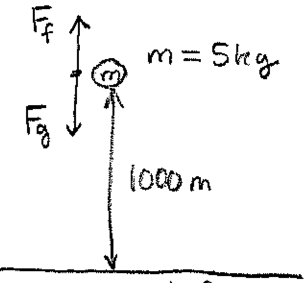
NEWTONIAN MECHANICS : §3.4

Classical Mechanics says that the motion of particles is governed by Newton's 2^{nd} law which states

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}}$$

for the momentum $\vec{P} = m\vec{V}$ and net-external force $\vec{F}$ with respect to a particle with mass m at position $\vec{r}$ and velocity $\vec{V} = \frac{d\vec{r}}{dt}$. If $\frac{dm}{dt} = 0$ then $\frac{d\vec{P}}{dt} = m\vec{a}$ where $\vec{a} = \frac{d\vec{V}}{dt}$. All of these vector quantities are measured with respect to a particular Cartesian coordinate system. We can also consider frames which are in motion. In §3.4 we'll consider essentially one-dimensional problems. In general $\frac{d\vec{P}}{dt} = \vec{F}$ is a <u>system</u> of three ODEs.

§3.4 #1



$y=0$ is ground for me the book may differ.

$$F_f = -\left(50\ \frac{Ns}{m}\right)\frac{dy}{dt}, \quad b = 50\ \frac{Ns}{m}, \quad V = \frac{dy}{dt}$$

$$F = F_f + F_g = -bV - mg$$

$$\frac{dm}{dt} = 0 \text{ thus } m\frac{dV}{dt} = -bv - mg$$

$$\int \frac{m\,dV}{bv + mg} = \int -\,dt$$

$$\frac{m}{b}\ln(bv + mg) = -t + c_1$$

$$bv + mg = \frac{b}{m} e^{-\frac{bt}{m}} e^{c_2}$$

$$\underline{V(t) = ke^{-\frac{bt}{m}} - mg/b}$$: velocity as function of time. However we know $V(0) = 0$ thus $k = mg/b$

Hence, $V(t) = \frac{mg}{b}\left(e^{-\frac{bt}{m}} - 1\right)$

Note $v = \frac{dy}{dt} = \frac{mg}{b}\left(e^{-\frac{bt}{m}} - 1\right)$

Continued

$$\frac{dy}{dt} = \frac{mg}{b}\left(e^{-\frac{bt}{m}} - 1\right)$$

$$y(t) = \frac{mg}{b}\left[-\frac{m}{b}e^{-\frac{bt}{m}} - t\right] + C_1 \quad \therefore \quad \frac{m}{b} = \frac{5}{50} = \frac{1}{10}$$

$$y(t) = -\frac{g}{100}e^{-10t} - \frac{gt}{10} + C_1$$

$$y(t) = -0.0981e^{-10t} - 0.981t + C_1$$

The text puts $y = 0$ mid-air which makes $C_1 = 0.0981$. I put $y = 0$ on the ground thus $y(0) = 1000$ then

$$1000 = -0.0981 + C_1 \quad \therefore \quad \underline{C_1 = 1000.0981}$$

$$\boxed{y(t) = -0.0981e^{-10t} - 0.981t + 1000.0981}$$

We can find the time it hits the ground by solving $0 = y(t)$,

$$0 = -0.0981e^{-10t} - 0.981t + 1000.0981$$

Notice $t > 0$ for the physically interesting sol$^{\underline{n}}$ and $e^{-10t} \approx 0$ relative to the other terms hence,

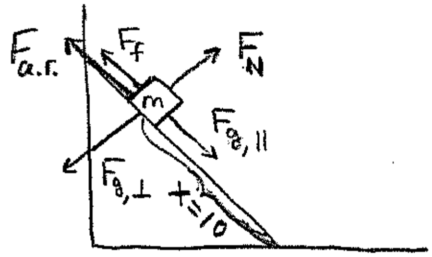
$$0.981t = 1000.0981$$

$$\Rightarrow t = \frac{1000.0981}{0.981} \cong 1019.47$$

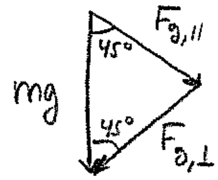
Notice that $e^{-10(1019.47)} \cong 1.78 \times 10^{-443}$ so my approximation is quite valid. Finally put in the units,

$$\boxed{t = 1019.47 \text{ seconds hits the ground}}$$

§3.4 #19 $m = 60\,\text{kg}$.



$$F_{g,\parallel} = \frac{mg}{\sqrt{2}}$$
$$F_{g,\perp} = \frac{mg}{\sqrt{2}}$$



Since m stays on plane must have $F_N = F_{g,\perp}$ thus $F_f = \mu F_N = \dfrac{0.05mg}{\sqrt{2}}$. $F_{a.r.} = -3V$ (air resistance)

