Charge is a fundamental quantity. Basic constituents of matter are all assigned particular charges. For example,

e = electron, is negatively charged

P = proton, is positively charged

N = nuetron, has zero charge

In modern physics $\exists$ particles with fractional charge but these are never observed directly. In practice directly observed charges come in integer multiples of e,

$$\boxed{Q_{observed} = ne \quad \text{for some } n \in \mathbb{Z}.}$$

The standard unit for charge is the COULOMB which we denote by C. It is ginormous,

$$C = 6.25 \times 10^{18}\, e$$

$$e = 1.60 \times 10^{-19}\, C$$

This means you need 6.25×10^18 electrons to obtain $Q = -1\,C$.

COULOMB (and others) observed that materials will attract or repel each other when rubbed with particular cloths and from this they deduced the existence of a new force which we call the ELECTRIC FORCE. COULOMB did experiments with a torsion balance to find the mathematical form of the new force. Interestingly it turns out to follow the same mathematical form as Newton's Universal Law of Gravitation. We'll return to the mathematics, but 1st let me settle a few qualitative issues.

I say <u>qualitative</u> not because $\nexists$ mathematics to describe the physics to follow, rather because this <u>course</u> does not provide such detail.

- <u>CONSERVATION OF CHARGE</u>: isolated system has a constant amount of total charge.

- A <u>CONDUCTOR</u> is a material which allows for the free flow of charge. (ideally)

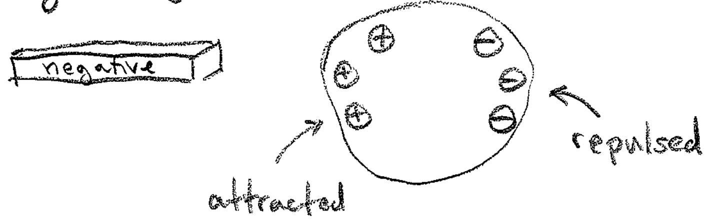
- An <u>INSULATOR</u> is a material which holds charges in place. In other words, it blocks the flow of charge.

- A "GROUND" is an essentially infinite source of charge. Typically denoted by ⏚

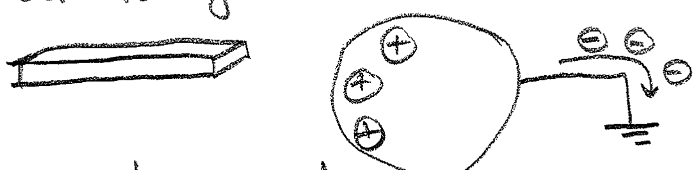
<u>Induction</u> is a process by which we may <u>induce</u> a net charge on a conductor. Here's how:

1.) nuetral conductor minding its business.

2.) bring negative rod near it

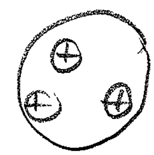


3.) connect to ground to drain off ⊖ charges



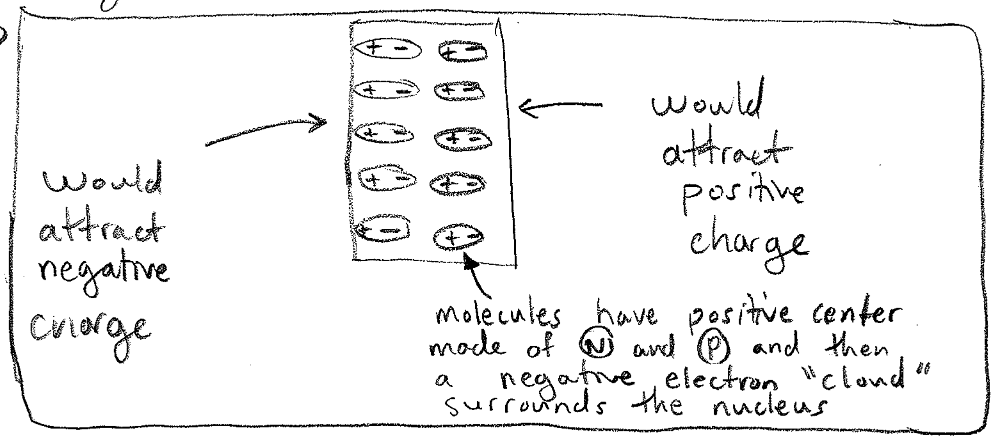
4.) disconnect ground.

