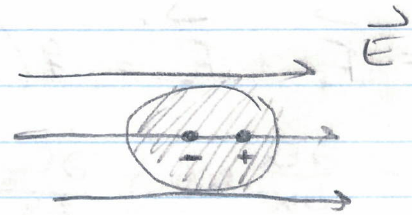
Test ~~→~~ 10, 12 instead of 8^{th}

— Chapter Four —

$\boxed{\text{§ 4.1 POLARIZATION}}$

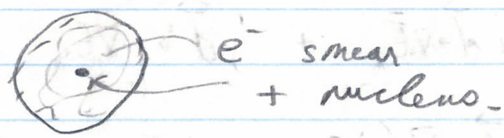


place an atom in $\vec{E}$

$$\vec{p} = \alpha \vec{E}$$

then there is a dipole on atom a seperation between positive and neg. center, we call α the atomic polarizability

— Crude Atomic Model

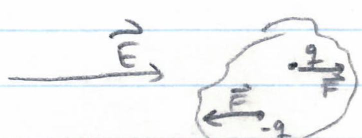


For molecules, it is more complicated, the polarizability depends on the relative direction of the applied field and the molecular axis.

$$P_{\perp} = \alpha_{\perp} \vec{E}_{\perp} + \alpha_{\parallel} \vec{E}_{\parallel}$$

$$\left\{ \begin{aligned} P_x &= \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z \\ P_y &= \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z \\ P_z &= \alpha_{zx} E_x + \alpha_{yz} E_y + \alpha_{zz} E_z \end{aligned} \right\} \leftarrow \alpha \text{ tensor}$$

For some molecules the center of negative charge does not coincide with the positive charge



$$\vec{N} = (\vec{r}_- \times \vec{F}_-) + (\vec{r}_+ \times \vec{F}_+)$$
$$= (-\vec{r}_- \times \vec{F}_+) + (\vec{r}_+ \times \vec{F}_+)$$
$$= (\vec{r}_+ - \vec{r}_-) \times \vec{F}_+ \quad = q(\vec{r}_+ - \vec{r}_-) \times \vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

If $\vec{E}$ is not uniform

$$\vec{F} = \vec{F_+} + \vec{F_-} = q(\vec{E_+} - \vec{E_-}) = q\,\Delta\vec{E}$$

$$\Delta E_x = \frac{\partial E_x}{\partial x}\Delta x + \frac{\partial E_x}{\partial y}\Delta y + \frac{\partial E_z}{\partial z}\Delta z = (\nabla E_x)\cdot\Delta l = (\nabla E_x)\cdot\vec{d}$$

$$\Delta\vec{E} = i\,\Delta E_x + j\,\Delta E_y + k\,\Delta E_z$$

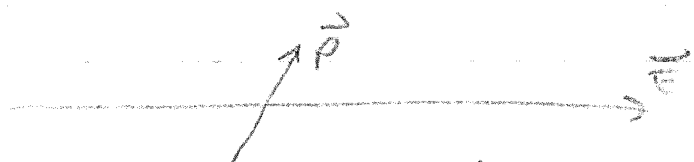
$$= i\,\vec{d}\cdot\nabla E_x + j\,\vec{d}\cdot\nabla E_y + k\,\vec{d}\cdot\nabla E_z$$

$$= \vec{d}\cdot\nabla(iE_x + jE_y + kE_z) =$$

$$F_{net} = q(d\cdot\nabla)\vec{E} = (\vec{P}\cdot\nabla)\vec{E}$$

$\alpha = 4\pi\epsilon_0 (0.667)\, 10^{-30}$ for Hydrogen

$$\vec{F_{net}} = (\vec{p}\cdot\nabla)\vec{E}$$

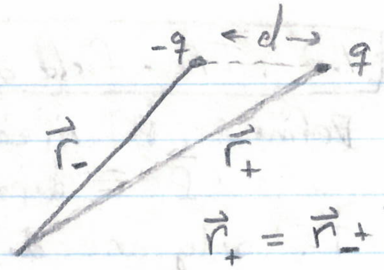


$$\vec{N} = \vec{P}\times\vec{E}$$

clock wise rotation ?

T (4,5)(4,6)

Potential Energy of a Dipole



$$\vec{r}_+ = \vec{r}_- + \vec{d}$$

$$U = -q\,V(\vec{r}_-) + q\,V(\vec{r}_+)$$

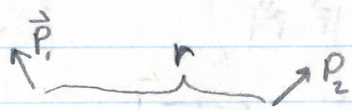
$$= q\left(V(\vec{r}+\vec{d}) - V(\vec{r})\right)$$

$$\cong q(\nabla V)\cdot\vec{d}$$

$$= -\vec{E}\cdot\vec{p} = -\vec{p}\cdot\vec{E} = \text{Potential energy of a dipole}$$

