Gauss's Law For $\vec{B}$

$$\Phi_{\vec{B}} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$

Ampere's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_c$$ for any <u>closed</u> curve C.

In terms of calculus III, Chpt. 17,

$$\oint_{\partial S = C} \vec{B} \cdot d\vec{l} = \iint_S (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_0 I_c = \mu_0 \iint_S \vec{J} \cdot d\vec{A}$$

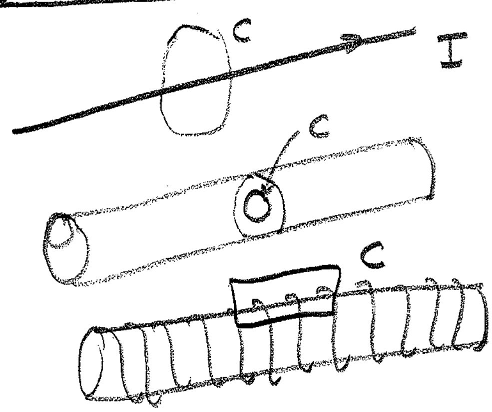
holds for any surface S hence,

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

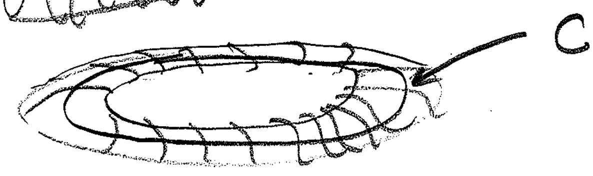
- Assuming <u>steady</u> currents an no charge accumulation. (otherwise need Maxwell's Correction).

Standard Examples & Choice of C

1.) long wire
2.) solid long wire
3.) Solenoid
4.) toroid

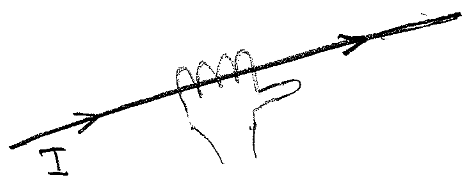


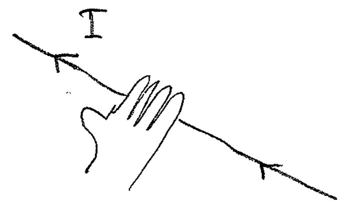
(we'll work these out)



Standard Examples of Ampere's Law

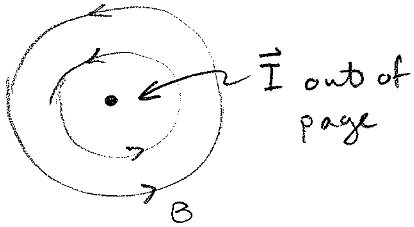
Ampere's Law is typically useful if the problem has a certain cylindrical symmetry. If this symmetry is present then we know by Biot-Savart that $\vec{B}$ is constant <u>magnitude</u> along a circle centered on the axis of symmetry. The magnetic field is not constant, it does have a changing <u>direction</u> for most examples. The direction is given by the RHR

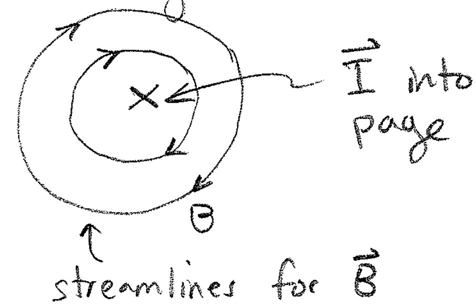




point thumb in direction of current then $\vec{B}$ wraps around wire just like fingers.