5.) BEHOLD, the positively charged conductor



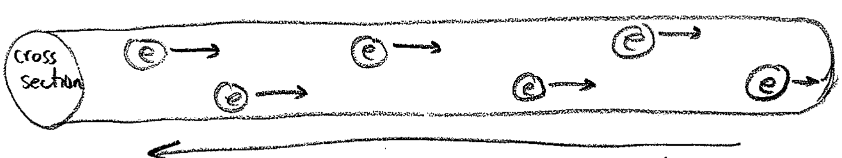
It is also possible to place charge on insulators.

<u>POLARIZATION</u>: in an insulator it may be the case that the constituent molecules line up as pictured below. The interesting feature is that despite the total charge being zero the object can still attract or repel charged objects.



<u>SEMICONDUCTORS</u>: behave as both conductors and insulators depending on the conditions in which they are placed. This means they can build switches. The physics of semiconductors is typically introduced in modern physics and explored in the solid state course. We need more student demand to start the physics minor. So, if you're interested speak up and be heard.

<u>Remark</u>: <u>Current</u> is the rate of flow of charge through some particular cross-section (wire usually). It is usually the case that e's move, <u>BUT</u>,



$$\text{Current } I = \frac{dQ}{dt}$$

the current, based on <u>positive</u> charge, goes the opposite direction. This is Benjamin Franklin's fault, he just made a choice which sets $Q_e < 0$. One could write physics where $Q_e > 0$, but we don't.

Coulomb's Law & Electric Fields.

Coulomb's experiments revealed that if one assigned charges to objects then the electric force between a point mass with charge q_1 at $\vec{r}_1$ and a point mass with charge q_2 at $\vec{r}_2$ was inversely proportional to the square of the distance between them and it was directed along $\vec{r}_2 - \vec{r}_1 = \vec{\mathscr{r}}$ such that it attracts for $q_1 q_2 < 0$ and repels for $q_1 q_2 > 0$. In particular, the <u>force on q_1 due to q_2</u> is given by (in vacuum, but to a good approx. air $\approx$ vacuum)

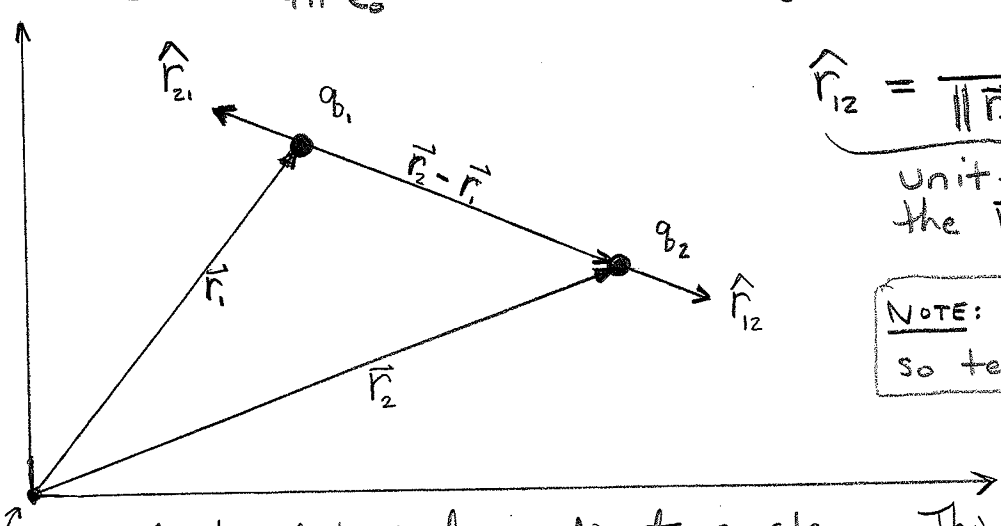
$$\boxed{\vec{F}_{21} = -\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{\|\vec{r}_2 - \vec{r}_1\|^2}\right)\hat{r}_{12}}$$

note, $\|\vec{A}\|^2 = \vec{A}\cdot\vec{A}$.

