<u>DETERMINANTS:</u>

The determinant of any *square* matrix is defined inductively on the basis of the formulas given below. There is a better definition, but I'm just trying to show you how to calculate it here. See a good linear algebra's treatment in terms of the wedge product for a better basis for what a "determinant" really is.

A number is a 1 x 1 matrix with

$$\det a = a$$

Then the determinant of a 2 x 2 is defined as follows:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Notice that this is formed by an alternating sum of the sub-determinants of the 1 x 1 matrix determinants $a \cdot \det(d) - b \cdot \det(c)$. I mean the sub-determinants are formed by excluding the row and column of a and then b respectively. Its not much of a pattern yet but wait until the next case. The determinant of 3 x 3 follows similarly:

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a \cdot \det \begin{pmatrix} e & f \\ h & i \end{pmatrix} - b \cdot \det \begin{pmatrix} d & f \\ g & i \end{pmatrix} + c \cdot \det \begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

You know how to expand the 2 x 2 determinants already so I'll leave it at that. What about a 4 x 4? It works the same way, we take an alternating sum of the top row times the sub-determinants formed through excluding the row and column of the top row entry multiplied,

$$\det \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} = a \cdot \det \begin{pmatrix} f & g & h \\ j & k & l \\ n & o & p \end{pmatrix} - b \cdot \det \begin{pmatrix} e & g & h \\ i & k & l \\ m & o & p \end{pmatrix} + c \cdot \det \begin{pmatrix} e & f & h \\ i & j & l \\ m & n & p \end{pmatrix} - d \cdot \det \begin{pmatrix} e & f & g \\ i & j & k \\ m & n & o \end{pmatrix}$$

now you already know how to compute 3 x 3 determinants so I'll leave it at this. I hope you can see how to do a 5 x 5 now. It would involve computing an alternating sum of 5 sub-determinants of 4 x 4 matrices. Its easy to see determinants are hard to calculate for large n systems. There are properties and tricks that make it easier but I'll leave that to your linear algebra course. (if you know tricks use them correctly)