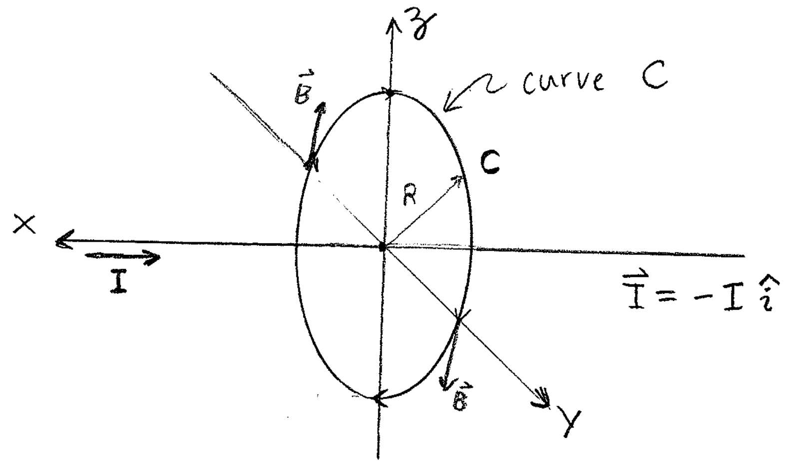
again point thumb in direction of I then fingers curl around wire just like magnetic field.



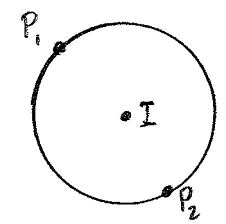


The starting point for Ampere's Law is understanding the directionality of $\vec{B}$ as described above. This is implicit within our later arguments...

$\boxed{E70}$ Find $\vec{B}$-field generated by steady line current which goes along a very, very, very ... very long wire.



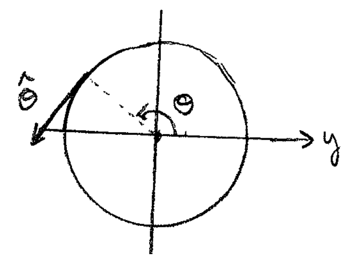
- no point on wire distinguished.
- no direction $\perp$ to wire distinguished



P_1 & P_2 should have same physics.

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_c$$

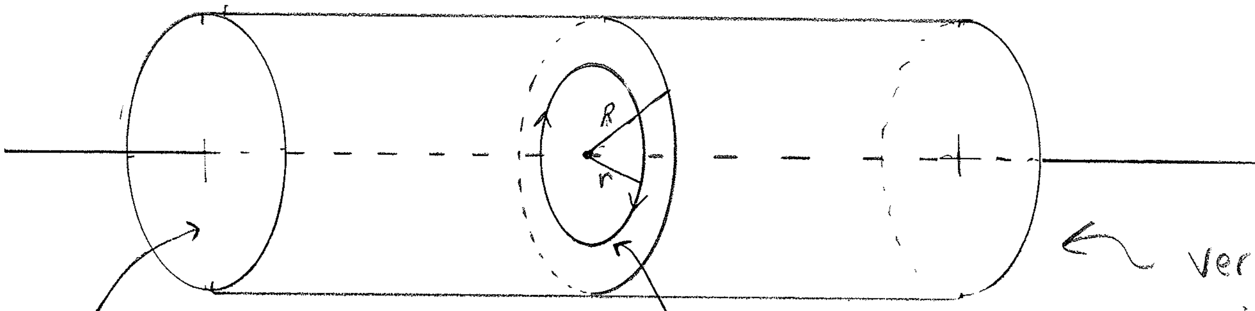
$$(2\pi R)B = \mu_0 I \quad \therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi R}(-\hat{\theta})$$



$\vec{B}$ field directed opposite $\hat{\theta}$ in this choice of coordinates.

<u>Remark:</u> Key idea here is that $B = \frac{\mu_0 I}{2\pi R}$ with a direction supplied by the <u>Right</u> <u>Hand</u> <u>Rule</u> (RHR)

$\boxed{\text{E71}}$ Long wire, current $\vec{I} = I\hat{z}$ as in $\boxed{\text{E70}}$ find $\vec{B}$ via Ampere's



very long wire, precise location of C shouldn't matter...

Current $\vec{I}$ <u>uniformly</u> distributed across core of wire from $0 \le r \le R$.