Likewise, the <u>force on q_2 due to q_1</u> is

$$\boxed{\vec{F}_{12} = -\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_2}{\|\vec{r}_1 - \vec{r}_2\|^2}\right)\hat{r}_{21}}$$

Where $\epsilon_0 = 8.8542 \times 10^{-12}\ C^2/Nm^2$ is the <u>permittivity</u> of free space. For most calculations you'd rather write $k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{Nm^2}{C^2}$ (called Coulomb's Constant)

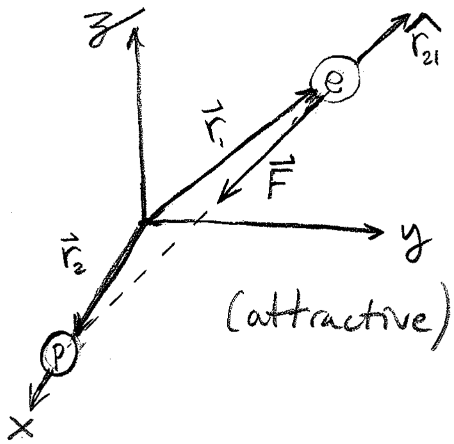


$$\underbrace{\hat{r}_{12} = \frac{1}{\|\vec{r}_2 - \vec{r}_1\|}(\vec{r}_2 - \vec{r}_1)}_{\text{unit-vector in the } \vec{r}_2 - \vec{r}_1 \text{ direction.}}$$

Note: $\hat{r}_{12} = -\hat{r}_{21}$ so text's Eq$^{\underline{n}}$ is same.

origin of the intertial coordinate system. Think back to Newton's Laws. That's our back drop here.

$\boxed{E1}$ find electric force on electron at $\vec{r}_2 = (0,1,1)\,m$ from a proton at $\vec{r}_1 = (1,0,0)\,m$.



$$\vec{r}_1 - \vec{r}_2 = \langle 1,-1,-1 \rangle m$$

$$\| \vec{r}_1 - \vec{r}_2 \|^2 = m^2\sqrt{1^2+1^2+1^2} = \sqrt{3}\, m^2$$

$$\underbrace{\hat{r}_{21} = \frac{1}{\sqrt{3}} \langle 1,-1,-1 \rangle}$$

note, unit vectors have no units in the sense of m, kg, s, C etc...

$$\vec{F}_e = \frac{-1}{4\pi\epsilon_0}\left(\frac{(e)(-e)}{\sqrt{3}\, m^2}\right)\frac{1}{\sqrt{3}}\langle 1,-1,-1 \rangle$$

$$= \left(8.99\times 10^9 \frac{Nm^2}{C^2}\right)\left(\frac{(1.6\times 10^{-19})^2}{3}\frac{C^2}{m^2}\right)\langle 1,-1,-1 \rangle$$

$$= \underbrace{(4.43\times 10^{-29} N)}_{\text{Magnitude}} \underbrace{\langle 1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3} \rangle}_{\text{direction}}.$$

We can contrast $\vec{F}_e$ to the gravitational force exerted on $m_e = 9.10938\times 10^{-31}\,kg$ by $m_p = 1.67262\times 10^{-27}\,kg$

$$F_g = \frac{G m_e m_p}{r^2} = \left(6.673\times 10^{-11}\frac{Nm^2}{kg^2}\right)\left(\frac{9.11\times 1.67\times 10^{-58}\,kg^2}{\sqrt{3}\, m^2}\right)$$

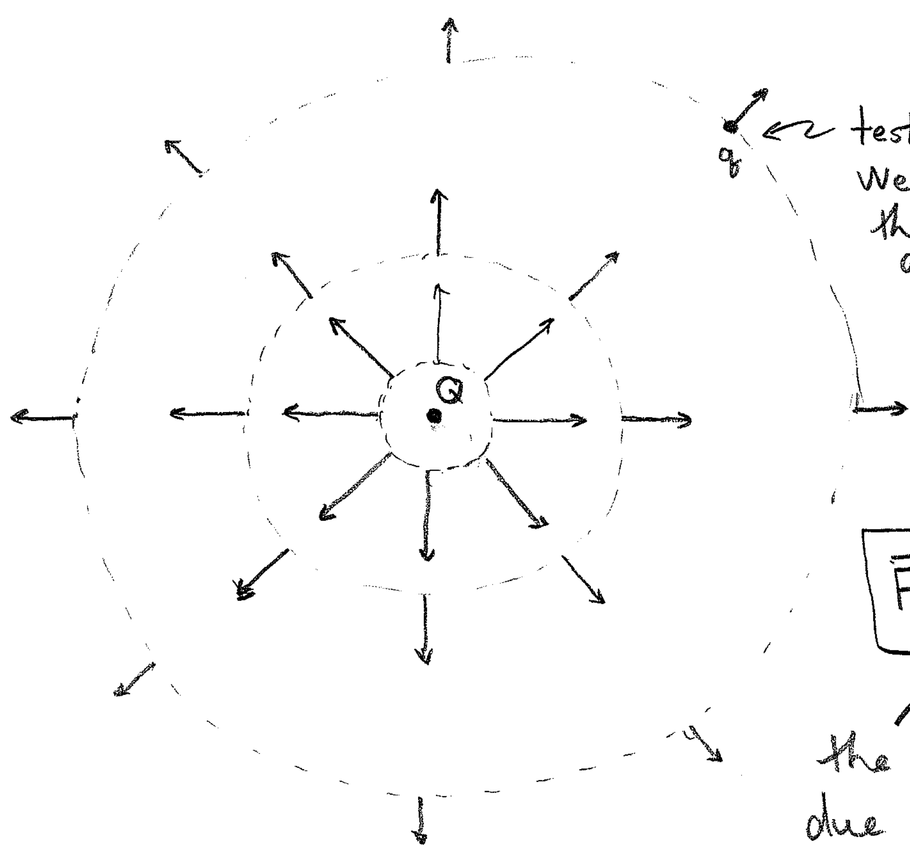
$$= \underline{5.86\times 10^{-68} N}.$$