Dipole / Dipole Interaction

then $U = \dfrac{q_1 q_2}{4\pi\epsilon_0}$



we know that $U = -\vec{p}\cdot\vec{E}$, the $\vec{E}$ for this situation is due to P_1 for P_2

$$\vec{E}_{\text{due to } \vec{P}_1 \text{ at } \vec{P}_2} = \frac{1}{4\pi\epsilon_0 r^3}\left(3(\vec{P}_1\cdot\hat{r})\hat{r} - \vec{P}_1\right)$$

$$U = -\vec{P}\cdot\vec{E} = -\vec{p}\cdot\left(\frac{1}{4\pi\epsilon_0 r^3}\right)\left(3(\vec{P}_1\cdot\hat{r})\hat{r} - \vec{P}_1\right) = \frac{1}{4\pi\epsilon_0 r^3}\left(\vec{P}_1\cdot\vec{P}_2 - 3(\vec{P}_1\cdot\vec{r})(\vec{P}_2\cdot\hat{r})\right)$$

§4.2 The field of a Polarized Object

Define : Polarization

$\vec{P} \equiv$ dipole moment per unit volume

If only one dipole $\Rightarrow$ $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\hat{\mathscr{r}} \cdot \vec{p}}{\mathscr{r}^2}$ $\quad \vec{\mathscr{r}} = \vec{r} - \vec{r}'$

If we have many dipoles then we must integrate the potentials of many infitestimal dipoles

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\tau'} \frac{\hat{\mathscr{r}} \cdot \vec{P} \, d\tau'}{\mathscr{r}^2}}$$

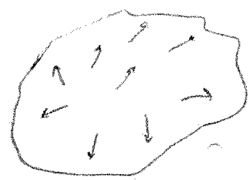
$$\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = \nabla' \frac{1}{\mathscr{r}} \quad \text{as} \quad \left(\nabla' \frac{1}{|\vec{r} - \vec{r}'|} = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} = -\nabla \frac{1}{|\vec{r} - \vec{r}'|} \right) \quad \hat{\mathscr{r}} = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \vec{P} \cdot \left(\nabla' \frac{1}{\mathscr{r}}\right) d\tau' = \frac{1}{4\pi\epsilon_0} \left[\underbrace{\int \nabla' \cdot \frac{\vec{P}}{\mathscr{r}} \, d\tau'} - \int \frac{1}{\mathscr{r}} \nabla' \cdot \vec{P} \, d\tau' \right]$$

$$\nabla (f\vec{A}) = \vec{A} \cdot \nabla f + f \nabla \vec{A} \qquad \oint \frac{\vec{P}}{\mathscr{r}} \cdot d\vec{a}' - \int \frac{1}{\mathscr{r}} \nabla' \cdot \vec{P} \, d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') \, d\tau'}{\mathscr{r}}$$

on surface some sort of chare



$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \, da'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0} \int \frac{\rho \, d\tau'}{\mathscr{r}}$$

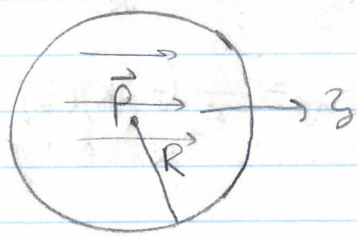
$$= \frac{1}{4\pi\epsilon_0} \int \frac{\vec{P}}{\mathscr{r}} \cdot d\vec{a}' - \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}} \nabla' \cdot \vec{P} \, d\tau'$$

$$\vec{P} \cdot d\vec{a}' = \vec{P} \cdot \hat{n} \, da' = \sigma \, da' \Rightarrow \sigma = \vec{P} \cdot \hat{n}$$

$$-\nabla \cdot \vec{P} = \rho$$

potential must be cont. at boundary

Find the electric field produced by a uniformly polarized sphere of radius R with $\vec{P} = P\hat{k}$



$$V_{in} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$V_{out} = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

$E_{\perp}^{bel} - E_{\perp}^{abo} = \frac{\sigma}{\epsilon_0}\hat{n}$ and at R $V_{in} = V_{out}$ So,

$$V_{in} = V_{out} \Rightarrow A_l R^l = \frac{B_l}{R^{l+1}} \Rightarrow B_l = A_l R^{2l+1}$$

$$\sigma = \vec{P}\cdot\hat{n} = P\cos\theta$$

$$E_{abo}^{\perp} - E_{bel}^{\perp} = -\frac{\partial V_{out}}{\partial r} + \frac{\partial V_{in}}{\partial r} = \frac{\sigma}{\epsilon_0} = \frac{P\cos\theta}{\epsilon_0} \qquad P_1 = \cos\theta.$$

notice then if $l \neq 1$ $A_l = B_l = 0$.

