Boundary Conditions

① V is continuous on boundaries $\quad V_{in} = V_{out}$

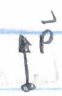
② $E^{\perp}_{abo} - E^{\perp}_{bel} = \dfrac{\sigma}{\epsilon_0}$ ← total charge density $\quad \sigma_b + \sigma_f = \sigma$

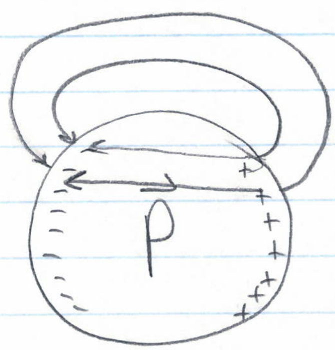
②' $D^{\perp}_{abo} - D^{\perp}_{bel} = \sigma_f \quad$ just free charge surface density

know how to use but don't have to memorize

$$\boxed{\vec{E}_{dipole} = \frac{1}{4\pi\epsilon_0 r^3}\left(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right)}$$

⊛ field point





$$\vec{E}_{in} = -\frac{P}{3\epsilon_0}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$
$$= -\frac{\vec{P}}{3} + \vec{P} = \frac{2}{3}\vec{P}$$

$$\oint \vec{D}\cdot d\vec{a} = q_f = 0$$

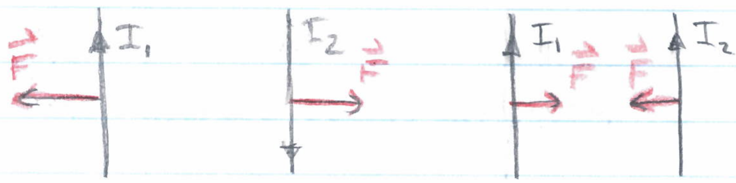
but $\vec{D}$ is not spherically symetric so we can just pull out D

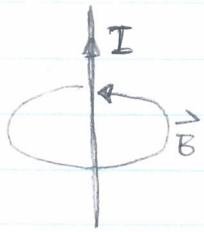
$$\boxed{\oint \vec{D}\cdot d\vec{a} = 4\pi r^2 D = q_f}$$

$$\frac{Q^2}{8\pi\epsilon_0}\left(\frac{b + a\,\chi_e}{ab(1+\chi_e)}\right)$$

CHAPTER 5 MAGNETOSTATICS

§5.1 The Lorentz Force Law

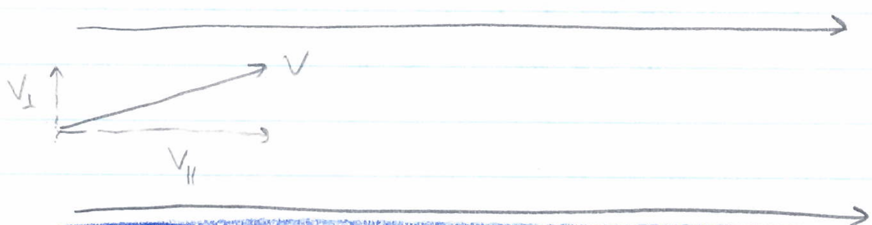




Lorentz Force Law

$$\vec{F} = Q(\vec{V} \times \vec{B})$$

$$F = Q V_\perp B = \frac{m V_\perp^2}{r} \qquad r = \frac{m V_\perp}{Q B}$$



If $\vec{E}$ field is also present then the Lorentz Force is

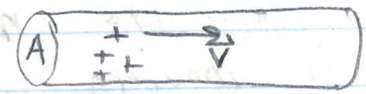
$$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$$

$\vec{E}$ tends to accelerate and do work
$\vec{B}$ tends to <u>never</u> do work

$$\int \vec{F}_{mag} \cdot d\vec{r} = \int \vec{F}_{mag} \cdot \frac{d\vec{r}}{dt}\, dt = \int \vec{F} \cdot v \, dt = q\int(\vec{V} \times \vec{B}) \cdot v \, dt = 0$$

