - 4y = 15$.

Problem 121: Given $f(0) = 144$, find the polynomial $f(x)$ of least degree with the graph below.



Problem 122: Solve $\log_3 3^x - 11 \log_3 9 = \log_3 3^{20}$.

Problem 123: Does $f(x) = 3x^4 + 14x^3 + 24x^2 + 18x + 10$ have a factor $g(x) = x^2 + 4x + 5$? Calculate $\frac{f(x)}{g(x)}$ via long-division of polynomials and answer the question.

Problem 124: Find the formula for the polynomial $f(x) = ax^2 + bx + c$ whose graph contains the points $(0, 4), (2, 10)$ and $(-2, 14)$

Problem 125: Solve the following system of equations.

$$\begin{aligned} 3x - 4y + 5z &= 42 \\ x + 3y - 2z &= 17 \\ x - 3y + 2z &= 1 \end{aligned}$$