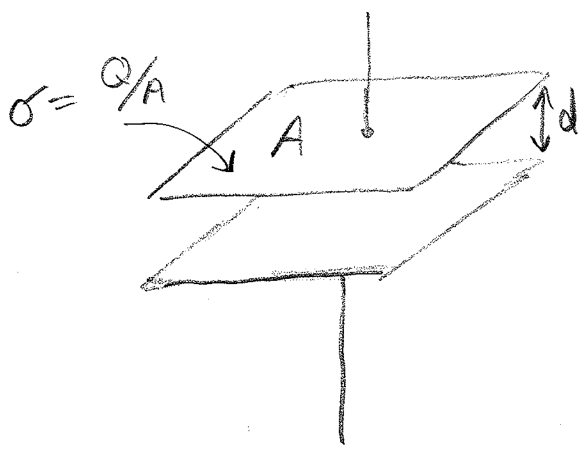
CAPACITANCE:

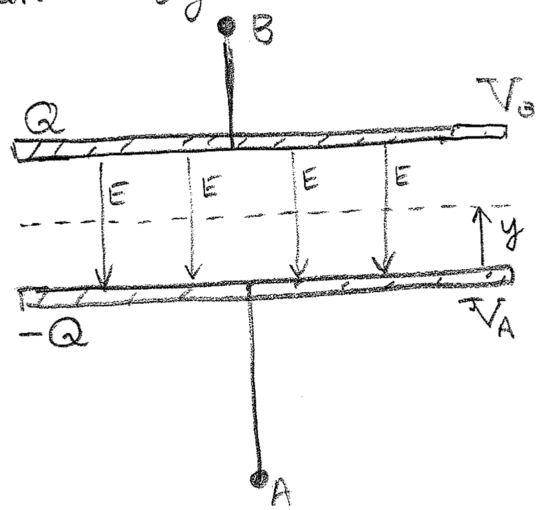
The capacitor is an important electronic device. Essentially the idea is this: we bring two conductors together, but not in contact, and the ratio of the charge on the conductor to the voltage between the plates is <u>defined</u> to be the capacitance:

$$\boxed{C = \frac{Q}{V}}$$

We assume Q on one conductor and $-Q$ on the other conductor so the net-charge on the capacitor is zero.

E39: parallell plate capacitor: two plates with area A, separated by distance d.





$$\vec{E} = \frac{-\sigma}{\epsilon_0}\hat{y} = -\frac{dV}{dy}$$

$$\Rightarrow V(y) = \frac{\sigma y}{\epsilon_0} + C_1$$

The freedom to choose C_1 is the freedom to set V_A or V_B to some particular value. Absolute potential only meaningful if a reference pt. for $V=0$ is already chosen. Otherwise, relative differences are what is measurable.

Continuing with our capacitor example,

$$V_B - V_A = V(d) - V(0)$$
$$= \left(\frac{\sigma}{\epsilon_0} d + c_1\right) - \left(\frac{\sigma}{\epsilon_0}(0) + c_1\right)$$
$$= \frac{\sigma d}{\epsilon_0} = \frac{Qd}{A\epsilon_0}$$

$$\therefore \quad \underbrace{V_B - V_A}_{\substack{\text{change} \\ \text{in voltage} \\ \text{across plates.}}} = Q \Big/ \left(\frac{A\epsilon_0}{d}\right) \implies \boxed{C = \frac{A\epsilon_0}{d}}$$