For $l = 1$

$$\frac{2B_1}{R^3}\cancel{P_1(\cos\theta)} + A_1 \cancel{P_1(\cos\theta)} = P\cancel{\cos\theta}/\epsilon_0 \quad , B_1 = A_1 R^{2l+1}$$

$$\frac{2B_1}{R^3} + A_1 = \frac{P}{\epsilon_0}$$

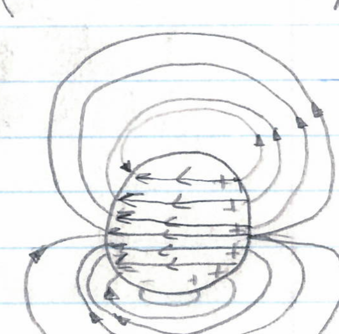
$$2A_1 + A_1 = P/\epsilon_0 \Rightarrow A_1 = P/3\epsilon_0$$

$$V_{in} = \frac{P}{3\epsilon_0} r\cos\theta = \frac{Pz}{3\epsilon_0} \Rightarrow E_{in} = -\frac{P}{3\epsilon_0}\hat{z}$$

$$V_{out} = \frac{PR^3}{3\epsilon_0}\frac{\cos\theta}{r^2} \quad , \vec{E}_{out} = -\frac{\partial V}{\partial r}\hat{r} - \frac{\partial V}{\partial\theta}\hat{\theta} = \frac{PR^3}{3\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$$E_{out} = \frac{R^3}{3\epsilon_0 r^3}\left[3(\vec{P}\cdot\hat{r})\hat{r} - \vec{P}\right] = \frac{1}{4\pi\epsilon_0}\left(3(\vec{P}_{tot}\cdot\hat{r})\hat{r} - \vec{P}_{tot}\right)$$

$$\frac{4}{3}\pi R^3 \cdot \vec{P} = \vec{P}_{tot.}$$



dipole per unit volume.

$\boxed{4.10}$ $\vec{P} = k\vec{r}$ sphere R find $\vec{E}_{in}$ and $\vec{E}_{out}$

$$\rho_B = -\nabla \cdot \vec{P} = -3k \qquad \sigma_B = \vec{P} \cdot \hat{n}\Big|_R = kR$$

$r < R$ $\quad \oint \vec{E}_{in} \cdot d\vec{a} = \frac{1}{\epsilon_0}\int \rho_B \, d\tau = 4\pi R^2 \cdot E_{in} = \frac{1}{\epsilon_0}(-3k)\left(\frac{4}{3}\pi r^3\right)$

$$\boxed{\vec{E}_{in} = \frac{-k}{\epsilon_0}\vec{r}}$$

$r > R$ $\quad \oint \vec{E}_{out} \cdot d\vec{a} = \frac{1}{\epsilon_0}\left[\int \rho_B \, d\tau + 4\pi R^2 \cdot \sigma_B\right] = \frac{1}{\epsilon_0}\left(-3k\frac{4}{3}\pi R^3 + 4\pi R^2 \cdot kR\right)$

$$E_{out} = 0$$

4.12, 4.13

$\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$ $\quad$ τ' is a sphere centered at origin

τ' an infinite clyinder, origin on ax.

$\frac{1}{4\pi\epsilon_0}\int \frac{\rho\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau' = \vec{E}$ obtain from Mr. Guass

4.3 The electric displacement

Polarization Charge $\rho_B = -\nabla \cdot \vec{P}$

Guass Law $\epsilon_0 (\nabla \cdot \vec{E}) = \rho$

There are 2 kinds of charge free and bound now guass law is modified for materials as to describe bound and free charge

$$\epsilon_0 (\nabla \cdot \vec{E}) = \rho = \rho_f + \rho_b = \rho_f - \nabla \cdot \vec{P}$$

$$\nabla \cdot (\epsilon_0 \vec{E} \cdot \vec{P}) = \rho_f \qquad \text{define } \epsilon_0 \vec{E} + \vec{P} = \vec{D}$$

THEN $\boxed{\nabla \cdot \vec{D} = \rho_f}$ this is good all that interests us is typically the free charge

In a typical problem ρ_f is known, ρ_B is not given, using $\vec{D}$ is more straight forward

But,

$$\nabla \times \vec{D} = \epsilon_0 \underbrace{(\nabla \times \vec{E})}_{=0} + (\nabla \times \vec{P}) \quad \text{(not always zero)}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho \hat{r} \, d\tau'}{r^2} \Longrightarrow \vec{D}(\vec{r}) \neq \frac{1}{4\pi\epsilon_0} \int \frac{\hat{r} \rho_f(\vec{r}') d\tau'}{r^2}$$

only sometimes does it work as $\vec{E}$

what make it a bound charge? are not all the e^-'s bound charges.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}_{\parallel}$$