Current Density: $\vec{J} = \rho \vec{v}$ wire has cross-section

$$\vec{J}\Delta A = \vec{I} = \Delta A \rho \vec{v} = \lambda \vec{v}$$

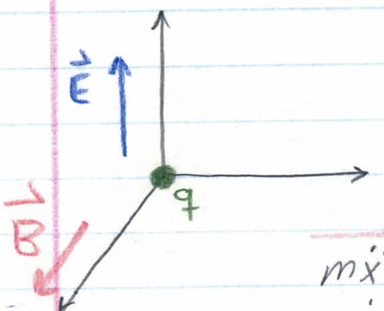


$$\vec{F}_{mag} = \int (\vec{v}\times\vec{B})\,dq = \int (\vec{v}\times\vec{B})\lambda\,dl = \boxed{\int (\vec{I}\times\vec{B})\,dl = \int I(dl\times\vec{B})}$$

Example: have $\vec{B} = Bi$ and $\vec{E} = Ek$

A charge is released at $(0,0,0)$ at $t=0$

Find the trajectory:



$$F = q(Ek + (\dot{x}i + \dot{y}j + \dot{z}k)\times Bi) = m(\ddot{x}i + \ddot{y}j + \ddot{z}k)$$

$$= q(Ek - \dot{y}Bk + \dot{z}Bj) =$$

$m\ddot{x} = 0 \Rightarrow \dot{x} = \text{constant}$, at $t=0$ $\dot{x}=0 \Rightarrow \dot{x} = 0$.

$\dot{x} = 0 \Rightarrow x = \text{const}$, $x(0)=0 \Rightarrow \boxed{x = 0}$

$m\ddot{y} = qB\dot{z} \Rightarrow m\dot{y} = qBz + C_1$ at $t=0$ $\dot{y}=0, z=0 \Rightarrow C_1 = 0$.

$\Rightarrow \dot{y} = \frac{qB}{m}z$

$$m\ddot{z} = qE - Bq\dot{y} = qE - qB\frac{qB}{m}z = qE - \frac{(qB)^2}{m}z$$

$$\ddot{z} + \frac{q^2B^2}{m^2}z = qE/m$$

$z = a_1\sin wt + b_1\cos wt + \frac{Em}{B^2q}$ $w = \frac{qB}{m}$

$z(0) = 0 \Rightarrow b_1 + \frac{Em}{B^2q} \Rightarrow b_1 = -\frac{Em}{B^2q} \Rightarrow \boxed{z = \frac{Em}{qB^2}(1-\cos wt)}$

$\dot{z}(0) = 0 \Rightarrow a_1 = 0$

$\dot{y} = \frac{E}{B}(1-\cos wt) \Rightarrow y = \frac{E}{Bw}\sin wt + C_2$, $y(0) = 0 \Rightarrow C_2 = 0$

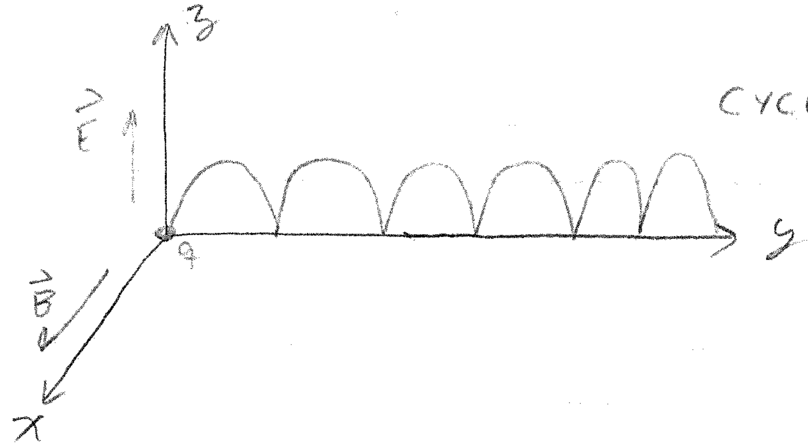
$\boxed{y = \frac{E}{Bw}\sin wt}$

$$\left\{\begin{aligned} z &= \frac{qE}{Bw}(1-\cos wt) \\ y &= \frac{qE}{Bw}(wt - \sin wt) \end{aligned}\right\}$$

from prev. Ex.