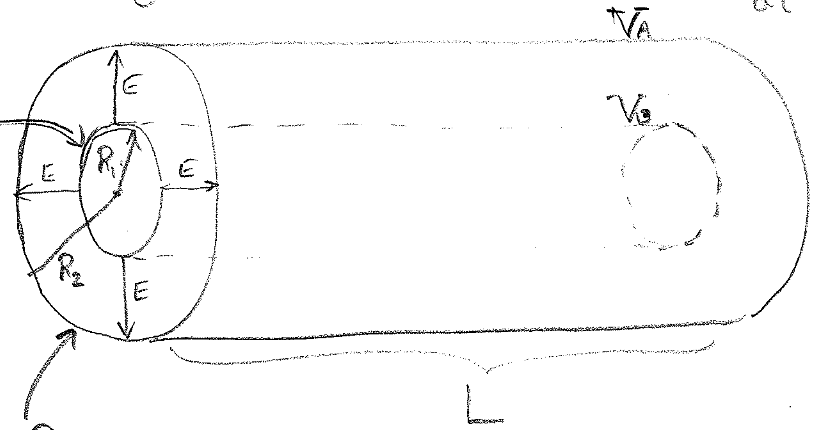
Remark: If the space between plates is replaced with dielectric then E-field between plates is modified and C is different. This formula is nearly correct if dry air is between plates.

Remark: $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m} = 8.85\,\text{p}\,F/m$ where p = pico $= 10^{-12}$ and F = FARAD in honor of Faraday who played a large role in the discovery and concept of the E-field.

E40 | $\underbrace{A = 0.001\,m^2 \text{ and } d = 1\,mm}_{\text{for a //-plate capacitor.}} \implies \underline{C = 8.85\,pF}$

E41: Cylindrical Capacitor: outer conductor at $s = R_2$. Inside conductor of radius R_1.

Q at $s = R_1$



$-Q$ (at $s = R_2$)

Assume $L \gg R_1, R_2$

So Gauss' Law applies! (nicely)

For $R_1 < s < R_2$ draw cylindrical Gaussian surface,

$$\Phi = \underbrace{(2\pi s L)}_{\substack{\text{area where} \\ E \text{ cuts surface, note} \\ \text{caps don't count.}}} E = \frac{Q}{\epsilon_0}$$

assume vacuum between conductors for now...

since $\vec{E} = E\hat{s}$ by symmetry!

$$E = \frac{Q}{2\pi s L \epsilon_0} = -\frac{dV}{ds}$$

$$\Rightarrow \quad \underline{V(s) = \frac{-Q}{2\pi\epsilon_0 L} \ln(s) + C_1}$$

for $R_1 < s < R_2$

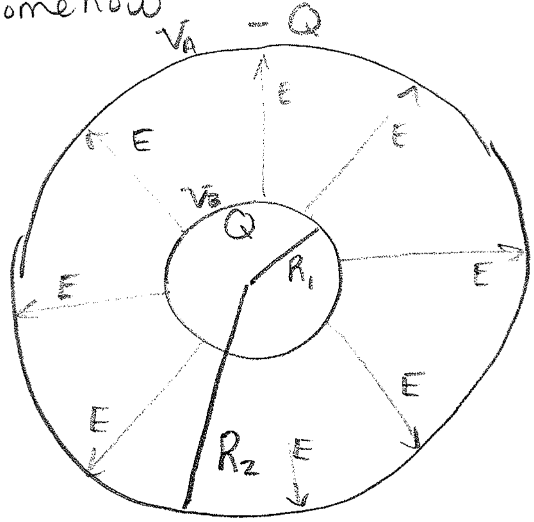
Hence,

$$\begin{aligned} V_B - V_A &= V(R_1) - V(R_2) \\ &= \frac{-Q}{2\pi\epsilon_0 L}\left(\ln(R_1) - \ln(R_2)\right) \\ &= \frac{Q}{2\pi\epsilon_0 L} \ln\left(R_2/R_1\right) \end{aligned}$$

$$\Rightarrow \quad \frac{Q}{V_B - V_A} = \boxed{\frac{2\pi\epsilon_0 L}{\ln(R_2/R_1)} = C_{\text{cylindrical cap}}}$$

Example: Spherical Capacitor

Suppose a pair of spherical conductors are separated as pictured below and somehow



$$E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

for $R_1 < r < R_2$

$$E = -\frac{dV}{dr}$$

as by symmetry $\vec{E} = E\hat{r}$

$$V(r) = \frac{Q}{4\pi\epsilon_0 r^2} + C_1$$

$$V_B - V_A = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{Q}{V_B - V_A} = \boxed{\frac{4\pi\epsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}} = C_{\text{spherical shell cap.}}}$$

Remark: the cylindrical cap. is more useful in applications. These spherical capacitors are kinda hard to build. The text by Serway ~~calculates~~ Viewing the Earth with atmosphere as cap. implies a capacitance $\approx 1\,F$ for the whole Earth / Atmospheric system.