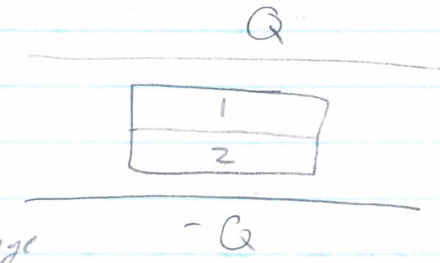
$$\boxed{\nabla \cdot \vec{D} = \rho_f} \qquad \boxed{\nabla \cdot \vec{E} = \rho/\epsilon_0}$$

$$\oint \vec{D} \cdot d\vec{a} = Q_{free}$$

Boundary Condition: $E^{\perp}_{abo} - E^{\perp}_{bel} = \dfrac{\sigma}{\epsilon_0}$

$$D^{\perp}_{abo} \cdot \Delta a - D^{\perp}_{bel} \cdot \Delta a = Q_{free}$$

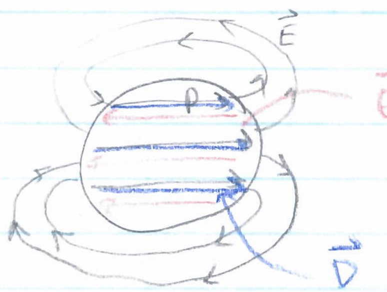
$$D^{\perp}_{abo} - D^{\perp}_{bel} = \frac{Q_{free}}{\Delta a} = \sigma_{free}$$



D stays same but E and $\vec{P}$ change

$$\vec{E}_{in} = -\frac{\vec{P}}{3\epsilon_0}$$

$$\vec{D} = \epsilon_0 \vec{E}_{in} + \vec{P} = -\frac{\vec{P}}{3} + \vec{P} = \frac{2}{3}\vec{P}$$



E discontinuous

D continuous

$$\boxed{\oint \vec{D} \cdot d\vec{a} = Q_{free}}$$

§4.4 LINEAR DIELECTRICS

Susceptibility, permitivity and dielectric constant.

When material placed in $\vec{E}$, a $\vec{P}$ is induced

$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$ where χ_e is susceptibility

Total field from everything (including induced $\vec{E}$)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} = \epsilon_0 (1 + \chi_e) \vec{E}$$

$\boxed{\epsilon_0(1 + \chi_e) = \epsilon = \text{permitivity}}$

relative permitivity $= \dfrac{\epsilon}{\epsilon_0} = 1 + \chi_e = \boxed{\epsilon_r = \text{dielectric constant}}$

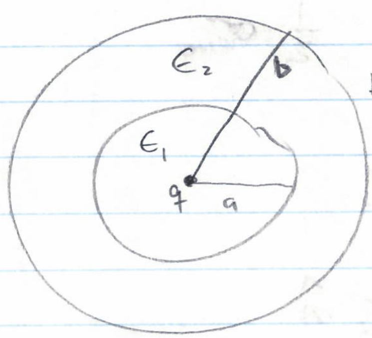
P $\quad \vec{D} = \epsilon_0 \vec{E}_{vacuum}$ with no dielectric

$$\vec{E} = \frac{1}{\epsilon} \vec{D} = \frac{1}{\epsilon_0 \epsilon_r} \vec{D} = \frac{1}{\epsilon_r} \left(\frac{\vec{D}}{\epsilon_0} \right) = \frac{1}{\epsilon_r} \left(\vec{E}_{vacuum} \right)$$

$$\boxed{\vec{E}_{dielectric} = \frac{E_{vacuum}}{\epsilon_r}}$$

Hence for a free charge in a vacuum $\vec{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{r^2} \hat{r}$

then with a dielectric $\vec{E} = \dfrac{1}{4\pi\epsilon} \dfrac{q}{r^2} \hat{r}$

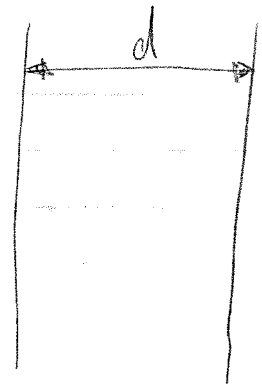


HE SAYS DISPLACEMENT IS CONSTANT.

and surface charge between a, b.

$\boxed{4.15,\ 4.16}$

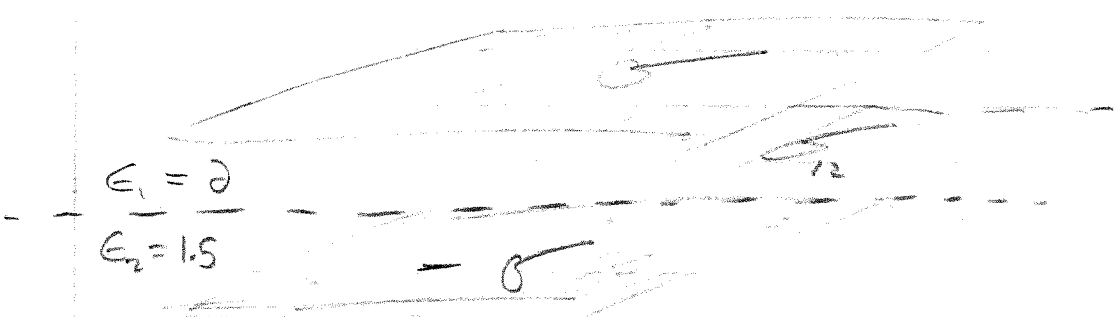
Ex// A parelell plate capacitor is filled with a material with ϵ_r, How Does Capacitance change?