In <u>some</u> situations the gravitational force is <u>far</u> weaker than the electric force. This is fortunate, if they we're of similar strength it would have been impossible for COULOMB to discover his law so easily & the same goes for NEWTON.

No action at a distance: Electric Fields & Photons

In $\boxed{E1}$ you might ask how the proton's charge exerts a force on the electron from a distance. The answer is provided in this course by the ELECTRIC FIELD. For $\boxed{E1}$ we imagine the proton's charge generates a field which permeates all space. Then the electron ⓔ is acted on by the field in a local manner. The modern quantum field theoretic idea is that ⓟ and (-e) throw photons back and forth. The electric field is merely a classical description of the quantum interchange of photons. (Photons are the quanta of light)

I know a good, non-technical, book to explain this better if you're interested. We care mostly about the electric field ($\vec{E}$)



test charge q. We idealize that the field due to q is negligable

$$\boxed{\vec{F}_Q = q\vec{E}_Q}$$

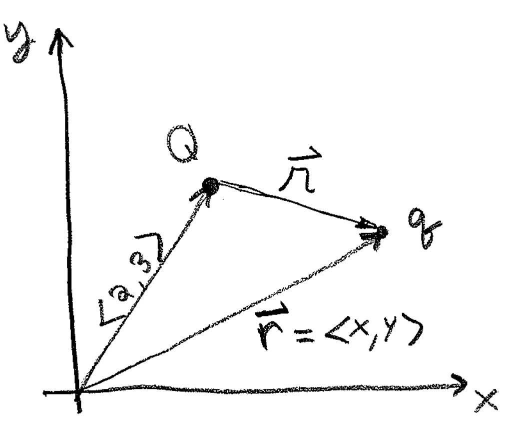
the electric field due to Q defined by this eq$\underline{^{n}}$.

- The field fills space, just waiting to communicate the Coulomb Force from Q to other charges.

Naturally you'll wonder, what if Q moves? What happens to the electric field $\vec{E}_Q$? Well, you're not allowed to ask that in this course. Sorry. In this course, for now, we study <u>static</u> charge configurations. In other words, we set up a particular geometry of charges (a charge distribution) then we study what happens to a test charge if it is subjected to the field of the static charges. (we don't ask about how the field of the test charge transforms as it accelerates because that's a <u>much</u> harder question & we can neglect it for now)

$\boxed{\text{E2}}$ Find electric field due to Q placed at the position $(2,3)$ m

Let $\vec{r} = \langle x, y \rangle$ denote the location of the hypothetical test charge,



$$\vec{\mathscr{r}} = \underbrace{\vec{r}}_{\text{field point}} - \underbrace{\langle 2, 3 \rangle}_{\text{source point}}$$
$$= \langle x, y \rangle - \langle 2, 3 \rangle$$
$$= \langle x-2, y-3 \rangle$$

$$\vec{F} = \frac{qQ}{4\pi\epsilon_0} \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = q\vec{E}$$

$$\Rightarrow \quad \underline{\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}}.$$

