$$V(s,\phi) = A_0 + b_0 \ln(s) + \sum_{m=1}^{\infty} s^{\pm m}\left(a_m \cos m\phi + b_m \sin m\phi\right)$$

Chapter 5

$\vec{F} = Q(\vec{E} + \vec{V}\times\vec{B})$ Lorentz Force

$\vec{F}_{mag.} = Q\vec{V}\times\vec{B}$ force on current $\vec{F}_{mag} = I\int d\vec{l}\times\vec{B}$

Continuity Equation $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$: conservation of charge

$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2}$ Biot - Savart Law.

$\mu_0 = 4\pi\times10^{-7}\ N/A^2$ B in Tesla know ϵ_0, μ_0

Ampere's Law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$ or $\nabla\times\vec{B} = \mu_0\vec{J}$ $\vec{B} = \mu_0 n I$ (solenoid)

$\nabla\cdot\vec{B} = 0$, $\vec{B} = \nabla\times\vec{A}$, A is vector potential

A is not unique

if $\nabla\cdot A = 0 \Rightarrow \nabla^2 A = -\mu_0\vec{J}$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')}{\mathscr{r}}d\tau'$ like electric potential just a vector.

Boundary Condition

$\vec{B}_{abo} - \vec{B}_{below} = \mu_0(\vec{K}\times\hat{n})$ $\vec{A}_{ABO} = \vec{A}_{BEL}$

Magnetic dipole from current loop, $\vec{m} = I\vec{a}$(area)

$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

Eight Problem's Choose 7. Most problems are straight forward.

2nd Semester PY 415

§6.1 Magnetization

$\vec{M}$ ~ dipole moment per unit volume

$\vec{P}$ in direction of $\vec{E}$ field applied but,

for $\vec{M}$ we have 2 types

(i) if $\vec{M} \parallel \vec{B}$ then the material is called paramagnetic

(ii) if $\vec{M} \nparallel \vec{B}$ then the material is called diamagnetic

He, Beryllium have paired orbitals, when we place magnetic field it creates $\vec{M}$ which is ______ to $\vec{B}$ applied.

Recall that $\vec{N} = \vec{P} \times \vec{E}$ similiarly

$\vec{N} = \vec{m} \times \vec{B}$ for a uniform $\vec{B}$ field

- In a uniform $\vec{B}$ field, the net force on $\vec{m}$ is zero where m is a magnetic dipole

Since

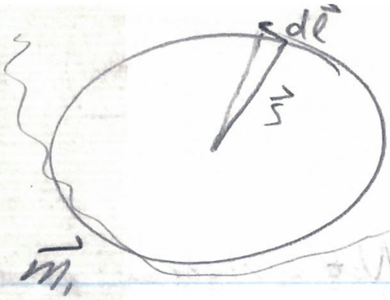
$$\vec{F}_{mag} = \oint I d\vec{l} \times \vec{B} = I \oint d\vec{l} \times \vec{B} \equiv 0$$

- If $\vec{B}$ is not uniform then $F_{mag} \neq 0$ rather,

as $U_E = -\vec{p} \cdot \vec{E}$

$U_m = -\vec{m} \cdot \vec{B}$

and so, $\vec{F} = -\nabla U = \nabla(\vec{m} \cdot \vec{B})$



$d\vec{l} \times \vec{s} = d\vec{a}$

$\oint_{\text{loop}} d\vec{l} \times \vec{s} = \text{area}$

$\boxed{\text{Ex. } 6.3}$ have to magnets $\vec{m}_1$ $\vec{m}_2$ $\longrightarrow z$