$$E = \frac{1}{\epsilon_r} E_0 \quad \text{for a fixed charge } Q$$

$$V = E \cdot d = E_0 d / \epsilon_r$$

$$C_r = Q/V = \frac{Q}{Ed} = \frac{Q}{V_0}\epsilon_r = \boxed{\epsilon_r C_0}$$



$\vec{P}_1 = ?$
$\vec{E}_1 = ?$
$\vec{P}_2 = ?$
$\vec{E}_2 = ?$
$\sigma_{12} = ?$

1^{st} note D is continuous

$$\vec{D} = \sigma_f = \vec{D}_1 = \vec{D}_2$$

$$\vec{E}_1 = \vec{D}/\epsilon_1 = \vec{D}/\epsilon_r\epsilon_0 = \frac{1}{2}\frac{\vec{D}}{\epsilon_1} = \frac{1}{2}\sigma_{free}$$

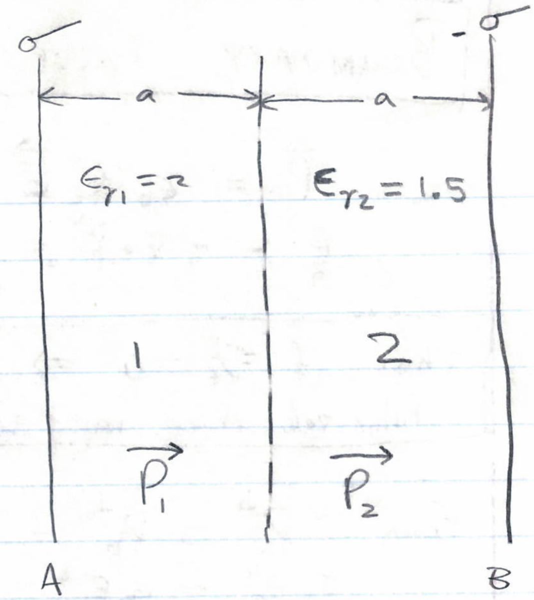
$$\vec{E}_2 = \vec{D}/\epsilon_2 = \frac{1}{1.5}\frac{\sigma_f}{\epsilon_0}$$

$$\vec{P}_1 = \vec{D}_1 - \epsilon_0\vec{E}_1 = \frac{1}{2}\sigma_f\,\hat{n}$$

$$\vec{P}_2 = \vec{D}_2 - \epsilon_0\vec{E}_2 = \sigma_f - \frac{1}{1.5}\sigma_f = \frac{1}{3}\sigma_f\,\hat{n}$$

$\boxed{4.22,\ 4.24}$

Example with $\boxed{\text{free charge } \sigma}$



(a) $\oint \vec{D} \cdot d\vec{a} = Q$

$\vec{D}_1 = \sigma \hat{n} \qquad \vec{D}_2 = \sigma \hat{n}$

(b) $\vec{E}_1 = \vec{D}_1 / \epsilon_1 = \dfrac{\vec{D}_1}{\epsilon_{r1} \epsilon_0} = \dfrac{\sigma \hat{n}}{2 \epsilon_0}$

$\vec{E}_2 = \dfrac{\vec{D}_2}{\epsilon_{r2} \epsilon_0} = \dfrac{\sigma \hat{n}}{1.5 \epsilon_0}$

(c) $\vec{P}_1 = \vec{D}_1 - \epsilon_0 \vec{E}_1 = \sigma \hat{n} - \dfrac{\epsilon_0 \sigma \hat{n}}{\epsilon_0 2} = \dfrac{\sigma \hat{n}}{2}$

$\vec{P}_2 = \vec{D}_2 - \epsilon_0 \vec{E}_2 = \sigma \hat{n} - \dfrac{2}{3} \dfrac{\epsilon_0 \sigma}{\epsilon_0} = \dfrac{1}{3} \sigma \hat{n}$

(d) $V_{AB} = E_1 a + E_2 a = \dfrac{\sigma}{2 \epsilon_0} a + \dfrac{\sigma}{1.5 \epsilon_0} = \dfrac{7}{6} \dfrac{\sigma a}{\epsilon_0}$

(e) $\sigma = \vec{P} \cdot \vec{n}$

$\sigma_{total\ A} = \sigma + \vec{P}_1 \cdot \hat{n}_A = \sigma - \dfrac{\sigma}{2} = \sigma/2$

$\sigma_{mid} = \vec{P}_1 \cdot \hat{n} + \vec{P}_2 \cdot \hat{n}_2 = \dfrac{\sigma}{2} - \dfrac{1}{3} \sigma = \dfrac{1}{6} \sigma$