[E2] Continued, let me expand $\hat{\boldsymbol{\mathscr{r}}}$ and $\mathscr{r}^2$ to make sure the notation is understood,

$$\vec{E}(x,y) = \frac{Q}{4\pi\epsilon_0}\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{(x-2)^2+(y-3)^2}\right)\frac{1}{\sqrt{(x-2)^2+(y-3)^2}}\langle x-2, y-3\rangle$$

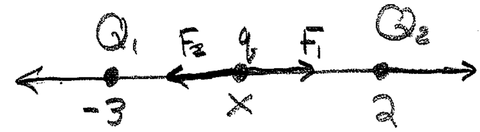
$$= \underline{\frac{Q}{4\pi\epsilon_0}\frac{1}{\left((x-2)^2+(y-3)^2\right)^{3/2}}\langle x-2, y-3\rangle}.$$

Remark: the electric field due to a point charge Q placed at source point $\vec{r}_0$ generates the COULOMB field

$$\boxed{\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0}\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} : \text{ where } \vec{\mathscr{r}} = \vec{r} - \vec{r}_0 \text{ and } \hat{\mathscr{r}} = \frac{1}{\|\vec{\mathscr{r}}\|}\vec{\mathscr{r}} \text{ as usual.}}$$

[E3] Suppose Q_1 is placed at $(-3,0,0)$m and Q_2 is placed at $(2,0,0)$m. Where should we put q to obtain static equilibrium? Assume Q_1 & Q_2 are positive

This is clearly a one-dimensional problem so we can omit vector notation and use our usual $(+)$ for $\langle 1,0,0\rangle$ and $(-)$ for $\langle -1,0,0\rangle$ convention.



$$F_{net} = \underbrace{\frac{kqQ_1}{(x+3)^2}}_{F_1} - \underbrace{\frac{kqQ_2}{(x-2)^2}}_{F_2}$$

$$= kq\left[\frac{Q_1}{(x+3)^2} - \frac{Q_2}{(x-2)^2}\right] = 0 \quad \underline{\text{for } a = 0}.$$

$$\Rightarrow \frac{Q_1}{(x+3)^2} = \frac{Q_2}{(x-2)^2} \Rightarrow Q_1[x^2-4x+4] = Q_2[x^2+6x+9]$$

$$\Rightarrow \underline{(Q_1-Q_2)x^2 - (4Q_1+6Q_2)x + 4Q_1 - 9Q_2 = 0}$$

Solve via quadratic eq$^{\underline{n}}$ to find sol$^{\underline{n}}$.

E3 Continued | We found $\vec{F}_1 + \vec{F}_2 = 0$ yields,

$$(Q_1 - Q_2)x^2 - (4Q_1 + 6Q_2)x + 4Q_1 - 9Q_2 = 0 \quad (*)$$

Easiest case is $Q_1 = Q_2$ then we have

$$-10x - 5 = 0 \quad \therefore \quad x = \frac{-5}{10} \Rightarrow \underline{x = -\tfrac{1}{2}\,m}.$$

(midpoint)

Other values do give a quadratic algebra problem.

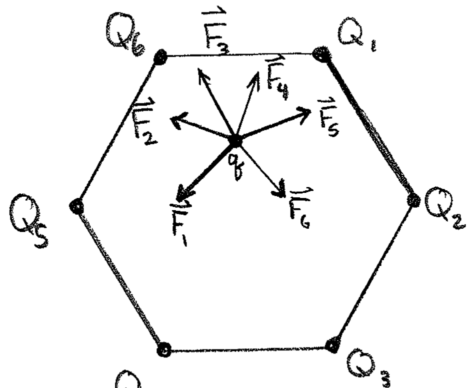
$$x = \left(\frac{4Q_1 + 6Q_2 \pm \sqrt{(4Q_1 + 6Q_2)^2 - 4(Q_1 - Q_2)(4Q_1 - 9Q_2)}}{Q_1 - Q_2} \right) m$$

Take the case of $Q_1 = Q$, $Q_2 = 2Q$ then returning to $(*)$ is easier for me, $-x^2 - 16x - 14 = 0$

$$\Rightarrow x^2 + 16x + 14 = 0 \Rightarrow \underline{x \approx -15.1m \text{ or } x = -0.92m}.$$

Now, only one of these answers is physically reasonable. Notice I assume $-3 < x < 2$ in setting up Newton's Eq^{n} ☆ $\therefore$ we find $\boxed{x = -0.92m}$

E4 Find equilibrium position for q given the hexagonal charge configuration below



q at $\vec{r}$

Q_j at $\vec{r}_j$ for $j = 1, 2, .., 6$.