$$\left(z - \frac{qE}{B\omega}\right)^2 + \left(y - \frac{qE}{B}t\right)^2 = \frac{q^2E^2}{B^2\omega^2}$$

see its moving in a circle whose center is moving with constant velocity and a radius $\frac{qE}{B\omega} = r$



CYCLOID Motion

5.4, 5.8

$$F_{mag} = Q(\vec{V} \times \vec{B})$$
$$= \int (\vec{V} \times \vec{B}) dq = \int (\vec{V} \times \vec{B}) \lambda dl = \int (\vec{I} \times \vec{B}) dl = \int I (d\vec{l} \times \vec{B})$$

For surface current $\vec{k} = \sigma \vec{V}$

$$\vec{F}_{mag} = \int (\vec{V} \times \vec{B}) dq = \int (\vec{V} \times \vec{B}) \sigma d\vec{a} = \int (\vec{k} \times \vec{B}) da \neq \int k(d\vec{a} \times \vec{B})$$

For volume charge

$$F_{mag} = \int (\vec{V} \times \vec{B}) dq = \int (\vec{V} \times \vec{B}) \rho d\tau = \int (\vec{J} \times \vec{B}) d\tau$$

Current Crossing a Surface

$$I = \int \vec{J} \cdot d\vec{a} \qquad \oint \vec{J} \cdot d\vec{a} = \text{net charge flow out/sec.}$$
$$= \text{net charge reduced in } \tau\text{/sec}$$
$$= -\frac{d}{dt} \int \rho d\tau = -\int \frac{\partial \rho}{\partial t} d\tau$$

$$\oint \vec{J} \cdot d\vec{a} = \int \nabla \cdot \vec{J} d\tau = -\int \frac{\partial \rho}{\partial t} d\tau \Rightarrow \boxed{\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

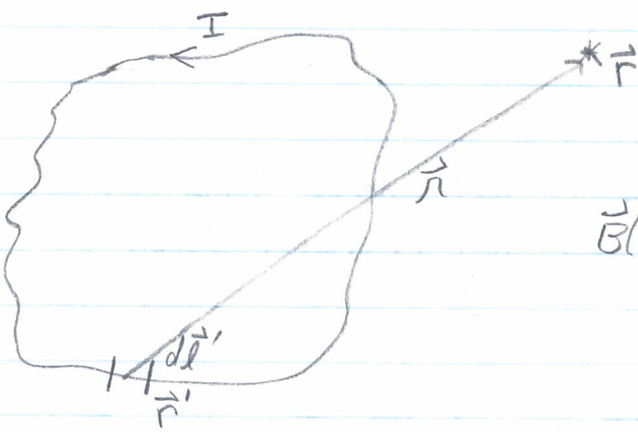
Continuity Equation

§5.2 Biot Savart Law

Stationary Charge $\Rightarrow$ constant electric field (in time)
$\Rightarrow$ ELECTROSTATICS

STEADY CURRENT $\Rightarrow$ Constant magnetic field (in time)
$\Rightarrow$ Magnetostatics

$$\boxed{\text{Define Steady Current as } \nabla \cdot \vec{J} = 0 \Rightarrow \frac{\partial \rho}{\partial t} = 0}$$



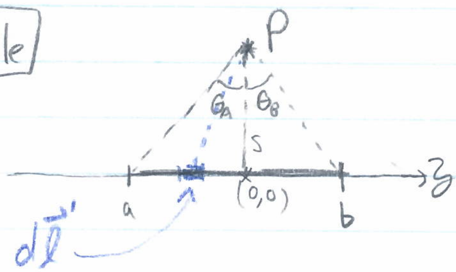
Biot Savart Experimental Law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