$\sigma_{total\ B} = -\sigma + \vec{P}_2 \cdot \vec{n}_B = -\sigma + \dfrac{1}{3} \sigma = \dfrac{-2}{3} \sigma$

Boundary Value Problems with Linear Dielectrics

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\rho_B = -\nabla\cdot\vec{P} = -\nabla\cdot\epsilon_0\chi_e\vec{E} = -\epsilon_0 \frac{\chi_e}{\underbrace{\epsilon}_{(1+\chi_e)\epsilon_0}}(\underbrace{\nabla\cdot\vec{D}}_{\rho_f}) = \frac{-\chi_e}{1+\chi_e}\rho_f$$

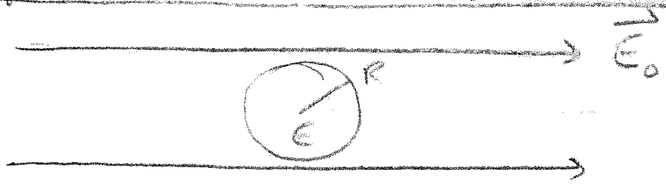
Thus if $\rho_f = 0 \Rightarrow \rho_B = 0$
Polarized charge must be on surface

From $D_a^{\perp} - D_b^{\perp} = \sigma_f$

$$\epsilon_a \underbrace{E_a^{\perp}}_{-\frac{\partial V_a}{\partial n}} - \epsilon_b \underbrace{E_b^{\perp}}_{-\frac{\partial V_b}{\partial n}} = \sigma_f$$

also $V_a = V_b$ at surface

~ Example to use or new tricks: Linear Dielectric Sphere in Uniform $\vec{E}_0$ ~



$$\left.\begin{aligned} V_{in} &= \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \\ V_{out} &= -E_0 r\cos\theta + \sum \frac{B_l}{r^{l+1}} P_l(\cos\theta) \end{aligned}\right\} \text{only } l=1 \text{ survive} \quad \left\{\begin{aligned} V_{in} &= A_1 r^1 \cos\theta \\ V_{out} &= -E_0 r\cos\theta + \frac{B_1}{r^2}\cos\theta \end{aligned}\right\}$$

$$\left\{\begin{aligned} &A_1 R = -E_0 R + B_1/R^2 \quad \text{(match at boundary)} \\ &-\epsilon\frac{\partial V_{in}}{\partial r} = -\epsilon_0 \frac{\partial V_{out}}{\partial r} \Rightarrow -\epsilon A_1 = \epsilon_0\left(E_0 + \frac{2B_1}{R^3}\right) \\ &\text{(as there is no free charge)} \end{aligned}\right\} \Rightarrow \quad A_1 = \frac{-3}{2+\epsilon_r}E_0$$

$$V_{in} = \frac{-3}{2+\epsilon_r}E_0 r\cos\theta \Rightarrow \vec{E}_{in} = -\frac{\partial V_{in}}{\partial z}\hat{k} = \frac{3E_0}{2+\epsilon_r}\hat{k}$$

we can go back and solve to get B_1 to find V_{out}.

Energy in Dielectric System

$$W = \frac{\epsilon_0}{2}\int E^2 \, d\tau$$

How does this Δ in Dielectrics

$$\Delta W = \int (\Delta \rho_f) \cdot V \, d\tau \qquad \Delta \rho_f \, d\tau = \Delta Q$$

$$\nabla \cdot \vec{D} = \rho_f \qquad \nabla \cdot (\Delta \vec{D}) = \Delta \rho_f$$

$$\Delta W = \int [\nabla \cdot (\Delta \vec{D})] V \, d\tau \qquad \nabla \cdot (\Delta \vec{D}) V = \nabla \cdot (\Delta \vec{D} V) - \Delta \vec{D} \cdot (\nabla V)$$

$$= \int \nabla \cdot (\Delta \vec{D} V) \, d\tau - \int \Delta \vec{D} \cdot (\nabla V) \, d\tau$$

$$= \oint \Delta \vec{D} V \cdot d\vec{a} + \int (\Delta \vec{D} \cdot \vec{E}) \, d\tau$$

over all space

dielectric charge localized

$\frac{1}{r^2}$ $\frac{1}{r}$ r^2 this goes to zero as $\frac{1}{r^2} \cdot \frac{1}{r} \cdot r^2 \to 0$ so the 1st term

thus drops out.