$\vec{m}_1 = m_1 \hat{z}$

$\vec{m}_2 = m_2 \hat{z}$

(a) Let $m_1 = m_2$ what is $\vec{F}$

$\underbrace{\vec{m}_1 \qquad\qquad \vec{m}_2}_{\vec{r}}$

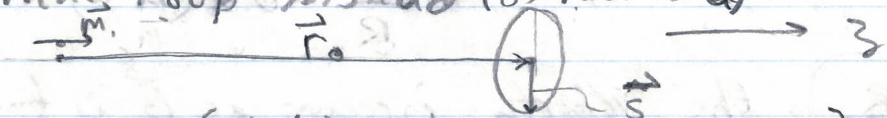
$$\vec{B}^* = \vec{B}_{\text{at 2 due to one}} = \frac{\mu_0}{4\pi r^3}\left(3(\vec{m}_1 \cdot \hat{r})\hat{r} - \vec{m}_1\right) = \frac{\mu_0}{4\pi r^3}(2\vec{m}_1)$$

$$\vec{F} = \nabla(\vec{m}_2 \cdot \vec{B}^*)$$

$$= \nabla\left(\frac{\mu_0 m_1 m_2}{2\pi r^3}\right) = \frac{\mu_0 m_1 m_2}{2\pi}\left(\frac{-3}{r^4}\hat{r}\right)$$

$$\boxed{\vec{F} = -\frac{3\mu_0 m_1 m_2 \hat{r}}{2\pi r^4}}$$

(b) if instead of a simply point dipole we have a current loop instead (of radius a)



$(r^2 = r_0^2 + s^2)$

$$\vec{B} = \frac{\mu_0}{4\pi r^5}\left\{\vec{m}_1 \cdot (\vec{r}_0 + \vec{s})(\vec{r}_0 + \vec{s}) - \vec{m}_1 r^2\right\}$$

$$= \frac{\mu_0}{4\pi r^5}\left\{m_1 r_0(\vec{r}_0 + \vec{s}) - m_1 r^2 \hat{z}\right\}$$

$$\vec{F} = I\oint d\vec{l} \times \vec{B} = \frac{\mu_0 I}{4\pi r^5}\oint d\vec{l} \times \left[3m_1 r_0(\vec{r}_0 + \vec{s}) - \underbrace{\vec{m}_1(r_0^2 + a^2)}_{\text{goes to } 0}\right]$$

$$= \frac{3m_1 r_0 \mu_0 I}{4\pi r^5}\oint d\vec{l} \times \vec{s}$$

$\boxed{(I)(\text{AREA}) = m_2}$

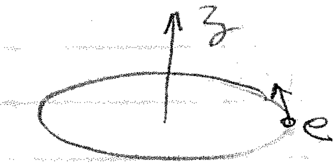
$$= -\frac{3m_1 r_0 \mu_0 I}{4\pi r^5}(2(\text{Area})) = \frac{-3\mu_0 m_1 m_2 \hat{r}}{2\pi r^4} \quad \text{as } r_0 \approx r$$

is we have a small loop it is quite reasonable to consider it to be a point dipole

$\boxed{6.1,\ 6.5}$

Diamagetism (Not a rigorous proof) Consider e^- in orbit about Nucleus,

$$I = \frac{e}{T} = \frac{ev}{2\pi R}$$



$$m = Ia = \frac{ev}{2\pi R}\pi R^2 = \frac{evR}{2}$$

$$\vec{m} = \frac{-evR}{2}\hat{z}$$

$$\Delta\vec{m} = \frac{-e\Delta v R}{2}\hat{z}$$

the centripetal force

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R} \quad \text{(if no } \vec{B} \text{ field)}$$

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + ev'B = m_e\frac{v'^2}{R} \quad \text{(with } \vec{B} \text{ and thus Lorentz force)}$$

$$ev'B = m_e\left(\frac{v'^2 - v^2}{R}\right) = \frac{m_e}{R}(v'+v)(v'-v)$$

$$\cong \frac{m_e}{R}\cdot 2v(\Delta v) \quad \text{as } v \approx v'$$

$\boxed{\Delta v = \frac{eBR}{2m_e}}$ the velocity increases as $\vec{B}$ is turned on

$$\Delta\vec{m} = -\frac{eR}{2}\frac{eBR}{2m_e}\hat{z} = \boxed{-\frac{e^2R^2}{4m_e}\vec{B} = \Delta\vec{m}}$$

