Chapter 6 $\longrightarrow$ Chapter 12 Review 4 Final

$\vec{M}$ $\quad \vec{m} = \vec{M} \cdot V$ Magnetic Dipole's

$\vec{N} = \vec{m} \times \vec{B}$ $\quad U = -\vec{m} \cdot \vec{B}$ Torque and PE for magnetic dipole

$\vec{F} = -\nabla(\vec{m} \cdot \vec{B})$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times \hat{r}}{r^2} d\tau = \text{after some calc...}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b}{r} d\tau' + \frac{\mu_0}{4\pi} \oint \frac{\vec{K}_b}{r} da'$$

Bound volume current $\qquad$ Bounded surface current

$$\vec{J}_B = \nabla \times \vec{M} \qquad \vec{K}_B = \vec{M} \times \hat{n}$$

Magnetic Field Intensity $\quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

$$\nabla \times \vec{H} = \vec{J}_f \quad \left(\text{if } \frac{\partial \vec{D}}{\partial t} = 0\right)$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad \vec{M} = \chi_m \vec{H}$$

$$\chi_m = \left\{ \begin{array}{ll} \chi_m > 0 & \text{paramagnetics} \\ \chi_m < 0 & \text{diamagnetic} \end{array} \right\} = \text{Magnetic Susecpitibility}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu \vec{H}$$

$\nabla \cdot \vec{B} = 0$, why not $\nabla \cdot \vec{H} = 0$, because μ discontinuous at boundary

$$\vec{J}_B = \nabla \times \vec{M} = \nabla \times (\chi_m \vec{H}) = \chi_m \nabla \times \vec{H} = \chi_m \vec{J}_f$$

$\boxed{\vec{J}_B = \chi_m \vec{J}_f}$ for Linear Media !

Ferromagnetism $\vec{M} \neq \chi_m \vec{H}$ (hysteresis loop)

$\boxed{\text{Chapter 7}}$

$$\vec{J} = \sigma \vec{E} \Longrightarrow V = IR$$

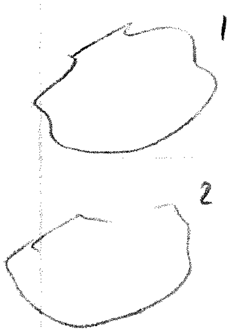
$$R = \frac{\rho D}{A} = \frac{\rho\ \text{lenth}}{\text{cross-section}}$$

$$P = IV = I^2 R = \frac{V^2}{R} = \frac{dU}{dt} = \frac{dU}{dq}\frac{dq}{dt} = V \cdot I \ !$$

(RC - Circuit, $Q = CV_0(1 - e^{-t/RC})$ $\quad Q = Q_0 e^{-t/RC}$)

$\mathcal{E} = \frac{-d\Phi}{dt}$ — changing the size of loop / moving magnetic flux / moving loop

☆ $\mathcal{E} = -\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi}{dt} \Longrightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

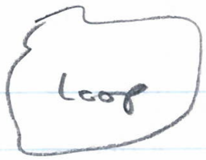


<u>Mutual Inductance</u>

$$M = \frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{r} \qquad \text{(Nueman Formula)}$$

$\Phi = MI$ (no matter which one we talk about $\Phi_1 = MI_2$, $\Phi_2 = MI_1$)

Now $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$

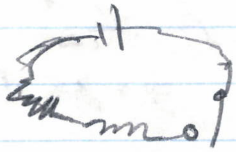


$$\Phi = L I$$

L ← self-inductance.

For Solenoid: $\frac{L}{\ell} = \mu_0 n^2 A$

RL Circuit: $I = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right)$ $\qquad \varepsilon = -L\frac{dI}{dt}$



have switch then close at $t=0$ . . .

$I(t=\infty) \longrightarrow \varepsilon_0/R$ ☺

Energy in Magnetic Field:

$$W_m = \frac{1}{2\mu_0}\int \vec{B}^2\, d\tau \qquad W_e = \frac{\epsilon_0}{2}\int \vec{E}^2\, d\tau$$

Maxwell's Eq's

$$\nabla\cdot\vec{E} = \rho/\epsilon_0$$
$$\nabla\cdot\vec{B} = 0$$
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{F} = q\left(\vec{E} + (\vec{v}\times\vec{B})\right)$$

$$\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

Maxwell's Eq's in Matter,

$$\vec{D} = \epsilon\vec{E} \qquad \vec{H} = \frac{1}{\mu}\vec{B}$$

$$\nabla\cdot\vec{D} = \rho_f$$
$$\nabla\cdot\vec{B} = 0$$
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

Boundary Condition's

(i) $D^{\perp}_{abo} - D^{\perp}_{bel} = \sigma_f$

(ii) $B^{\perp}_{abo} - B^{\perp}_{bel} = 0$

(iii) $E^{\parallel}_{abo} = E^{\parallel}_{bel}$

(iv) $H^{\parallel}_{abo} - H^{\parallel}_{bel} = \vec{K}_f \times \hat{n}$

Derive every one of these guys!