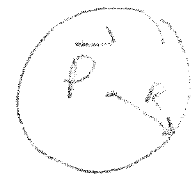
$$= \int (\Delta \vec{D} \cdot \vec{E}) \, d\tau' = \int \epsilon \, \Delta \vec{E} \cdot \vec{E} \, d\tau$$

$$\Rightarrow \boxed{W = \frac{1}{2}\int \epsilon E^2 \, d\tau = \frac{1}{2}\int \vec{D} \cdot \vec{E} \, d\tau}$$

Example// the energy of a sphere of uniform $\vec{P}$

$$\vec{E}_{in} = \frac{-\vec{P}}{3\epsilon_0} \qquad \vec{E}_{out} = \frac{R^3}{3\epsilon_0 r^3}\left[3(\vec{P}\cdot\hat{r})\hat{r} - \vec{P}\right]$$

$$= \frac{PR^3}{3\epsilon_0}\left(\frac{2\cos\theta\,\hat{r}}{r^3} + \frac{\sin\theta\,\hat{\theta}}{r^3}\right)$$



$$\vec{D}_{out} \cdot \vec{E}_{out} = \epsilon_0 \left|\vec{E}_{out}\right|^2$$

$$W_{out} = \frac{1}{2}\int \epsilon_0 \frac{P^2 R^6}{9\epsilon_0^2}\left(\frac{4\cos^2\theta}{r^6} + \frac{\sin^2\theta}{r^6}\right) r^2 dr \sin\theta d\theta\, d\phi$$

$$= \frac{P^2 R^6}{2\cdot 9\epsilon_0}\int (3\cos^2\theta + 1) \sin\theta d\theta d\phi \left. \frac{-1}{3r^3}\right|_R^\infty$$

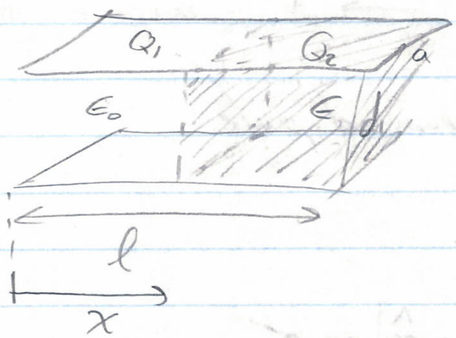
$$= \frac{4\pi P^2 R^3}{27\epsilon_0}$$

$$\vec{D}_{in} = \epsilon_0 \vec{E}_{in} + \vec{P} = \frac{2\vec{P}}{3}$$

$$\frac{1}{2}\int \vec{D}_{in}\cdot\vec{E}_{in}\, d\tau = \frac{1}{2}\int\left(\frac{2\vec{P}}{3} \cdot \frac{-\vec{P}}{3\epsilon_0}\right)d\tau = \frac{-P^2}{9\epsilon_0}\frac{4\pi}{3}R^3 = -\frac{4\pi}{27\epsilon_0}P^2R^3$$

$$W_{in} + W_{out} = 0$$

How does capacitance change when part of sphere is filled with dielectrics



$$Q = Q_1 + Q_2$$

$$\vec{E} \neq \frac{\sigma_1}{\epsilon_0} = \frac{\sigma_2}{\epsilon}$$

$$= \frac{Q_1}{x_a \epsilon_0} = \frac{Q_2}{(l-x)a\epsilon}$$

$$\frac{Q_2 x \epsilon_0}{(l-x)\epsilon} + Q_2 = Q \Rightarrow \frac{Q}{Q_2} = \frac{x\epsilon_0 + \epsilon(l-x)}{\epsilon(l-x)} = \frac{(\epsilon_0 - \epsilon)x + \epsilon l}{\epsilon(l-x)}$$

$$C_d = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q\epsilon a(l-x)}{Q_2 d} = \frac{a[(\epsilon_0 - \epsilon)x + \epsilon l]}{d}$$

$$W = \frac{1}{2} Q^2/C_d \qquad F_Q = -\left(\frac{\partial W}{\partial x}\right)_Q = \frac{1}{2}\frac{Q^2}{C_D^2}\frac{\partial C_d}{\partial x}$$

$$Q = C_d V \qquad W = \frac{1}{2} C_d V^2 \qquad \cancel{F_V = -\left(\frac{\partial W}{\partial x}\right)_V = \frac{-1}{2}V^2\frac{\partial C_d}{\partial x} = \frac{-1}{2}\frac{Q^2}{C_d^2}\frac{\partial C_d}{\partial x}}$$

right but wrong

F_Q OPPOSES F_V, why is that
the dielectric doesn't know if Q, or V is constant, what is wrong?
we cannot keep plates at constant potential on there own
we must supply energy from battery

$$dW = \underbrace{F_{ext}\,dx}_{\text{work from external force}} + \underbrace{V\,dQ}_{\text{work from battery}} = (-F)dx + VdQ$$

$$F_V = -\left(\frac{dW}{dx}\right)_V + V\frac{dQ}{dx} = \frac{-1}{2}\frac{Q^2}{C_d^2}\frac{dC_d}{dx} + V^2\frac{dC_d}{dx} = \frac{1}{2}\frac{Q^2}{C_d^2}\frac{dC_d}{dx}$$

MULTIPOLE EXPANSION ???!!