$\vec{B}$ is in Tesla $\quad 1T = 1N/\mathrm{Amp}(m)$

Example



$d\vec{l}' = dz'\, k$

$\vec{\mathscr{r}} = s\,i - z'\,k$

$\mathscr{r} = \sqrt{z'^2 + s^2}$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{(dz'\,k) \times (-z'k + s\,i)}{(z'^2 + s^2)^{3/2}} \qquad \text{as } \hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{|\mathscr{r}|}$$

$$= \frac{\mu_0 I}{4\pi}\, j \int \frac{s\, dz'}{(z'^2 + s^2)^{3/2}} \qquad \text{let } z' = s\tan\theta, \quad dz' = s\sec^2\theta\, d\theta$$

$$= \frac{\mu_0 I s^2}{4\pi s^3}\, j \int_{-\theta_A}^{\theta_B} \cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\, j \left\{ \sin(\theta_B) - \sin(\theta_A) \right\} = j\,\frac{\mu_0 I}{4\pi s}\left(\sin\theta_B + \sin\theta_A\right)$$

as $b - a \to \infty$, $\theta_B, \theta_A \to \pi/2$ and $\vec{B} \to \frac{\mu_0 I}{2\pi s}\, j$

Comparison of Magnetostatics and Electrostatics

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \\ \\ \nabla \times \vec{E} = 0 \end{array} \right\} \qquad \left\{ \begin{array}{l} \nabla \cdot \vec{B} = 0 \\ \\ \nabla \times \vec{B} = \mu_0 \vec{J} \end{array} \right\}$$

§5.4 Magnetic Vector Potential

$$\nabla \times \vec{E} = 0 \quad \Rightarrow \quad E = -\nabla V$$
$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \nabla \times \vec{A}$$

- Note that V is not unique, if we take $V' = V + f$ as long as $\nabla f = 0$ then $\vec{E} = -\nabla V = -\nabla V'$

You can add any function to change potential provided the potential satisfies the same $\vec{E}$ field.

Ex// In magnetic case

$$\nabla \times \vec{B} = \mu_0 \vec{J} = \nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

note that any $\vec{A}' = \vec{A} + \nabla C$ will lead to the same $\vec{B}$

Let us choose C such that $\nabla \cdot \vec{A}' = 0$

then $\nabla \times \vec{B} = \boxed{\mu_0 \vec{J} = -\nabla^2 \vec{A}'}$ 3D Poissons equation

remember $\nabla^2 V = -\rho/\epsilon_0 \Rightarrow V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{\mathscr{r}} d\tau'$

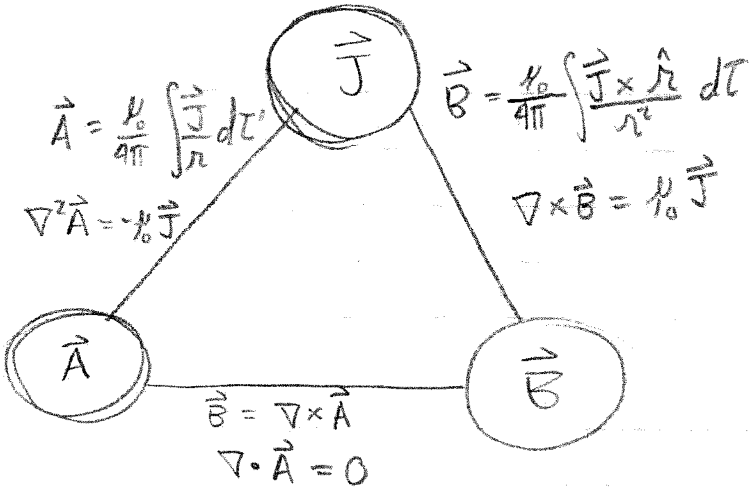
likewise,

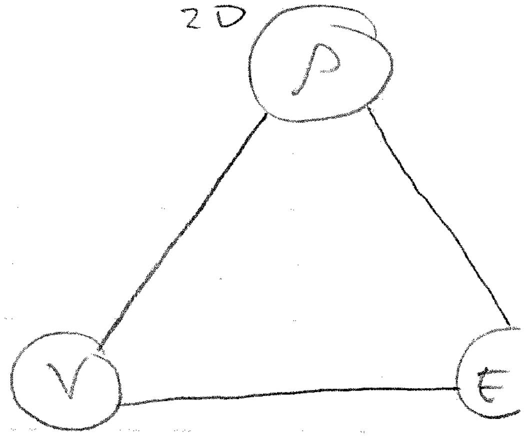
$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\mathscr{r}} d\tau'$$ the vector potential $\vec{A}$.