Chapter 8

Conservation of Energy:

$$\frac{dW_{mech}}{dt} + \frac{dU_{em}}{dt} = \oint \vec{S} \cdot d\vec{a} \quad \leftarrow \text{energy's}$$

$$\frac{\partial u_{mech}}{\partial t} + \frac{\partial u_{em}}{\partial t} = -\nabla \cdot \vec{S} \quad \leftarrow \text{energy densitys}$$

Poynting vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$ $\qquad u_{em} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$

Maxwell's Stress Tensor

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right)$$

$$\vec{F} = \oint \overleftrightarrow{T} \cdot d\vec{a} - \epsilon_0 \mu_0 \frac{d}{dt}\int \vec{S} \cdot d\tau = \frac{d\vec{p}}{dt}$$

must be some sort of momentum density.

$\vec{P}_{em} = \mu_0 \epsilon_0 \vec{S}$ ⟵ momentum density

$$\frac{\partial \vec{P}_{mech}}{\partial t} + \frac{\partial \vec{P}_{em}}{\partial t} = \nabla \cdot \overleftrightarrow{T}$$

compare with $\frac{\partial \rho}{\partial t} = - \nabla \cdot \vec{J}$

thus $- \nabla \cdot \overleftrightarrow{T}$ is divergence of momentum density

$- \overleftrightarrow{T}$: flow of momentum density

Linear Mom. Density $\vec{P}_{em} = \mu_0 \epsilon_0 \vec{S} = \epsilon_0 \vec{E} \times \vec{B}$

Angular Mom. Density $\vec{l}_{em} = \vec{r} \times \vec{P} = \epsilon_0 \vec{r} \times (\vec{E} \times \vec{B})$

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Chapter 9

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$ eq's like $g(z-vt) + h(z+vt)$ will satisfy wave eq.

Sinusoidal wave, $f = A\cos[k(z-vt)+\delta]$

more conviently $\tilde{f} = \tilde{A}e^{i(kz-\omega t)}$

Basic Relations

$$k = \frac{2\pi}{\lambda} \qquad kv = \omega = \frac{2\pi}{T} = 2\pi\nu$$

Superposition of All these waves,

$$\tilde{f}(z,t) = \int \tilde{A}(k)e^{i(kz-\omega t)}\,dk$$

At interface we gets some boundary conditions,

$$\tilde{f}(0^-,t) = \tilde{f}(0^+,t)$$

$$\left.\frac{\partial f}{\partial z}\right|_{0^-} = \left.\frac{\partial f}{\partial z}\right|_{0^+}$$

$$\tilde{\vec{E}} = \tilde{\vec{E}}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{k}\cdot\tilde{\vec{E}}_0 = 0 \qquad \vec{B} = \frac{\vec{k} \times \tilde{\vec{E}}_0}{\omega} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$v = \frac{1}{\sqrt{\epsilon\mu}} \cong \frac{c}{\sqrt{\epsilon_r}} \qquad v = \frac{c}{n} \qquad n = \sqrt{\epsilon_r}$$

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) \Rightarrow \epsilon_0 E^2 \qquad u_e = u_B$$

$$S = \frac{1}{\mu}(\vec{E} \times \vec{B})$$

$$S = \frac{1}{\mu}\frac{E^2}{v}$$

$$\langle S \rangle_{time} = \frac{1}{2}\epsilon v E_0^2$$

$$\tilde{E}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{E}_{0I} \qquad \tilde{E}_{0T} = \frac{2 v_2}{v_1 + v_2}\tilde{E}_{0I}$$

$$\left.\begin{aligned} R &= \text{reflection coefficient} = \frac{I_R}{I_{in}} \\ T &= \text{transmission coefficient} = \frac{I_T}{I_{in}} \end{aligned}\right\} \underline{R + T = 1}$$

Oblique ~~Inc~~ incidence : ① k_i, k_R, k_T on same plane
② $\theta_i = \theta_R$
③ $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\beta = \frac{\mu_1 V_1}{\mu_2 V_2} \qquad \alpha = \frac{\cos\Theta_T}{\cos\Theta_I}$$

For polarization on k_i, R, T

$$E_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)E_{0I} \qquad E_{0T} = \frac{2}{\alpha+\beta}E_{0I}$$

$$E_{0R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)E_{0I} \qquad E_{0T} = \frac{1}{1+\alpha\beta}E_{0I}$$

E/M Waves in Metal

$$\nabla^2 \vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}(z,t) = \tilde{E}_0 e^{i(\tilde{k}z-\omega t)}$$

$$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega \qquad \tilde{k} = k + i\kappa$$