Remark: given another geometry the method to calculate C should be clear. Find V for given $Q/-Q$ and calculate Q/V. Usually find V from E by integration.

CAPACITANCE SUMMARY:

When two conductors are separated by an insulator then we can show the charge Q on one of the conductors is <u>proportional</u> to the change in voltage ΔV across the plates. In particular,

$$\boxed{C = \frac{Q}{\Delta V}}$$ ← def$^{\underline{n}}$ of capacitance for a particular circuit element.

Usually C depends on the physical attributes of the circuit element like: area of conductors, separation between conductors, geometry of conductors, the <u>dielectric</u> <u>constant</u> κ for the insulator etc...

I worked out the paralell plate, coaxial cable and spherical cases under the assumption $\Phi = \frac{Q_{enc}}{\epsilon_0}$ which <u>means</u> I'm assuming the electric field is spreading out through <u>free</u> <u>space</u> (vacuum).

Air is very close to a vacuum so far as <u>electrostatics</u> is concerned because $\epsilon_{air} \cong \epsilon_0$.

If you're curious,

$$\kappa_{media} = \frac{\epsilon_{media}}{\epsilon_0}$$

$\boxed{\epsilon_{DIAMOND} = 5.7\epsilon_0}$

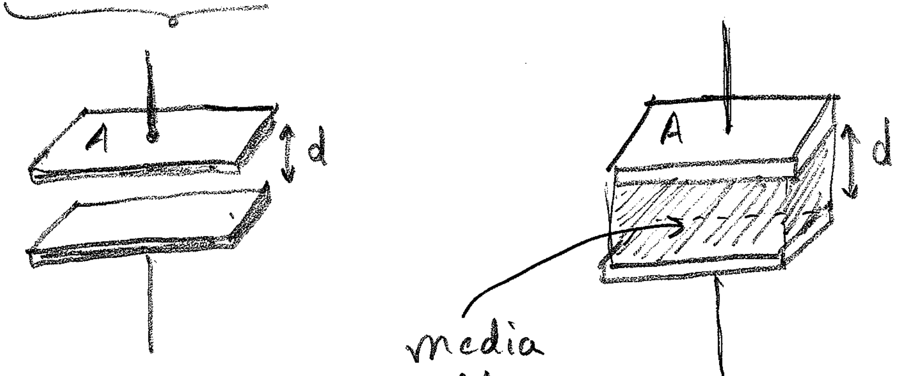
$\kappa_{vacuum} = 1$	$\kappa_{DIAMOND} = 5.7$
$\kappa_{air} = 1.00054$	$\kappa_{water} = 80.1$
$\kappa_{water\ vapor} = 1.00587$	$\kappa_{ICE} = 99$ $(-30°C)$
$\kappa_{helium} = 1.000065$	$\kappa_{KTaNbO_3} = 34{,}000$ $(0°C)$

(FROM GRIFFITH'S E&M, 3rd Ed, p. 180 which credits the Handbook of Chemistry & Physics 1997..)

So, to summarize, the polarization of a linear media has the effect of opposing the $\vec{E}$-field permeating the media. The neat thing is that we can still pretend like it's a vacuum and use $\Phi = \frac{Q_{enc}}{\epsilon}$ to encapsulate the net-effect of polarization. Thus, we can use our earlier derivations in vacuum to find formulas for capacitors with same geometry and differing insulators. You just replace ϵ_0 with ϵ.