$\rho \longrightarrow \sigma$

$\vec{J} \longrightarrow K$

3D 2D





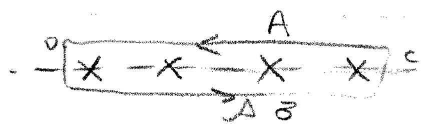
Vector Potential is a mathmatical tool not a physical reality

- In the case of surface charge

$$E_\perp^{abo} - E_\perp^{bel} = \frac{\sigma}{\epsilon_0} \quad \text{and} \quad E_\parallel^{abo} = E_\parallel^{bel}$$

- In the case of surface current

$$\oint_p \vec{B}\cdot d\vec{l} = \left(B_{abo}^{\parallel} - B_{bel}^{\parallel}\right)\cdot l = \mu_0 I_{enc}$$



$$\Rightarrow \boxed{B_{abo}^{\parallel} - B_{bel}^{\parallel} = \mu_0 K}$$

c and b go away as we shrink them to zero

From $\nabla\cdot\vec{B} = 0 \longrightarrow \oint \vec{B}\cdot d\vec{a} = 0$

shrink down so we only have top and bottom surfaces

$$\Rightarrow B_\perp^{abo}\cdot da = B_\perp^{bel}\cdot da \Rightarrow \boxed{B_{abo}^{\perp} = B_{bel}^{\perp}}$$

Combine the $\parallel$ and $\perp$ conditions : $\boxed{\vec{B}_{above} - \vec{B}_{below} = \mu_0 (\vec{K}\times\hat{n})}$

From $\nabla \cdot \vec{A} = 0 \implies A^{\perp}_{bel} = A^{\perp}_{abo}$

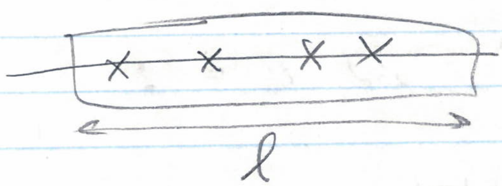
From

$$\int \vec{B} \cdot d\vec{a} = \Phi_B = \int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

let area $\rightarrow$ zero then $\Phi_B \longrightarrow 0$

we get $A^{\parallel}_{abo} = A^{\parallel}_{below}$

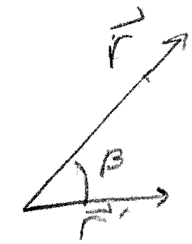
thus Combining 2 thoughts $\vec{A}_{above} = \vec{A}_{below}$



Multipole Expansion

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{1}{r} + \frac{r'}{r^2}P_1(\cos\beta) + \frac{r'^2}{r^3}P_2(\cos\beta) + \dots$$

$$= \sum_{l=0}^{\infty} \frac{r'^l}{r^{l+1}} P_l(\cos\beta)$$



$|\vec{r}\,'| < |r|$ for convience

Now Consider Magnetic Potential

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\,d\tau'}{\mathscr{r}}$$

note that $P_1(\cos\beta) = \cos\beta \doteq \hat{r}\cdot\hat{r}'$ and $P_0 = 1$

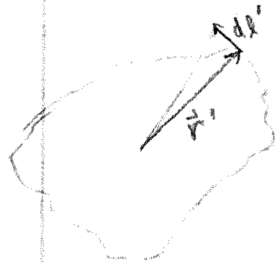
$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\int \vec{J}(\vec{r}\,')\,d\tau'}{r} + \underline{\frac{\mu_0}{4\pi}\int \frac{(\vec{r}\cdot\vec{r}\,')}{r^3}\vec{J}(\vec{r}\,')\,d\tau'} + \dots$$

hard to get but here is result.