DIAMAGNETICS

$\boxed{6,5}$

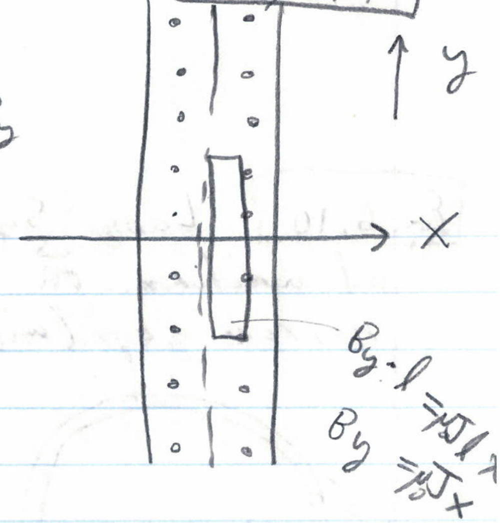
$\boxed{6,8,\ 6,9}$

as current is in $\hat{z}$ no B_z

from ∞ plane symetry each x has same B_y

$\nabla \cdot \vec{B} = 0 \Rightarrow B_x = 0$

$\Rightarrow B_y(x)$



§6.2 The Field of Magnetised Object

From $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\imath}}{\imath^2}$ (for large # of $\vec{m}$) $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times \hat{\imath}}{\imath^2} d\tau'$

$$\nabla \frac{1}{\imath} = -\frac{\hat{\imath}}{\imath^2} = -\nabla' \frac{1}{\imath} \quad \Rightarrow \quad \vec{A} = \frac{\mu_0}{4\pi} \int \left(\vec{M} \times \nabla' \frac{1}{\imath} \right) d\tau'$$

$$\nabla' \times \left(\frac{\vec{M}}{\imath} \right) = \frac{1}{\imath} \nabla' \times \vec{M} - \vec{M} \times \nabla' \frac{1}{\imath}$$

so exchanging ' for 2 terms

$$\vec{A} = \frac{\mu_0}{4\pi} \left[\int \left(\frac{1}{\imath} \nabla' \times \vec{M} \right) d\tau' - \underline{\int \left(\nabla' \times \frac{\vec{M}}{\imath} \right) d\tau'} \right.$$

note $\nabla \cdot (\vec{C} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{C}) - \vec{C} \cdot (\nabla \times \vec{B})$

let $\vec{C}$ be a constant then $\nabla \times \vec{C} = 0$ thus

$$\nabla \cdot (\vec{C} \times \vec{B}) = -\vec{C} \cdot (\nabla \times \vec{B})$$

$$\int \nabla \cdot (\vec{C} \times \vec{B}) d\tau = \oint (\vec{C} \times \vec{B}) \cdot d\vec{a} = \oint (B \times d\vec{a}) \cdot \vec{C}$$

$$-C \cdot \int (\nabla \times \vec{B}) d\tau = \oint (B \times d\vec{a}) \cdot \vec{C} \quad \Rightarrow \quad \boxed{\int \nabla \times \vec{B} \, d\tau = -\oint (\vec{B} \times d\vec{a})}$$

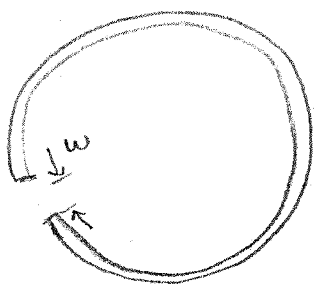
$$\vec{A} = \frac{\mu_0}{4\pi} \left[\int \frac{1}{\imath} (\nabla' \times \vec{M}) d\tau' + \oint \frac{\vec{M}}{\imath} \times d\vec{a}' \right]$$

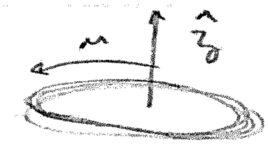
Define $\vec{J}_b = \nabla \times \vec{M}$ and $\vec{K}_b = \vec{M} \times \hat{n}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J_b(\vec{r}') d\tau'}{\imath} + \frac{\mu_0}{4\pi} \oint \frac{\vec{K}_b(\vec{r}')}{\imath} da'$$