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left(1+\sqrt{1+\sigma^2/(\epsilon\omega)^2}\right)^{1/2}$$

$$\kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left(\sqrt{1+\sigma^2/(\epsilon\omega)^2}-1\right)^{1/2}$$

B field lags behind electric field
good cond. has lag of 45°

$d = \frac{1}{\kappa}$ length to drop strength e

In dielectric with absorption;

$$\tilde{\epsilon}_r = 1 + \frac{Nq^2}{n\epsilon_0}\sum_i \frac{f_i}{\omega_j^2 - \omega^2 - i\gamma_j\omega} \qquad n = \frac{ck}{\omega}$$

f_i = # of e^- with eigen f. ω_i

$$k \cong \frac{\omega}{c}\left(1 + \frac{Nq^2}{2m\epsilon_0}\sum \frac{f_i}{\omega_j^2-\omega^2-i\gamma_j\omega}\right)$$

absorption coefficient $\alpha = 2\kappa = \frac{Nq^2\omega^2}{m\epsilon_0 c}\sum \frac{f_i\gamma_i}{(\omega_j^2-\omega^2)^2+\gamma_j^2\omega^2}$

anomalous dispersion $(\omega \cong \omega_j$

$$\left.\begin{aligned}\vec{\tilde{E}} &= \vec{\tilde{E}}_0(x,y)e^{i(kz-\omega t)} \\ \vec{\tilde{B}} &= \vec{\tilde{B}}_0(x,y)e^{i(kz-\omega t)}\end{aligned}\right\} \text{———} \quad k \neq \omega/c$$

$$E_z, B_z \Rightarrow \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega}{c}\right)^2 - k^2\right]\frac{B_z}{E_z} = 0$$

(Max. Eq's $\Uparrow$)

$E^{\parallel} = 0,\ B^{\perp} = 0$: good conductor

$E_z = 0 \Rightarrow$ a transverse electric wave, TE

$B_z = 0 \Rightarrow$ a transverse magnetic wave, TM

both TEM.

$(a, b, a > b)$

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)}$$

cut-off frequency $\omega_{mn} = \left[\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right]\pi c$

For TE lowest $\omega_{10} = \frac{\pi c}{a}$

For TM lowest $\omega_{11} =$

One Conductor : No TEM possible and $k \neq \omega/c$
2 Conductor : TEM possible but $k = \omega/c$ possible
if not req

$\boxed{\text{Chapter 10}}$

given $\rho(\vec{r},t)$, $\vec{J}(\vec{r},t)$ $\vec{E} = ?$, $\vec{B} = ?$

$V(\vec{r},t)$, $\vec{A}(\vec{r},t)$

$$\boxed{\begin{aligned}\vec{E} &= \nabla V - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \nabla \times \vec{A}\end{aligned}}$$

Use Maxwell's equation's to find what V and A must satisfy.

Lorentz Gauge : $\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$

Coulomb Guage : $\nabla \cdot \vec{A} = 0$

In Lorentz Gauge $\Box^2 \vec{A} = \mu_0 \vec{J}$

$\Box^2 V = -\rho/\epsilon_0$

Retarded Potentials

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r},t)}{\mathscr{r}} d\tau' \qquad t_r = t - \mathscr{r}/c$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r},t)}{\mathscr{r}} d\tau'$$

Leinard Weichart Potentials

$$V = \frac{1}{4\pi\epsilon_0} \frac{qc}{\mathscr{r}c - \vec{V} \cdot \vec{\mathscr{r}}} \Bigg|_{t=t_r}$$

$$\vec{A} = \frac{\vec{v}}{c^2} V_{LW}$$

By "simple" Differentiation can find $\vec{E}$ and $\vec{B}$ from V and $\vec{A}$.

$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\vec{\mathscr{r}} \cdot \vec{u})^3} \Big[\underset{\substack{\uparrow \\ \text{velocity field}}}{(c^2 - v^2)\vec{u}} + \underbrace{\vec{\mathscr{r}} \times (\vec{u} \times \vec{a})}_{\substack{\uparrow \\ \text{radiation field}}} \Big]$$

$$\vec{B}(\vec{r},t) = \frac{1}{c} \hat{\mathscr{r}} \times \vec{E}(\vec{r},t)$$

If $v \ll c$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 \mathscr{r}} \Bigg(\underset{\substack{\uparrow \\ \text{Coulomb field}}}{\frac{\hat{\mathscr{r}}}{\mathscr{r}}} + \underset{\substack{\uparrow \\ \text{radiation field}}}{\frac{\hat{\mathscr{r}}(\hat{\mathscr{r}} \times \vec{a})}{c^2}} \Bigg)$$

Chapter 11 / only 1st Sec

Chapter 12 /
- Lorentz Transformation
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- Defn of Momentum, Energy