E42 <u>Paralell Plate Capacitor:</u>

$$\underbrace{C_1 = \epsilon_0 \frac{A}{d}} \qquad\qquad C_2 = \epsilon \frac{A}{d}$$



media with permitivity ϵ

(dielectric constant $K = \frac{\epsilon}{\epsilon_0}$)

$\epsilon > 1 \Rightarrow C_2 > C_1$, so if we have media with big ϵ we can increase effective capacitance.

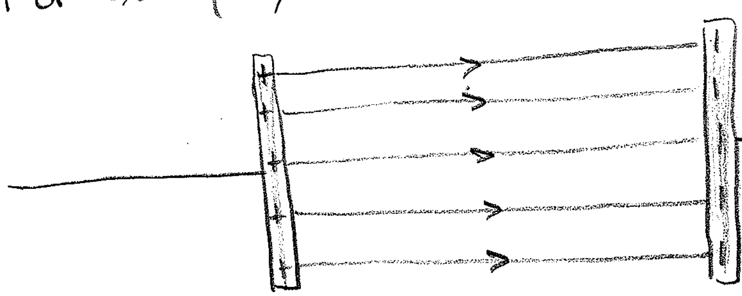
Summary of Electrostatics in Linear Media with permitivity $\epsilon = K\epsilon_0$:

Still apply Gauss' Law and analyze via symmetry, potentials etc... except, replace ϵ_0 with ϵ.

$$\Phi = \frac{Q_{enc}}{\epsilon}$$

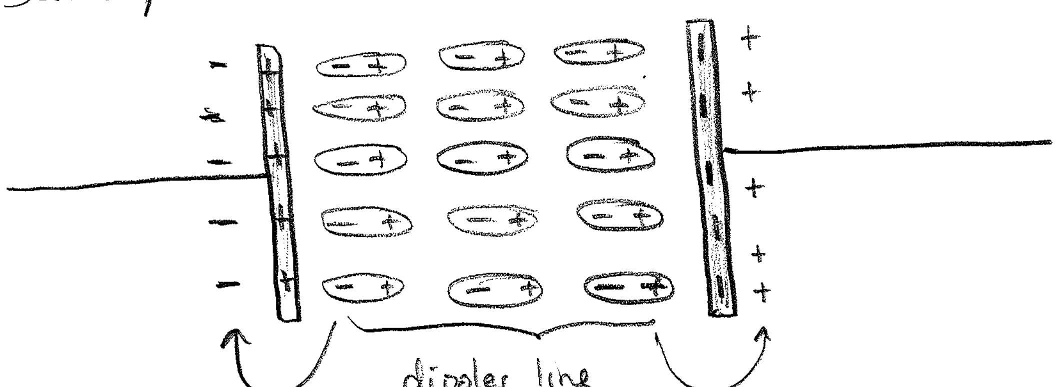
This means the flux will be smaller for a given charge configuration if we replace the vacuum with some medium. The reason for this is the polarization of the media by the electric field.

For example, in a capacitor with paralell plates,



$\vec{E}$ - field flows from (+) to (−) plates through vacuum ($\epsilon = \epsilon_0$)

Same, but with dielectric inserted,



dipoles line up with $\vec{E}$ - field. Net result is to induce an opposite charge at plates $\Rightarrow$ smaller $\vec{E}$ - field.

DIELECTRIC BREAKDOWN:

Air, water, diamond etc... if subjected to too large an electric field will break-down and ionize, burn etc... so it behaves as a conductor. Tables are available which explain how many Volts/meter are allowed before a breakdown may occur.

$$\text{Glass} \approx 1\times 10^{7}\ \frac{V}{m}$$

$$\text{Polystyrene} \approx 2\times 10^{7}\ \frac{V}{m}$$

$$\text{Air (dry)} \approx 3\times 10^{6}\ \frac{V}{m}$$ (see p. 669)

Energy Stored in Capacitor

Work to move dq from one plate to another is:

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

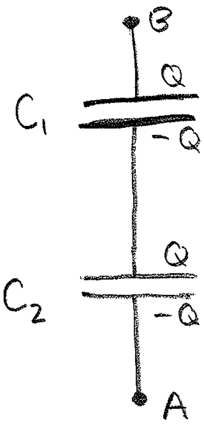
$$\Rightarrow W = \int \frac{q\, dq}{C} = \frac{1}{2}\frac{Q^2}{C}$$

$$\therefore\ U = \frac{Q^2}{2C}$$ = potential energy stored in capacitor with charge Q and capacitance C.

