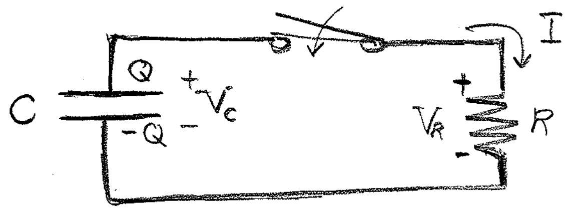
RC CIRCUITS:

We begin with the problem of discharging a capacitor C through a resistor R. Assume the capacitor has charge $Q(t)$ at time t. In particular $Q(0) = Q_0$.

ES7



close switch at $t = 0$

$$I_0 = \frac{V_0}{R} = \frac{Q_0}{RC} \quad \text{(current at } t = 0\text{)}$$

$$I = -\frac{dQ}{dt}$$

(minus because charge is leaving the capacitor and we're using the capacitor to index the charge)

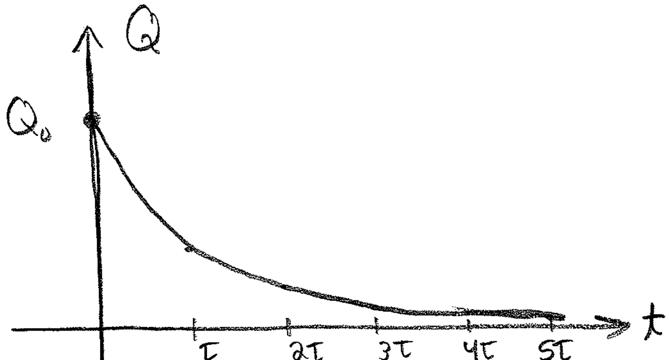
Note also $V_c = V_R$

but $V_c = Q/C$ whereas $V_R = IR$,

$$\frac{Q}{C} = IR = -R\frac{dQ}{dt}$$

$$\Rightarrow \frac{dQ}{Q} = \frac{-dt}{RC} \Rightarrow \ln|Q| = \frac{-t}{RC} + c_1$$

$$\Rightarrow \boxed{Q(t) = Q_0 \exp\left(-t/RC\right)}$$



$\tau = RC$ = time constant for circuit. After 5τ about 99% of Q_0 is discharged.

$$\left(\lim_{t \to \infty} Q(t) = 0 \quad \underline{\text{BUT}} \quad Q(t) \neq 0 \ \forall t \in \mathbb{R}\right)$$

We found $Q(t) = Q_0 e^{-t/\tau}$ for $\tau = RC$

The current in the circuit is found by differentiation since $I = -dQ/dt$,

$$I(t) = -\frac{d}{dt}\left(Q_0 e^{-t/\tau}\right)$$

$$= -Q_0 e^{-t/\tau} \frac{d}{dt}\left(-t/\tau\right)$$

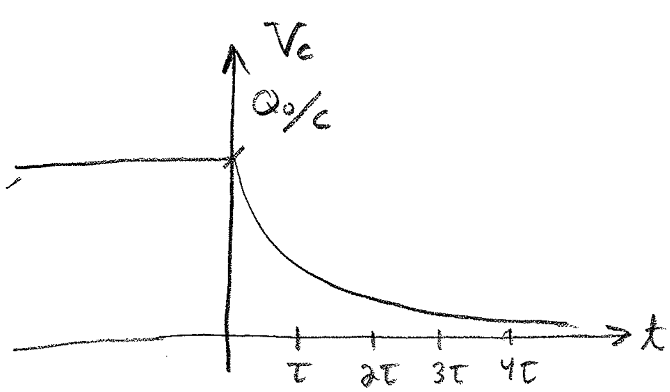
$$= \boxed{\frac{Q_0}{RC} e^{-t/RC} = I(t)}$$

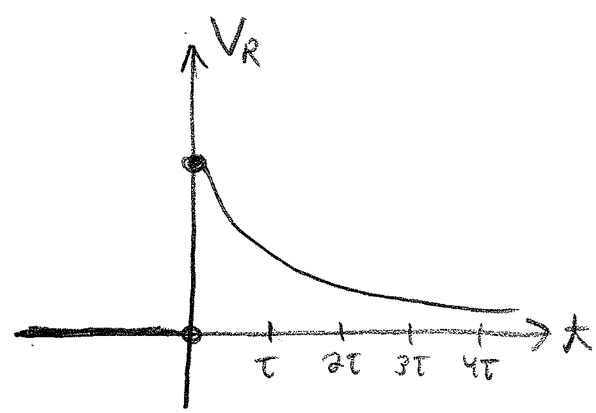
We can also write the voltage across R and C,

$$V_R(t) = \begin{cases} 0 & : t < 0 \\ \frac{Q_0}{C} e^{-t/RC} & : t \geq 0 \end{cases}$$