Test Three Review

$\vec{p} = q\vec{d}$ going from $-\longrightarrow+$ dipole moment

$U = -\vec{p}\cdot\vec{E}$ potential from dipole

$\vec{N} = \vec{p}\times\vec{E}$

$\vec{\underline{P}} = -\nabla\cdot\vec{P} = \rho_B$ $\qquad P\cdot\hat{n}\Big|_{\text{Boundary}} = \sigma_B$

$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$ (Dielectric)

$\epsilon = \epsilon_r\epsilon_0 = (1+\chi_e)\epsilon_0$

$\dfrac{\vec{E}_{VAC}}{\epsilon_r} = \vec{E}_{(\text{with dielectric})}$

$W = \dfrac{1}{2}\int\vec{D}\cdot\vec{E}\,d\tau$

Laplaces Equation

WRITE SOLUTION IN TERMS OF ∞ SUM OF ORTHOGONAL SETS

① IS SPHERICAL WITH AZIMUTHAL SYMETRY

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) + \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

③ CARTESIANS:

$$V(x,y,z) = e^{\pm kx}\ {\sin \atop \cos} k'y\ \ {\sin \atop \cos} k''z$$

③ CYLINDRICAL:

$$V(s,\phi) = A_0 + b_0\ln(s) + \sum_{m=1}^{\infty} s^{\pm m}(a_m\sin m\phi + b_n\cos m\phi)$$

Boundary Conditions

① V is continuous on boundaries $\quad V_{in} = V_{out}$

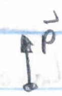
② $E^{\perp}_{abo} - E^{\perp}_{bel} = \frac{\sigma}{\epsilon_0} \leftarrow$ total charge density $\quad \sigma_b + \sigma_f = \sigma$

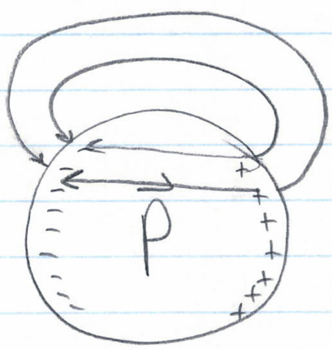
②' $D^{\perp}_{abo} - D^{\perp}_{bel} = \sigma_f \quad$ just free charge surface density

know how to use but don't have to memorize

$$\boxed{\vec{E}_{dipole} = \frac{1}{4\pi\epsilon_0 r^3}\left(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right)}$$

⊛ field point





$$\vec{E}_{in} = -\frac{P}{3\epsilon_0}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$
$$= -\frac{\vec{P}}{3} + \vec{P} = \frac{2}{3}\vec{P}$$

$\oint \vec{D}\cdot d\vec{a} = q_f = 0$

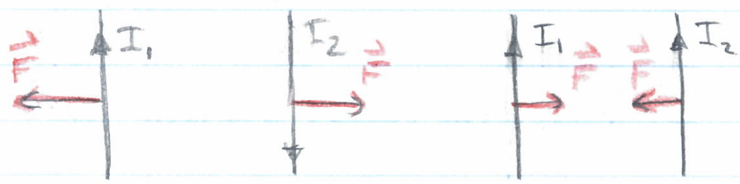
but $\vec{D}$ is not spherically symetric so we can just pull out D

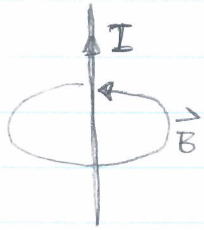
$$\boxed{\oint \vec{D}\cdot d\vec{a} = 4\pi r^2 D = q_f}$$

$$\frac{Q^2}{8\pi\epsilon_0}\left(\frac{b + a\,\chi_e}{ab(1+\chi_e)}\right)$$

Chapter 5 Magnetostatics

§5.1 The Lorentz Force Law

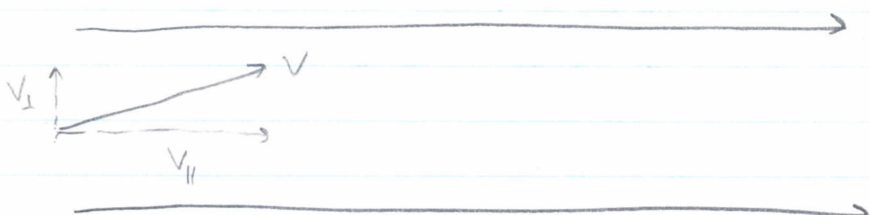




Lorentz Force Law

$$\boxed{\vec{F} = Q(\vec{V} \times \vec{B})}$$

$$F = Q V_\perp B = \frac{m V_\perp^2}{r} \qquad r = \frac{m V_\perp}{Q B}$$



If $\vec{E}$ field is also present then the Lorentz Force is

$$\boxed{\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})}$$

$\vec{E}$ tends to accelerate and do work
$\vec{B}$ tends to <u>never</u> do work

$$\int \vec{F}_{mag} \cdot d\vec{r} = \int \vec{F}_{mag} \cdot \frac{d\vec{r}}{dt}\, dt = \int \vec{F} \cdot v\, dt = q\int (\vec{V} \times \vec{B}) \cdot v\, dt = 0$$