(§4.4 #1) no t^{-1} is not allowed, derivatives of t^{-1} are unending!

(§4.4 #2) sure.

(§4.4 #3) yep, just remember $3^t = e^{\ln(3^t)} = e^{\ln(3)t}$.

(§4.4 #4) yep $\sin(x)/e^{4x} = e^{-4x}\sin(x)$, derivatives close back on themselves.

<u>Remark</u>: When solving $aY'' + bY' + cY = g(x)$ the method of undetermined coefficients will work so long as the function $g(x)$ and it's derivatives $g'(x), g''(x), \dots$ form a finite set of functions upto linear independence. The text explains the possibilities more explicitly as you should discover.

(§4.4 #8) sure, but it would be horrible.

(§4.4 #9)
$$Y'' + 3Y = -9$$
$$\lambda^2 + 3 = 0 \Rightarrow \lambda = \pm\sqrt{3}\,i \Rightarrow Y_h = C_1\cos(\sqrt{3}\,x) + C_2\sin(\sqrt{3}\,x)$$

Clearly there is no overlap of Y_1 and Y_2 with $g(x)$ thus guess $Y_p = A$

$$Y_p'' + 3Y_p = -9 \Rightarrow 3A = -9 \Rightarrow \underline{A = -3} \quad \therefore \boxed{Y_p = -3}$$

The general sol^{n} would then be $Y = Y_h + Y_p$.

(§4.4 #12)
$$2x' + x = 3t^2 \Rightarrow Y_p = At^2 + Bt + C$$
$$2\lambda + 1 = 0 \Rightarrow \lambda = -1/2 \Rightarrow Y_h = C_1 e^{-\frac{1}{2}t}$$ (no overlap with Y_p;

Substituting X_p into DEq^n: $2x_p' + x_p = 2(2At + B) + At^2 + Bt + C = 3t^2$

$$\underline{At^2 + (4A + B)t + (2B + C) = 3t^2} \quad (*)$$

Equate Coefficients of like powers of t in $(*)$ to get 3 eq^{n}'s below,

t^2 : $A = 3$

t^1 : $4A + B = 0 \Rightarrow B = -12$

t^0 : $2B + C = 0 \Rightarrow C = 24$ $\quad \therefore \boxed{X_p = 3t^2 - 12t + 24}$

(§4.4 #14)
$$Y'' + Y = 2^x = e^{\ln(2)\cdot x}$$

Note $\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i \Rightarrow Y_h = C_1\cos(x) + C_2\sin(x)$.

Thus no overlap with Y_p (naive), simply use $Y_p = Ae^{\ln(2)x} = A2^x$

$$\left.\begin{aligned} Y_p' &= \ln(2)\,A2^x \\ Y_p'' &= (\ln(2))^2 A 2^x \end{aligned}\right\} \Rightarrow (\ln(2))^2 A2^x + A2^x = 2^x$$
$$A([\ln(2)]^2 + 1)\,2^x = 2^x$$
$$\Rightarrow A = \frac{1}{[\ln(2)]^2 + 1}$$
$$\therefore \boxed{Y_p = \frac{1}{(\ln(2))^2 + 1}\,2^x}$$

Remark: §6.3 adds further insight on the meaning of "overlap"

§4.4 #16 $\Theta'' - \Theta = t\sin t$

$$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow \Theta_h = c_1 e^t + c_2 e^{-t}$$ (no overlap here)

$$\Theta_p = t(A\sin t + B\cos t) + C\sin t + D\cos t$$ (follows from $t\sin t$ term)

$$\Theta_p' = A\sin t + B\cos t + t(A\cos t - B\sin t) + C\cos t - D\sin t$$ (used prod. rule)

$$= \sin t[A - D] + \cos t[B + C] + t[A\cos t - B\sin t]$$

$$\Theta_p'' = \cos t[A - D] - \sin t[B + C] + [A\cos t - B\sin t] + t[-A\sin t - B\cos t]$$

$$= \cos t[A - D + A] + \sin t[-B - C - B] + t[-A\sin t - B\cos t]$$

Now substitute Θ_p, Θ_p' & Θ_p'' into $\Theta_p'' - \Theta_p = t\sin t$ to obtain,

$$\Theta_p'' - \Theta_p = t\sin t = \cos t[2A - 2D] + \sin t[-2B - 2C] + t\cos t[-B - B] + t\sin t[-A - A]$$

Note the functions $t\sin t$, $t\cos t$, $\sin t$, $\cos t$ are LI. so we can equate coefficients to find,

$$\left.\begin{array}{lll} t\sin t: & 1 = -2A & \Rightarrow A = -1/2 \\ t\cos t: & 0 = -2B & \Rightarrow B = 0 \\ \sin t: & 0 = -2B - 2C & \Rightarrow C = 0 \\ \cos t: & 0 = 2A - 2D & \Rightarrow D = -1/2 \end{array}\right\} \quad \boxed{\Theta_p = -\frac{1}{2}t\sin t - \frac{1}{2}\cos t}$$

§4.4 #36 Find particular sol^n to $y'''' - 3y'' - 8y = \sin(t)$. Really should find y_h to insure no overlap. Here the characteristic eq^n will be 4th order $\Rightarrow$ 4 complex sol^n's.

$$\lambda^4 - 3\lambda^2 - 8 = 0$$ let $\lambda^2 = \delta$ to reduce to quadratic,

$$\delta^2 - 3\delta - 8 = 0 \Rightarrow \delta = \frac{3 \pm \sqrt{9+32}}{2} = \frac{3 \pm \sqrt{41}}{2}$$

Hence $\delta = \lambda^2 = \frac{3}{2} \pm \frac{\sqrt{41}}{2} \Rightarrow \lambda = \pm\sqrt{\frac{3}{2} \pm \frac{\sqrt{41}}{2}}$ where the $\pm$ are not connected but instead give 4 outcomes.

$$\lambda_1 = \sqrt{\frac{3+\sqrt{41}}{2}}, \quad \lambda_2 = \sqrt{\frac{3-\sqrt{41}}{2}}, \quad \lambda_3 = -\lambda_1 \text{ and } \lambda_4 = -\lambda_2$$

Thus the auxillary or homogeneous sol^n would be

$$y_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3 e^{\lambda_3 t} + c_4 e^{\lambda_4 t}$$ (no overlap)

Now we find y_p as usual, guess $y_p = A\sin t + B\cos t$

$$\left.\begin{array}{l} y_p' = A\cos t - B\sin t \\ y_p'' = -A\sin t - B\cos t = -y_p \\ y_p''' = -y_p' = -A\cos t + B\sin t \\ y_p'''' = -y_p'' = -(-y_p) = y_p \end{array}\right\} \quad \begin{array}{l} y_p'''' - 3y_p'' - 8y_p = \sin t \\ y_p - 3(-y_p) - 8y_p = 12y_p = \sin t \\ 12A\sin t + 12B\cos t = \sin t \end{array}$$

Comparing coefficients of $\cos t$ and $\sin t$ yields $12A = 1$ & $12B = 0$

Therefore, $\boxed{y_p = \frac{1}{12}\sin(t)}$ The general sol^n would be $y = y_h + y_p$.