CIRCUITS & CAPACITORS

A <u>circuit</u> is a collection of elements connected by wires. It is assumed the wires or more generally conductors have no "resistance" and serve as perfect equipotentials. Charge is free to flow through the circuit from one element to the next as pictured. However, capacitor circuits are less exciting if we just considered the totally charged case. The symbol for capacitance is 
(in honor of the ∥ plate capacitor naturally)

SERIES:



$Q_1 = Q_2$: same charge on both.

$C_1V_1 = C_2V_2$: since $C_1 = \frac{Q_1}{V_1}$ & $C_2 = \frac{Q_2}{V_2}$

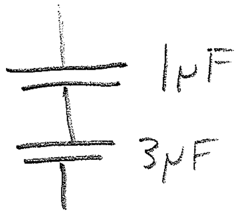
$V_B - V_A = \underset{\substack{\text{voltage}\\ \text{change}\\ \text{on } C_1}}{V_1} + \underset{\substack{\text{voltage}\\ \text{change}\\ \text{on } C_2}}{V_2}$: additivity of voltage.

$$\underbrace{C_{eq} = \frac{Q}{V_B - V_A}}_{\text{defines equivalent capacitance for this Series Circuit.}} = \frac{Q}{V_1+V_2} \Rightarrow \frac{V_1+V_2}{Q} = C_{eq}$$

$$\Rightarrow \frac{V_1}{Q} + \frac{V_2}{Q} = C_{eq}$$

$$\Rightarrow \boxed{\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}}$$

E43 Let $C_1 = 1\mu F$ and $C_2 = 3\mu F$ then find C_{eq} for series circuit with C_1 & C_2



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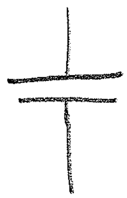
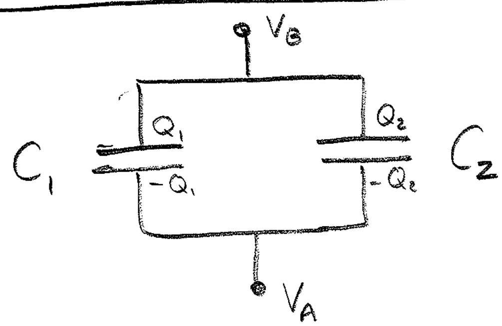


$$C_{eq} = \frac{1}{\frac{1}{1\mu F} + \frac{1}{3\mu F}} = \frac{1}{\frac{4}{3\mu F}} = \boxed{0.75\,\mu F}$$

PARALLEL CAPACITORS



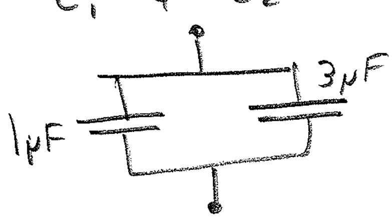
$V_1 = V_2$: same voltage drop on both.