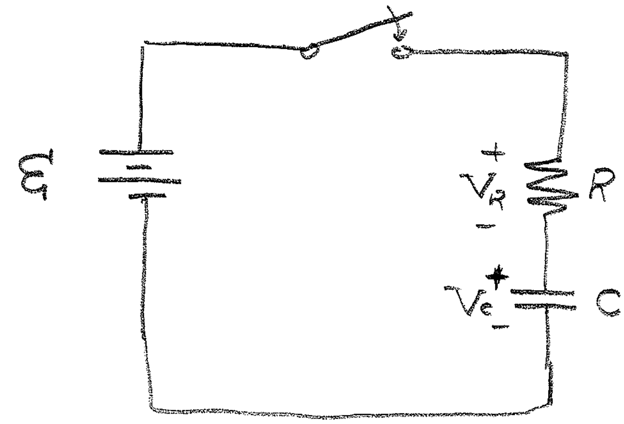
$$V_C(t) = \frac{Q(t)}{C} = \begin{cases} \frac{Q_0}{C} & : t < 0 \\ \frac{Q_0}{C} e^{-t/RC} & : t \geq 0 \end{cases}$$

Note: the voltage across R jumps at $t=0$, it is discontinuous. However, the V_C is continuous at $t = 0$.





E58) Close switch at $t=0$ and find current I and V_R and V_C as function of time for circuit pictured below



Let $Q(t)$ be charge on C at time t.
Kirchoff's Voltage Law around circuit yields:

$$\mathcal{E} - \underset{V_R = IR}{V_R} - \underset{V_C = \frac{Q}{C}}{V_C} = 0$$

Here $I > 0 \Rightarrow Q$ increases thus $I = \dfrac{dQ}{dt}$.
We find that,

$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

Algebra $\Rightarrow \dfrac{dQ}{dt} + \dfrac{1}{RC}Q = \dfrac{\mathcal{E}}{R}$

Calculus $\Rightarrow \underline{Q(t) = ke^{-t/RC} + C\mathcal{E}}$. ($\int$-factor method)

Now, $Q(0) = 0$ hence $Q(0) = k + C\mathcal{E}$ so we find $k = -C\mathcal{E}$ and hence,

$$\boxed{Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)}$$

Continuing, now that we've calculated $Q(t)$ it's easy to find I, V_R and V_C.

$$V_C = \frac{Q}{C} = \underline{\mathcal{E}\left(1 - e^{-t/RC}\right), \text{ for } t \geq 0}.$$

Differentiate, $I = \frac{dQ}{dt}$,

$$I = \frac{d}{dt}\left[C\,\mathcal{E}\left(1 - e^{-t/RC}\right)\right]$$

$$= -C\,\mathcal{E}\left(\frac{-1}{RC}e^{-t/RC}\right)$$

$$= \underline{\frac{\mathcal{E}}{R}e^{-t/RC}, \text{ for } t \geq 0}.$$

Finally, $V_R = IR = \underline{\mathcal{E}e^{-t/RC} \text{ for } t \geq 0}$.

To summarize:

$$V_C(t) = \begin{cases} 0 & : t < 0 \\ \mathcal{E}\left(1 - e^{-t/RC}\right) & : t \geq 0 \end{cases}$$

no jump in V_C at $t = 0$

$$V_R(t) = \begin{cases} 0 & : t < 0 \\ \mathcal{E}e^{-t/RC} & : t \geq 0 \end{cases}$$

jumps at $t = 0$

$$I(t) = \begin{cases} 0 & : t < 0 \\ \frac{\mathcal{E}}{R}e^{-t/RC} & : t \geq 0 \end{cases}$$

jumps at $t = 0$

(Remark: CAPACITORS RESIST **CHANGE** IN VOLTAGE. SINCE $V_C = Q/C$ and instantaneous $\Delta V_C \Rightarrow$ instantaneous change in Q $\Rightarrow$ infinite current.)