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6$$

where $\vec{F}_j = \dfrac{kqQ_j}{\|\vec{r} - \vec{r}_j\|^3}(\vec{r} - \vec{r}_j)$

Once we're given values for Q_j and $\vec{r}_j$ then we can solve

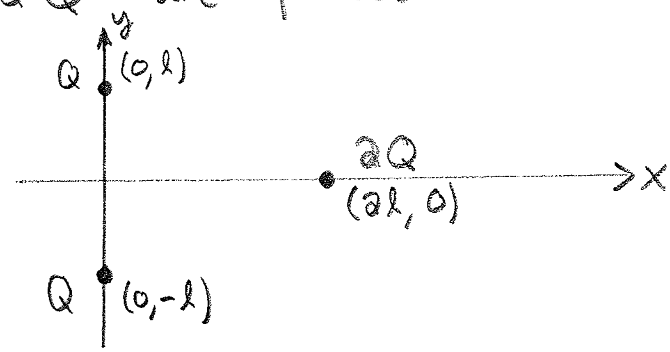
$$\vec{F}_{total} = 0. \text{ for } \vec{r} = (x, y).$$

Two Eq^{n}'s & two unknowns. (Ugly!)

If regular polygon & $Q_1 = Q_2 = \cdots = Q_6$ then easy answer by symmetry!

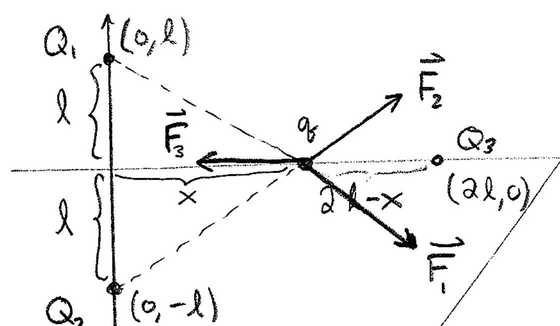
— SKIP TO PAGE (14) —

E5 Where should we place q to obtain static equilibrium given $\ell > 0$ and charges Q, Q and $2Q$ are placed as shown below:



Sol^{n}: By symmetry it is clear q must be placed on the x-axis. Suppose it is at $(x, 0)$. Moreover, it is physically clear it has $0 < x < 2\ell$ so I'll write Newton's Law in that context.

WHY IS THIS WRONG?



$$\vec{F}_3 = \frac{-kqQ_3}{(2\ell - x)^2}\,\hat{i}$$

note F_{2y} cancels F_{1y} by symmetry. We only need to find x-comp.

We need $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ for equilbrium (why?)
Let's use the non-unit-vector form of Coulomb's Law,

$$\vec{F}_1 = \frac{kqQ_1}{\left(\sqrt{x^2 + \ell^2}\right)^3}\langle x, -\ell \rangle$$

$$\vec{F}_2 = \frac{kqQ_2}{\left(\sqrt{x^2 + \ell^2}\right)^3}\langle x, \ell \rangle$$

$$\vec{F}_3 = \frac{kqQ_3}{\left(\sqrt{(2\ell - x)^2}\right)^3}\langle 2\ell - x, 0 \rangle$$

if $Q_1, q > 0$ then $\vec{F}_1$ points like $\searrow \vec{F}_1$ whereas $\vec{F}_2$ points like $\nearrow \vec{F}_2$

$\vec{F}_3$ is horizontal $\xrightarrow{\hspace{1.5em}} \vec{F}_3$

E5 Continued | I used the observation that

$$\vec{F} = \underbrace{\left(\frac{kqQ}{\|\vec{r}-\vec{r}_j\|^2}\right)}_{\text{magnitude}} \underbrace{\widehat{\vec{r}-\vec{r}_j}}_{\text{unit vector}} = \underbrace{\frac{kqQ}{\|\vec{r}-\vec{r}_j\|^3}}_{\text{scalar}} \underbrace{(\vec{r}-\vec{r}_j)}_{\text{vector}}$$

not the magnitude of $\vec{F}$ but I find this formula is nice for some problems.