$$\int (\vec{r}\cdot\vec{r}\,')\,\vec{J}(\vec{r}\,')\,d\tau' = -\frac{1}{2}\vec{r}\times\int \vec{r}\,'\times\vec{J}(\vec{r}\,')\,d\tau'$$

Which we can look up in J.D. Jackson's Classical Electrodynamics 2' Wiley Pg. 18

for current loop $\quad \frac{1}{2}\int \vec{r}\,'\times\vec{J}\,d\tau' = \frac{1}{2}\int \vec{r}\,'\times\vec{I}\,dl' = \frac{I}{2}\int \vec{r}\,'\times d\vec{l}\,'$

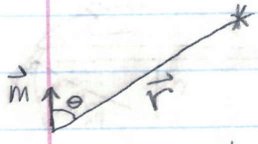


$|\vec{r}\,'\times d\vec{l}\,'|$ = area of parallelogram $\Rightarrow$ this integration gets area of loop thus

$$\frac{1}{2}\int r'\times J'\,d\tau' = I(\vec{a}\,(\text{area})) = \vec{m}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3} \text{ (due to a current loop)} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

If we place $\vec{m}$ at the origin and along $\hat{z}$



$$\vec{A} = \frac{\mu_0}{4\pi} \frac{m \sin\theta}{r^2} \hat{\phi}$$

$$k \times (i \sin\theta \cos\phi + j \sin\theta \sin\phi + k \cos\theta)$$
$$= \sin\theta (j \cos\phi - i \sin\phi) = \sin\theta \, \hat{\phi}$$

$$\vec{B}_{dip} = \nabla \times \vec{A} = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)\right]\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}(r A_\phi)\hat{\theta}$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{2\cos\theta}{r^3}\hat{r} + \frac{1}{r^3}\sin\theta\,\hat{\theta}\right] = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

which if recall the electrostatic dipole's form this suggests we call <u>m the magnetic dipole</u>

this is the same as $\vec{B}_{dip} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$

Example. Suppose we have a uniformly charged disk of density σ, radius R is rotating with ω, $\vec{m} = ?$

in solving this proble we are trying to find $\Leftrightarrow I$

Solid $I = \rho v A$, $dI = \rho v \, dA$ (Example)

disk $dI = \sigma v \, dr = \sigma \omega r \, dr$

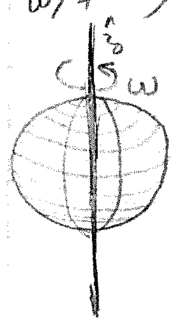
and area for that current loop is just πr^2

$$dm = dI \cdot \pi r^2 = \pi \sigma \omega r^3 dr$$

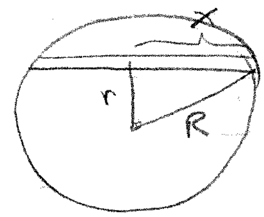
$$m = \int_0^R dm = \boxed{\frac{1}{4}\pi\sigma\omega R^4 = m}$$

$$dI = \sigma v \, dr$$
$$= \frac{dq}{dA}\frac{dx}{dt} dr$$

Suppose we have spinning uniformly charged Sphere w/ $+\rho$, R, ω then whats $\vec{m}$?