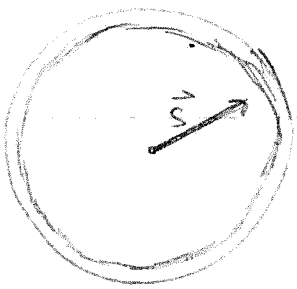
$\boxed{\text{Ex. 6.10}}$ Long Square cross section bar (side length a) of uniform $\vec{M}$ is bent into a circle with a very narrow gap (width w). Find $\vec{B}$ at the center of gap

(fill in gap use amp. law to find $\vec{B}$ then remove $\vec{B}$ from tiny gap (width w))





If complete circle



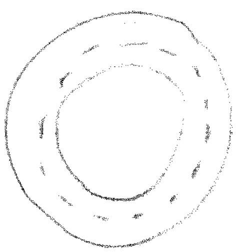
$\vec{M} = M\hat{\phi}$

$K_b^{s,\,out} = \vec{M}\times\hat{n} = M\hat{\phi}\times\hat{s} = -M\hat{z}$

$K_b^{s,\,in} = \vec{M}\times\hat{n} = M\hat{\phi}\times -\hat{s} = M\hat{z}$

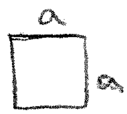
$K_b^{up} = \vec{M}\times\hat{n} = M\hat{\phi}\times\hat{z} = M\hat{s}$

$K_b^{down} = \vec{M}\times\hat{n} = -M\hat{s}$



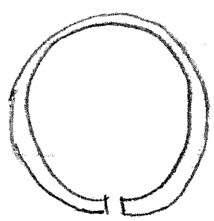
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = B_\phi \cdot 2\pi R = \mu_0 K_B 2\pi R$$

$$\vec{B} = \mu_0 M\hat{\phi} = \mu_0\vec{M}$$



Total current = Mw

$\boxed{5-8} \Rightarrow B_{\text{center of } \square^a} = \dfrac{\sqrt{2}\,\mu_0 I}{\pi\, a/2}$



$\vec{B} = \mu_0 M - \dfrac{\sqrt{2}\, M w \mu_0}{\pi\, a/2}$ in the same direction as $\vec{M}$

§6.3 The $\vec{H}$ field — "Why do you have to live in you're metal world"

$\boxed{\begin{matrix} 6.12 \\ 6.16 \end{matrix}}$

From $\nabla \times \vec{B} = \mu_0 \vec{J} = \mu_0 \left(\vec{J}_f + \vec{J}_B \right)$

$\vec{J}_B = \nabla \times \vec{M}$

$$\Rightarrow \nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = J_f \Rightarrow \boxed{\frac{\vec{B}}{\mu_0} - \vec{M} \equiv \vec{H}}$$ would be a good def$^{\underline{n}}$ to make

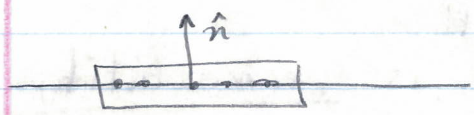
$$\nabla \times \vec{H} = \vec{J}_f$$

$$\int (\nabla \times \vec{H}) \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l} = \int \vec{J}_f \cdot d\vec{a} = I_{f enc}$$

Boundary Conditions

$\nabla \cdot \vec{B} = 0$  $\Rightarrow B^{\perp}_{abo} - B^{\perp}_{bel} = 0$

$\vec{B} = (\vec{M} + \vec{H}) \mu_0 \Rightarrow H^{\perp}_{abo} - H^{\perp}_{bel} = -\left(M^{\perp}_{abo} - M^{\perp}_{bel} \right)$

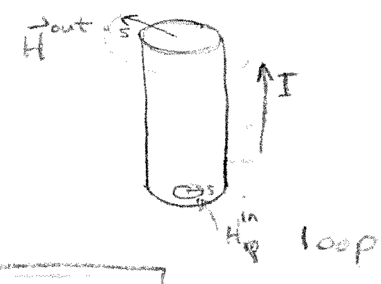
 $\Rightarrow \left(H^{\parallel}_{abo} - H^{\parallel}_{bel} \right) \cdot \Delta l = I_f$