Returning to the problem, the y-components in $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ clearly cancel and we're left with the following from the x-component eq$^{\underline{n}}$, of $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, cancel kq, recall $Q_1 = Q_2 = Q$ & $Q_3 = 2Q$,

$$\frac{x}{\left(\sqrt{x^2+l^2}\right)^3} + \frac{x}{\left(\sqrt{x^2+l^2}\right)^3} + \frac{2(2l-x)}{\left(\sqrt{(2l-x)^2}\right)^3} = 0$$

Now it's "just" algebra.

$$\frac{2x}{[x^2+l^2]^{3/2}} = \frac{2(x-2l)}{[(2l-x)^2]^{3/2}}$$

$$x[2l-x]^3 = [x^2+l^2]^{3/2}(x-2l)$$

$$\Rightarrow \underbrace{(x-2l)}_{\text{gives sol}^{\underline{n}}\ x = 2l \text{ (not physical)}}\left[[x^2+l^2]^{3/2} - x[2l-x]^2\right] = 0$$

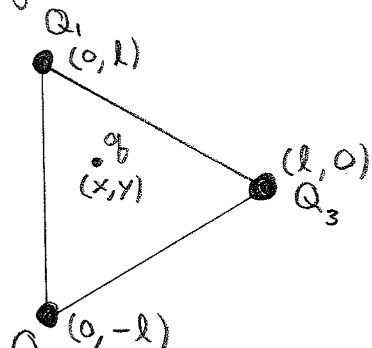
Thus $[x^2+l^2]^{3/2} = x[2l-x]^2$

$$\Rightarrow (x^2+l^2)^3 = x^2[2l-x]^4$$

$$\vdots$$

I'll leave the conclusion of the previous example to the reader. Notice I made the quiz question easier to avoid the difficulty ...

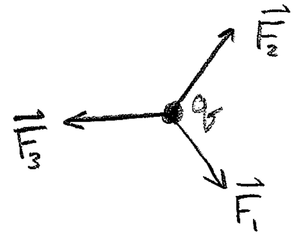
$\boxed{\text{E12}}$ Given the set of three fixed, stuck, glued immoveable, ... charges where should we place a fourth charge if we want it to be in equilbrium?



Goal: find (x,y) such that q is at rest if placed at (x,y) with zero velocity.

$(Q_1 = Q_2 = Q_3 = Q)$

It all goes back to Physics I. We begin with the freebody diagram on the mass with charge q,

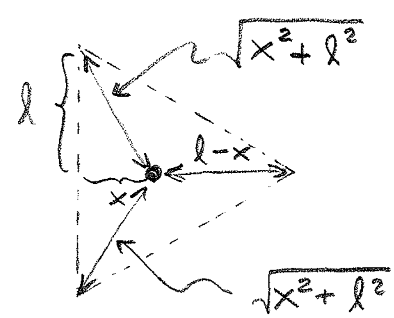


By symmetry $y = 0$. Again for convenience I'll use the form of Coulomb's Law I discussed in $\boxed{\text{E11}}$.

$$\vec{F}_1 = \frac{kqQ}{\left(\sqrt{x^2+\ell^2}\right)^3}\langle x, -\ell\rangle$$

$$\vec{F}_2 = \frac{kqQ}{\left(\sqrt{x^2+\ell^2}\right)^3}\langle x, \ell\rangle$$

$$\vec{F}_3 = \frac{kqQ}{\left(\sqrt{(\ell-x)^2}\right)^3}\langle -(\ell-x), 0\rangle$$



you need to find <u>distance</u> from source charge to test point.

E12 Continued

For equilibrium $\vec{F}_{net} = m\vec{a} = 0$. The net-force is found from a vector sum.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \quad (\star)$$

This gives two eq^n's,

$$x: \quad \frac{kqQx}{\left(\sqrt{x^2+l^2}\right)^3} + \frac{kqQx}{\left(\sqrt{x^2+l^2}\right)^3} - \frac{kqQ(l-x)}{\left(\sqrt{(l-x)^2}\right)^3} = 0$$

$$y: \quad \underbrace{\frac{-kqQl}{\left(\sqrt{x^2+l^2}\right)^3} + \frac{kqQl}{\left(\sqrt{x^2+l^2}\right)^3} + 0 = 0}$$

See our assumption that symmetry $\Rightarrow$ $y=0$ for q_3 is at least consistent with Newton's Eq^k's. I don't usually bother to write this step!

TO COMPLETE THIS EXAMPLE WE NEED TO SOLVE $F_{1x} + F_{2x} + F_{3x} = 0$ for x.

This is easier to say than to do.

For now, I leave it to the reader and DR. SKOUMBOURDIS.