$$m_{disk} = \frac{1}{4}\pi\sigma\omega R^4 = \frac{1}{4}Q\omega R^2$$



$$dm = \frac{1}{4}dQ\cdot\omega x^2$$ where dQ is charge on disk

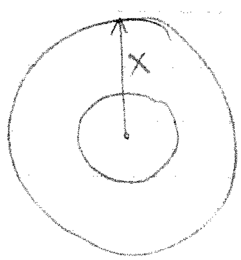
$$dQ = \rho\, d\tau = \rho\pi x^2\, dr$$

$$r^2 + x^2 = R^2$$

$$dm = \frac{1}{4}\rho\pi\omega x^4\, dr \Rightarrow M = \int dm = \int_0^R 2\cdot\frac{1}{4}\rho\pi\omega x^4\, dr$$

$$\int_0^R x^4\, dr = \int_0^R (R^2-r^2)^2\, dr = \int_0^R (R^4 - 2R^2r^2 + r^4)\, dr = R^5 - \frac{2}{3}R^5 + \frac{1}{5}R^5 = \frac{2}{5}R^5$$

$$\boxed{\vec{M} = \frac{\rho\pi\omega}{5}R^5\,\hat{z}}$$

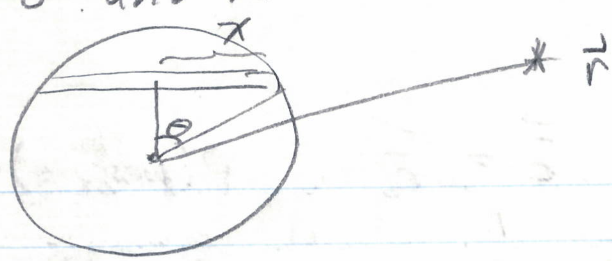


$$m \equiv I\int d\vec{a}$$

$$I = \frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \lambda v$$

$$dq\,\omega = \frac{dq\, v}{x}$$

SPHERICAL SHELL WITH σ and R
rotating with ω.



$$\vec{m} = I\vec{a}$$

$$dI = \sigma v R d\theta$$
$$dm = dI \pi x^2 = \sigma x \omega R d\theta \ \pi x^2$$
$$= \sigma \omega \pi R^4 \sin^3\theta \, d\theta$$

$$M = \int dm = \sigma \omega \pi R^4 \int_0^{\pi} \sin^3\theta \, d\theta = \sigma \pi \omega R^4 \int_0^{\pi} \sin\theta (1-\cos^2\theta) d\theta$$

$$\vec{M} = \frac{4}{3}\sigma \omega \pi R^4 \hat{z}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} = \frac{\mu_0 \sigma \omega R^4}{3r^2} \sin\theta \hat{\phi}$$

- Chapter One Unlikely
- Probably 2 problems from each chapter

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$
$$\nabla^2 \frac{1}{r} = -4\pi \delta(\vec{r})$$

Coulomb $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}} \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i \Rightarrow \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')\hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$

Gauss $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \nabla \cdot \vec{E} = \rho/\epsilon_0$

$$\vec{E} = -\nabla V$$
$$\nabla \times \vec{E} = 0$$
$$\rho = 0 \Rightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \nabla^2 V = 0$$
$$\nabla^2 V = -\rho/\epsilon_0 \Rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,') d\tau'}{\mathscr{r}}$$

$$\vec{E}_{Abo} - \vec{E}_{Bel} = \frac{\sigma}{\epsilon_0}\hat{n} \qquad V_{ABOVE} = V_{BELOW}$$

$$W = q(V_B - V_A) \Rightarrow W = \sum_{i<j} \frac{q_i q_j}{\mathscr{r}_{ij}} \longrightarrow W = \frac{\epsilon_0}{2} \int E^2 d\tau$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} \qquad P(\text{pressure}) = \frac{\epsilon_0}{2}E^2$$

Capacitors $\quad C = Q/V$

Solving Laplace's Eq. $\quad V_{center} = \frac{1}{4\pi R^2}\oint V\,da$

Method of images

SEPARATION OF VARIABLES

WRITE DOWN SOLUTION (EXCEPT CONSTANTS)

CART., CYLIND, SPHERICALS,

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^{\infty} \frac{r'^l}{r^{l+1}} P_l(\cos\beta) \qquad r > r'$$

MULTIPOLE EXPANSION

$$\vec{p} = q\vec{d} \qquad V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\vec{N} = \vec{p}\times\vec{E}$$

