TEST ONE REVIEW SHEET // Chapter ONE.

$\vec{A}\cdot\vec{B} \equiv AB\cos\theta = \vec{B}\cdot\vec{A}$

$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

$\vec{A}\cdot\vec{A} = A^2$

$\vec{A}\times\vec{B} \equiv AB\sin\theta\,\hat{n}$ where $\hat{n}$ is given by right hand rule.

$\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{C})$.

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\vec{A}\times\vec{A} = \vec{0}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & B_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$, $\vec{A}\cdot(\vec{C}\times\vec{B}) = \vec{B}(\vec{A}\times\vec{C}) = \vec{C}\cdot(\vec{B}\times\vec{A})$.

$$A\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$A\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$\vec{r} = xi + yj + zk$, where (x,y,z) are the coordinates and $\vec{r}$ is the position vector.

$r = \sqrt{x^2+y^2+z^2}$, distance from origin

$d\vec{l} = i\,dx + j\,dy + k\,dz$, infitesimal displacement vector.

$\vec{\imath} \equiv \vec{r} - \vec{r}'$ where $\vec{r}$ is the field point and $\vec{r}'$ is the source point.

$\hat{\imath} = \dfrac{(x-x')i + (y-y')j + (z-z')k}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$ $\hat{\imath}$ is unit vector pointing from source to field point.

$\overline{A_i} = \sum_{j=1}^{3} R_{ij}A_j$ or more familiarly $x' = Rx$. // vector transform under matrix.

$dT = \left(\frac{\partial T}{\partial x}i + \frac{\partial T}{\partial y}j + \frac{\partial T}{\partial z}k\right)\cdot(dx\,i + dy\,j + dz\,k) = (\nabla T)\cdot d\vec{l}$

$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$, $\nabla T = i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}$ and ∇T points to maximum increase in $T(x,y,z)$ where $|\nabla T|$ gives slope of increase

∇T is gradiant of T (scalar)

$\nabla\cdot\vec{V}$ is divergence of $\vec{V}$ (vector)

$\nabla\times\vec{V}$ is curl of $\vec{V}$ and $= \begin{vmatrix} i & j & k \\ \nabla_x & \nabla_y & \nabla_z \\ V_x & V_y & V_z \end{vmatrix}$ (rules for product and quotient ∇'s on page 21)

$\nabla^2 T = \nabla\cdot(\nabla T) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\left(i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}\right) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$, LAPLACIAN of T.

$\nabla\times(\nabla T) = 0$ the curl of a gradiant is always zero.

$\nabla\cdot(\nabla\times\vec{V}) = 0$ the divergence of a curl is always zero.

$$\int_{a\,\mathscr{P}}^{b} \vec{v}\cdot d\vec{l}$$

$\int_S \vec{v}\cdot d\vec{a}$ $\quad d\vec{a}$ = small small patch of area with vector normal to that area, on surface "S"

$\int_{\mathcal{V}} T\,d\tau$ $\quad d\tau = dx\,dy\,dz$ for the volume integral on scaler function T.

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

$\int_{a\,P}^{b} (\nabla T)\cdot d\vec{l} \equiv T(b) - T(a)$ $\quad$ fundamental Theorem for gradients.
note this integral is independent of path and so $\oint = 0$.

$\int_{\mathcal{V}} (\nabla\cdot\vec{v})\,d\tau = \oint_S \vec{v}\cdot d\vec{a}$ $\quad$ this is Guass' Theorem aka the divergence theorem
the integral of the divergence over a volume $\mathcal{V}$ = the integral of the divergedvector dotted into $d\vec{a}$ which is part of S which surrounds the volume $\mathcal{V}$.

$\int_S (\nabla\times\vec{v})\cdot d\vec{a} = \oint_P v\cdot d\vec{l}$ $\quad$ $\int(\nabla\times\vec{v})d\vec{a}$ depends only on boundary not the surface.
$\oint(\nabla\times\vec{v})\cdot d\vec{a} = 0$ $\quad$ This is Stokes Theorem.