$I_{f enc}$ from loop $\Rightarrow H^{\parallel}_{abo} - H^{\parallel}_{bel} = K_f$

$I_{f enc} = \oint \vec{H} \cdot d\vec{l}$ $H \perp$ to ~~$\hat{n}$ and $\vec{K}_f$~~

thus $\boxed{\vec{H}^{\parallel}_{abo} - \vec{H}^{\parallel}_{bel} = \vec{K}_f \times \hat{n}}$

$\boxed{Ex}$ A long Cylinder carries uniform current I. Find $\vec{H}$ inside and outside?



$$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

$$H_\varphi^{out} \cdot 2\pi s = I_f \Rightarrow \boxed{H_\varphi^{out} = \frac{I_f}{2\pi s}}$$

$$H_\varphi^{in} \cdot 2\pi s = \int \vec{J}_f \cdot d\vec{a} = \frac{I_f}{\pi R^2} \cdot \pi s^2 \Rightarrow \boxed{H_\varphi^{in} = \frac{I_f\, s}{2\pi R^2}}$$

$$\left\{ \frac{\vec{B}}{\mu_0} - \vec{M} = \vec{H} \right\} \Rightarrow \begin{array}{l} \vec{B}_{out} = \mu_0 \vec{H} + \vec{M}\mu_0 = \mu_0 \vec{H} = \frac{\mu_0 I_f}{2\pi s} \\ \vec{B}_{in} = ???\ \text{without more about } \vec{M}. \end{array}$$

§6.4 Linear and Non-Linear Media

LINEAR

Customarily $\vec{M} = \chi_m \vec{H}$

χ_m is magnetic susceptibility > 0 paramagnetic
< 0 diamagnetic

In linear media $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H}$

thus define $\boxed{\mu \equiv \mu_0(1 + \chi_m)}$ μ is <u>permeability</u>

which allow us to write $\boxed{\vec{B} = \mu \vec{H}}$

In Vacuo $\mu = \mu_0(1 + \underbrace{\chi_m}_{0}) = \mu_0$ = permeability of vacuum

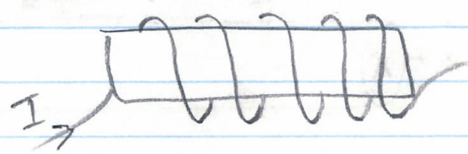
Although $\nabla \cdot \vec{B} = 0$ we do not necessarily have $\nabla \cdot \vec{H} = 0$

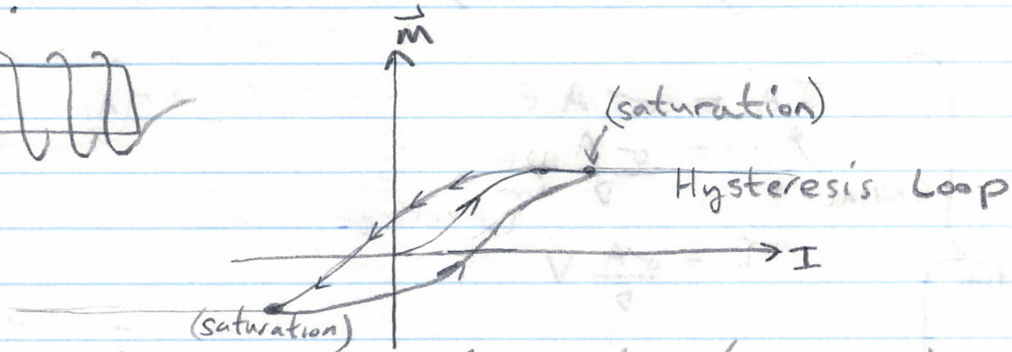
$\nabla \cdot \vec{H} \neq 0$ because $\vec{M}$ changes at boundary

$$\vec{J}_B = \nabla \times \vec{M} = \nabla \times (\chi_m \vec{H}) = \chi_m (\nabla \times \vec{H}) = \chi_m \vec{J}_f \quad \text{(linear materials)}$$