Dielectrics

$$\vec{P} \equiv \frac{\text{dipole moment}}{\text{unit volume}} \qquad \rho_B = -\nabla\cdot\vec{P} \qquad \vec{P}\cdot\hat{n} = \sigma_B$$

$$\vec{D} \equiv \text{Displacement} = \epsilon_0\vec{E} + \vec{P} \quad , \quad \vec{P} = \chi_e\epsilon_0\vec{E}$$

$$\vec{D} = \epsilon\vec{E} \quad \text{and} \quad \epsilon = \epsilon_r\epsilon_0 = (1+\chi_e)\epsilon_0$$

$$W = \frac{1}{2}\int(\vec{D}\cdot\vec{E})\,d\tau$$

BOUNDARY CONDITIONS

$$E^{\perp}_{abo} - E^{\perp}_{bel} = \frac{\sigma_{total}}{\epsilon_0} \quad \text{but also} \quad \oint\vec{D}\cdot d\vec{a} = Q_f \Rightarrow D^{\perp}_{abo} - D^{\perp}_{bel} = \sigma_f$$

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) + \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad , \quad P_l(1) = 1$$

$$V(x,y,z) = e^{\pm kx} \begin{matrix}\sin k'y \\ \cos k''y\end{matrix} \begin{matrix}\sin k'' \\ \cos k''\end{matrix}$$

$$V(s,\phi) = A_0 + b_0 \ln(s) + \sum_{m=1}^{\infty} s^{\pm m}\left(a_m \cos m\phi + b_m \sin m\phi\right)$$

Chapter 5

$\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$ Lorentz Force

$\vec{F}_{mag.} = Q\vec{V}\times\vec{B}$ force on current $\vec{F}_{mag} = I\int d\vec{l}\times\vec{B}$

Continuity Equation $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$: conservation of charge

$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$ Biot-Savart Law.

$\mu_0 = 4\pi\times10^{-7}\ N/A^2$ B in Tesla know ϵ_0, μ_0

Ampere's Law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$ or $\nabla\times\vec{B} = \mu_0\vec{J}$ $\vec{B} = \mu_0 n I$ (solenoid)

$\nabla\cdot\vec{B} = 0$, $\vec{B} = \nabla\times\vec{A}$, A is vector potential

A is not unique

if $\nabla\cdot A = 0 \Rightarrow \nabla^2 A = -\mu_0\vec{J}$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')}{\mathscr{r}}d\tau'$ like electric potential just a vector.

Boundary Condition

$\vec{B}_{abo} - \vec{B}_{below} = \mu_0(\vec{K}\times\hat{n})$ $\vec{A}_{ABO} = \vec{A}_{BEL}$

Magnetic dipole from current loop, $\vec{m} = I\vec{a}$(area)

$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

Eight Problem's Choose 7. Most problems are straight forward.

§6.1 Magnetization

$\vec{M}$ ~ dipole moment per unit volume

$\vec{P}$ in direction of $\vec{E}$ field applied but,

for $\vec{M}$ we have 2 types

(i) if $\vec{M} \parallel \vec{B}$ then the material is called paramagnetic

(ii) if $\vec{M} \nparallel \vec{B}$ then the material is called diamagnetic

He, Beryllium have paired orbitals, when we place magnetic field it creates $\vec{m}$ which is ________ to $\vec{B}$ applied.

Recall that $\vec{N} = \vec{P} \times \vec{E}$ similiarly

$\vec{N} = \vec{m} \times \vec{B}$ for a uniform $\vec{B}$ field

- In a uniform $\vec{B}$ field, the net force on $\vec{m}$ is zero where m is a magnetic dipole

Since

$$\vec{F}_{mag} = \oint I d\vec{l} \times \vec{B} = I \oint d\vec{l} \times \vec{B} \equiv 0$$

- If $\vec{B}$ is not uniform then $F_{mag} \neq 0$ rather,

as $U_E = -\vec{P} \cdot \vec{E}$

$U_m = -\vec{m} \cdot \vec{B}$

and so, $\vec{F} = -\nabla U = \nabla(\vec{m} \cdot \vec{B})$