Let the mass initial position be at $x = 0$ and let $V = \dfrac{dx}{dt}$ where x is the distance pictured. Newton's 2nd Law for m in the x-direction

$$m\frac{dV}{dt} = \frac{mg}{\sqrt{2}} - \frac{0.05\,mg}{\sqrt{2}} - 3V$$

$$60\frac{dV}{dt} = \frac{0.95}{\sqrt{2}}(60)(9.81) - 3V = 395.4 - 3V$$

$$\int \frac{20\,dV}{131.8 - V} = \int dt$$

$$-20\ln(131.8 - V) = t + C_1$$

$$131.8 - V = C_2\exp(-t/20)$$

$$V(t) = 131.8 - C_2\exp(-t/20)$$

$V(0) = 0 \Rightarrow C_2 = 131.8$ thus $V(t) = 131.8\left(1 - e^{-t/20}\right)$

$$V(t) = \frac{dx}{dt} = 131.8\left(1 - e^{-t/20}\right)$$

$$x(t) = \int_0^t 131.8\left(1 - e^{-u/20}\right)du$$

$$= 131.8t + 20(131.8)\left[e^{-t/20} - 1\right]$$

We need to solve $10 = 131.8t + 2636e^{-t/20} - 2636$ this gives $2646 = 131.8t + 2636e^{-t/20}$ [note we cannot neglect $e^{-t/20}$ here].

Thus, $\boxed{x(t) = 131.8t + 2636e^{-t/20} - 2636}$ and graphing or numerical solver $\longrightarrow$ $\boxed{t = 1.768\,s}$

§3.4 #25

$r = \varkappa$

$\varkappa$ = distance from center, $\varkappa > 0$ by def^{n}.

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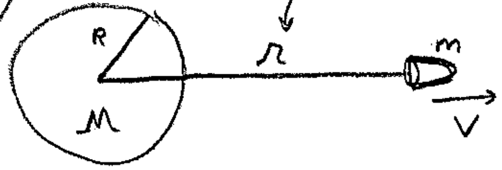

$F_g = \frac{GMm}{\varkappa^2}$ in direction to attract m & M

a.) $ma = F_{net} = -\frac{GMm}{\varkappa^2} = m\frac{d^2\varkappa}{dt^2}$

divide by m $\Rightarrow -\frac{GM}{\varkappa^2} = \frac{d}{dt}\left(\frac{d\varkappa}{dt}\right) = \frac{d}{dt}(v) = \frac{dv}{dt}$

Now at the surface of the earth $\varkappa = R$ and by def^{n} of g we have $F_{gravity} = \cancel{m}g = \frac{G\cancel{m}M}{R^2}$ thus we find $G = gR^2/M$ then

$$\frac{dv}{dt} = -\frac{GM}{\varkappa^2} = -\frac{gR^2}{M}\frac{M}{\varkappa^2} = \boxed{-\frac{gR^2}{\varkappa^2} = \frac{dv}{dt}}$$

b.) $\frac{dv}{dt} = \frac{d\varkappa}{dt}\frac{dv}{d\varkappa}$ (assuming radial motion!)

$= \boxed{v\frac{dv}{d\varkappa} = \frac{-gR^2}{\varkappa^2}}$ using (a.)

c.) $\int_{v_0}^{v_f} v\,dv = \int_{\varkappa_0}^{\varkappa_f} -gR^2\frac{d\varkappa}{\varkappa^2} \Rightarrow \frac{1}{2}(v_f^2 - v_0^2) = gR^2\left(\frac{1}{\varkappa_f} - \frac{1}{\varkappa_0}\right)$

seperated variables and integrated from intial to final velocities & positions; note $\varkappa_0 = R$, $v_0 = v_0$, $v_f = v$ and $\varkappa_f = \varkappa$ in the langauge of the problem set up.

$$\frac{1}{2}(v^2 - v_0^2) = gR^2\left(\frac{1}{\varkappa} - \frac{1}{R}\right)$$

$$v^2 = v_0^2 + 2gR^2\left(\frac{1}{\varkappa} - \frac{1}{R}\right)$$

$$= \boxed{v_0^2 - 2gR + 2gR^2/\varkappa = v^2}$$

d.) For velocity to become negative (although strictly speaking since $\varkappa$ = distance $\Rightarrow \frac{d\varkappa}{dt}$ = speed, but we could have taken $\varkappa$ to be position along the direction of motion) we need $v > 0 \longrightarrow v < 0$ hence $v = 0$ for some $\varkappa_e$, but $KE = \frac{1}{2}mv_e^2 = 0$ for $\varkappa_e$ notice that

$\frac{1}{2}mv^2 \stackrel{?}{=} \frac{1}{2}m(v_0^2 - 2gR + 2gR^2/\varkappa)$

§ 3.4 # 25 continued $\quad K.E. = \frac{1}{2}m\left(V_o^2 - 2gR + 2gR^2/r\right)$

And if V is to become negative it must be zero for some r as we assume $V > 0$ to begin. Notice that KE then will also be zero if that could happen. So how can KE be zero,

$$KE = \frac{1}{2}m\left(V_o^2 - 2gR + \frac{2gR^2}{r}\right)$$

As $r \longrightarrow \infty$ notice $KE \longrightarrow \frac{1}{2}m\left(V_o^2 - 2gR\right)$
if this is positive then were free from concern that $KE \to 0$ (when r is smaller that just adds to KE.)

Thus need $V_o^2 - 2gR > 0$ to insure $KE > 0$ for $r \to \infty$.