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\underbrace{C_{eq} = \frac{Q_1 + Q_2}{V_B - V_A}}_{\text{def}^{\text{n}}\text{ of equivalent capacitance for this circuit.}} = \frac{Q_1}{V_1} + \frac{Q_2}{V_2} = C_1 + C_2$$

$$\therefore \boxed{C_{eq} = C_1 + C_2}$$

E44 Let $C_1 = 1\mu F$ and $C_2 = 3\mu F$ then find C_{eq} for parallel combination of C_1 & C_2



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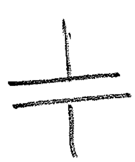
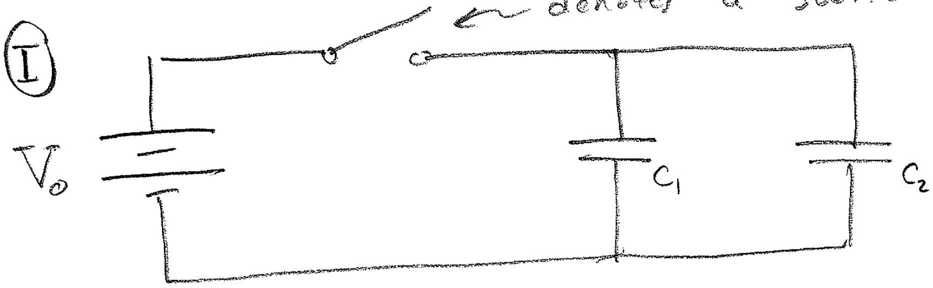


$$\boxed{C_{eq} = 4\mu F}$$

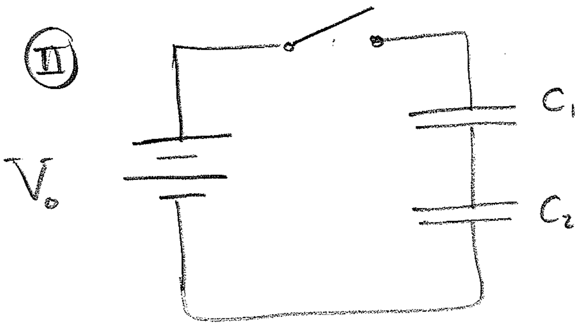
How to charge capacitors:

(I)



← denotes a switch which we can close thus supplying voltage to these circuits which moves charge thus storing electric potential energy in the E-fields between the capacitor plates.

(II)



Question: what is V_1 & V_2 for the series circuit?

Sol$^{\underline{n}}$/ $$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{Q}{V_0} \Rightarrow Q = \frac{V_0}{\frac{1}{C_1}+\frac{1}{C_2}}$$

Thus $$V_1 = \frac{Q}{C_1} = \frac{1}{C_1}\left[\frac{V_0}{\frac{1}{C_1}+\frac{1}{C_2}}\right] = \underline{\frac{V_0}{1+C_1/C_2} = V_1}$$

and $$V_2 = \frac{Q}{C_2} = \frac{1}{C_2}\left[\frac{V_0}{\frac{1}{C_1}+\frac{1}{C_2}}\right] = \underline{\frac{V_0}{C_2/C_1+1} = V_2}.$$

We should hope $V_1 + V_2 = V_0$ let's check,

$$V_1 + V_2 = \frac{V_0}{1+C_1/C_2} + \frac{V_0}{C_2/C_1+1}$$

$$= \frac{C_2 V_0}{C_2+C_1} + \frac{C_1 V_0}{C_2+C_1}$$

$$= V_0\left(\frac{C_2+C_1}{C_2+C_1}\right)$$

$$= V_0.$$

(phew!)

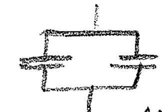
E45
<u>Question</u>: what are the charges on c_1 and c_2 in circuit I from 64 ?

parallel capacitors both at $V_1 = V_2 = V_0$

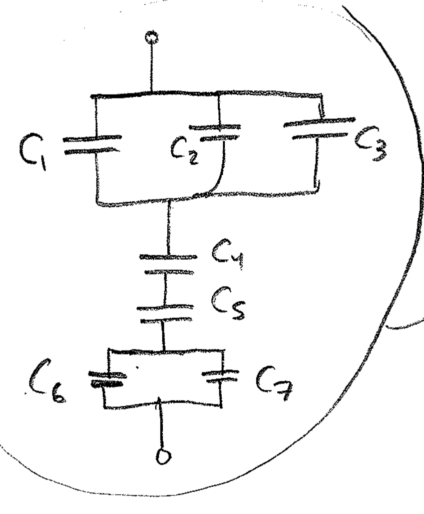
hence $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V_0$. Thus,