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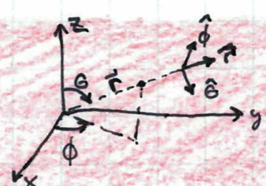
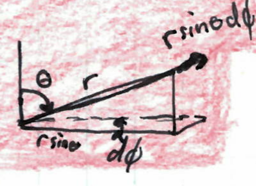
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$



$\hat{r} = (\sin\theta\cos\phi)i + (\sin\theta\sin\phi)j + (\cos\theta)k$
$\hat{\theta} = (\cos\theta\cos\phi)i + (\cos\theta\sin\phi)j - (\sin\theta)k$
$\hat{\phi} = (-\sin\phi)i + (\cos\phi)j$



note: $dl_r = dr$
$dl_\theta = r\,d\theta$
$dl_\phi = r\sin\theta\,d\phi$ $\Rightarrow$ Thus $d\vec{l} = (dr)\hat{r} + (r\,d\theta)\hat{\theta} + (r\sin\theta\,d\phi)\hat{\phi}$
$d\tau = dl_r\,dl_\theta\,dl_\phi = r^2\sin\theta\,d\theta\,d\phi\,dr$.
$d\vec{a} = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$ for $d\vec{a}$ of sphere
$d\vec{a} = dl_r\,dl_\phi\,\hat{\theta} = r\,dr\,d\phi\,\hat{\theta}$ for $d\vec{a}$ of $(x-y)$ plane.

$x = s\cos\phi$ $\quad \hat{s} = (\cos\phi)i + (\sin\phi)j$ $\quad dl_s = ds$
$y = s\sin\phi$ $\quad \hat{\phi} = (-\sin\phi)i + (\cos\phi)j$ $\quad dl_\phi = s\,d\phi$
$z = z$ $\quad \hat{z} = k$. $\quad dl_z = dz$.
$d\vec{l} = (ds)\hat{s} + (s\,d\phi)\hat{\phi} + (dz)k$
$d\tau = s\,ds\,d\phi\,dz$

$$\vec{F} = Q(\vec{E} + (\vec{v} \times \vec{B}))$$

$$\vec{F} = \int (\vec{v} \times \vec{B})\, dq = \int (\vec{v} \times \vec{B}) \lambda\, dl = \int (\vec{I} \times \vec{B})\, dl = \int I (d\vec{l} \times \vec{B})$$

$K \equiv \dfrac{dI}{dl_\perp}$ or if a mobile charge of σ has velocity $\vec{v}$, $\vec{K} = \sigma \vec{v}$

$F = \int (\vec{v} \times \vec{B}) \sigma\, da = \int (\vec{K} \times \vec{B})\, da$ for a surface ribbon like flow

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$J \equiv \dfrac{dI}{da_\perp}$ or if a mobile charge of ρ has velocity $\vec{v}$, $\vec{J} = \rho \vec{v}$

$F = \int (\vec{v} \times \vec{B}) \rho\, d\tau = \int (\vec{J} \times \vec{B})\, d\tau$ for a genuine 3D flow of charge.

$d\vec{I} = \vec{J}\, da_\perp$ or $d\vec{I} = \vec{J} \cdot d\vec{a}$ thus $I = \int \vec{J} \cdot d\vec{a}$

Consider some Volume then $I = \int_S \vec{J} \cdot d\vec{a} = \int_{\mathcal{V}} (\nabla \cdot \vec{J})\, d\tau$

$$\int_{\mathcal{V}} (\nabla \cdot \vec{J})\, d\tau = -\frac{d}{dt} \int_{\mathcal{V}} \rho\, d\tau = -\int_{\mathcal{V}} \left(\frac{\partial \rho}{\partial t}\right) d\tau \implies \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

Continuity Equation

Magnetostatics: For a steady line of current the Biot Savart law says:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dl' = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$