Ferromagnetism (Fe, Ni)

each piece has random $\vec{M}$, when we apply $\vec{B}$ the molecule is favored to happen the boundary, unfavored domains get eaten up by favored domain as $\vec{B}$ is increased, as we decrease $\vec{B}$ field we don't go back to original situation although we do lose a little of favored $\vec{M}$.





now in a linear material we would expect the material to follow same curve back as I decreased. The history of a Ferromagnetic material matters that is One value of $\vec{I}$ maps to many $\vec{I}$

Chapter Seven – Electrodynamics

HWK. 7.1, 7.3

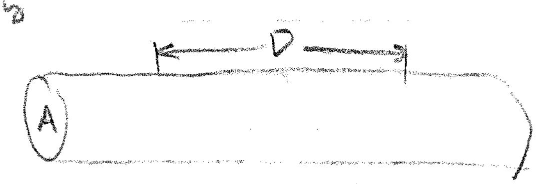
§ 7.1 Electomotive Force

- Ohm's law $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$ where $\boxed{\sigma \equiv \text{conductivity}}$

  for all pratical purposes $\vec{v} \times \vec{B} \longrightarrow$ zero

  thus $\vec{J} = \sigma \vec{E}$

- $\sigma \equiv \frac{1}{\rho}$ where $\rho \equiv$ resistivity



ohm's law derivation

$$JA = \sigma A E$$

current

$$= \frac{\sigma A}{D} \underbrace{DE}_{\text{voltage}}$$

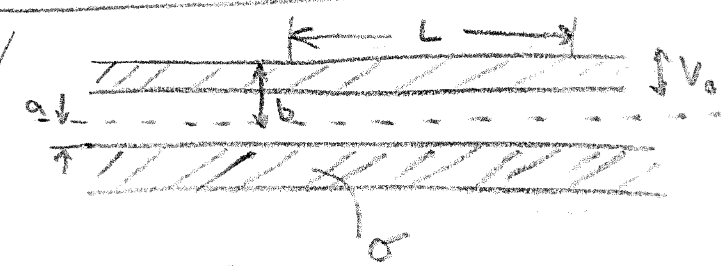
$$I = \frac{\sigma A}{D} V$$

Now Def^{n} $\frac{\rho D}{A} \equiv R \quad \therefore \quad I = \frac{A}{\rho D} V = \frac{V}{R} \Rightarrow V = IR.$

$$dw = dq\, V$$

$$\frac{dw}{dt} = \frac{dq}{dt} V = IV \quad \Longrightarrow \quad P = IV = I^2 R$$

Ex//



two long cylindrical surfaces with $\Delta V = V_0$

what is current in L $\quad I = \int \vec{J} \cdot d\vec{a} = \int J_s \cdot dl \cdot s\, d\phi$

$$I = \sigma \int E_s\, dl \cdot s\, d\phi \quad \text{note} \quad E_s = \frac{\lambda}{2\pi\epsilon_0 s} \Rightarrow I = \frac{\sigma\lambda}{2\pi\epsilon_0} \int dl\, d\phi = \frac{\sigma\lambda}{2\pi\epsilon_0} 2\pi L$$

$$I = \frac{\sigma\lambda}{\epsilon_0} L. \quad \text{Also} \quad V_0 = -\int_b^a \vec{E} \cdot d\vec{s} = \int_a^b E_s\, ds = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$\frac{\lambda}{\epsilon_0} = \frac{I}{\sigma L} \Rightarrow V_0 = \frac{1}{2\pi} \frac{I}{\sigma L} \ln\left(\frac{b}{a}\right) \Rightarrow \boxed{R = \frac{1}{2\pi\sigma L} \ln\left(\frac{b}{a}\right)} \quad \text{and} \quad I = \frac{2\pi\sigma L V_0}{\ln(b/a)}$$