$$\boxed{Q_1 = C_1 V_0} \quad \& \quad \boxed{Q_2 = C_2 V_0}$$

Again the total charge $Q_1 + Q_2 = (C_1 + C_2) V_0$
So it is as if we have $C_{eq} = C_1 + C_2$
with charge $Q_1 + Q_2$ at voltage V_0.

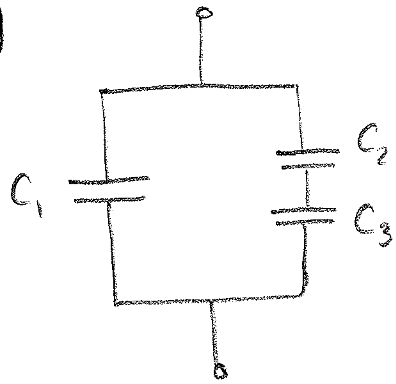
<u>Remark</u>: we can derive results that follow below by def$^{\underline{n}}$ of capacitance and equipotentials and conservation of charge <u>but</u> I'll just cut straight to the method for finding C_{eq}. Rest assured arguments like those just given for  (series) and  (parallel.) can be offered

<u>E46.)</u> Find C_{eq} for following:



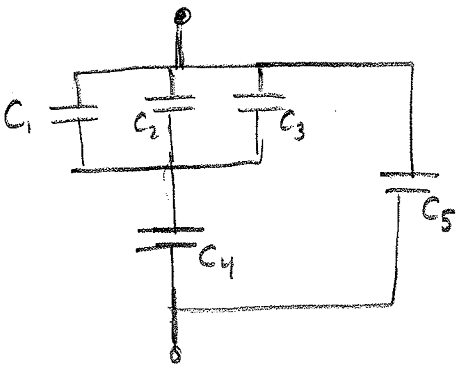
$$C_{eq} = \cfrac{1}{\underbrace{\cfrac{1}{C_1 + C_2 + C_3}}_{\text{for the three in } \|} + \underbrace{\cfrac{1}{C_4} + \cfrac{1}{C_5}}_{\text{plain old series with others}} + \underbrace{\cfrac{1}{C_6 + C_7}}_{C_6 \,\&\, C_7 \text{ in } \| \text{ again.}}}$$

E47



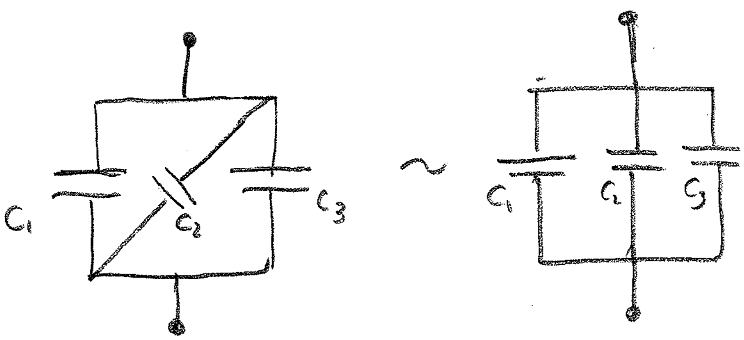
$$C_{eq} = C_1 + \underbrace{\frac{1}{\frac{1}{C_2} + \frac{1}{C_3}}}_{\text{effective capacitance of the series combo.}}$$

E48



$$C_{eq} = \underbrace{\frac{1}{\frac{1}{C_1+C_2+C_3} + \frac{1}{C_4}}}_{\text{this mess is in } \parallel \text{ with } C_5.} + C_5$$

E49



$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

<u>Remark</u>: we could go on, the possibilities are nearly endless. I hope this suffices for you to do my quiz etc...