Curve C at radius $r \le R$

$$\underbrace{\vec{J} = \frac{1}{\pi R^2}\vec{I}}_{} \quad \Rightarrow \quad I_c = \pi r^2 \vec{J} = \frac{r^2}{R^2} I$$

<u>constant</u> current density

Ampere's Law $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_c$ yields

$$B(2\pi r) = \mu_0 r^2 I / R^2$$

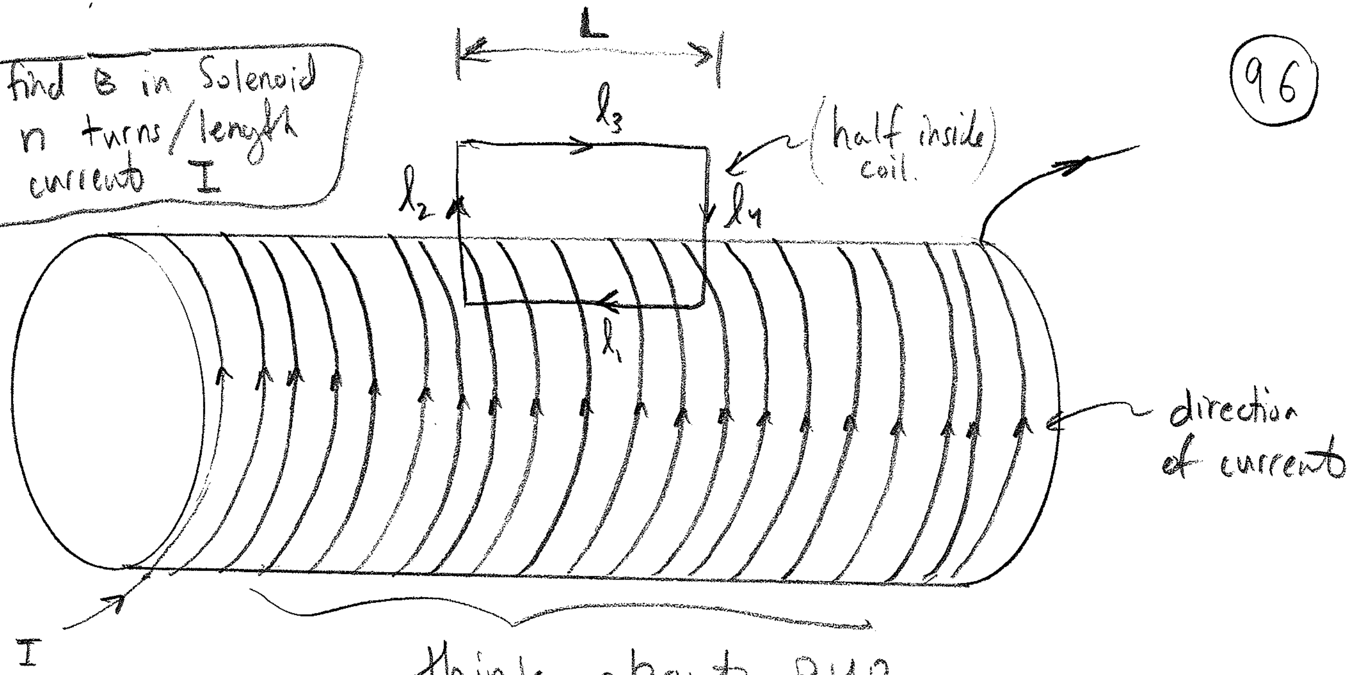
$$\therefore \boxed{B = \frac{\mu_0 I r}{2\pi R^2} \text{ for } r \le R. \text{ in the } -\hat{\theta} \text{ direction (see E71)}}$$

Outside wire $I_{enc} = I$ for circle of radius $r \ge R$ as pictured (still centered about axis of wire)

$$B = \begin{cases} \dfrac{\mu_0 I r}{2\pi R^2} & 0 \le r \le R \\ \dfrac{\mu_0 I}{2\pi r} & r \ge R. \end{cases}$$

(in the direction encircling the wire as prescribed by RHR.)

$\boxed{E72}$ find B in Solenoid with n turns/length and current I



think about RHR it will lead you to conclude $\vec{B}$ points left from the given I

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_c \qquad C = l_1 \cup l_2 \cup l_3 \cup l_4$$

$$\int_{l_1} \vec{B}\cdot d\vec{l} + \underbrace{\cancel{\int_{l_2} \vec{B}\cdot d\vec{l}}^{\,0} + \cancel{\int_{l_3} \vec{B}\cdot d\vec{l}}^{\,0} + \cancel{\int_{l_4} \vec{B}\cdot d\vec{l}}^{\,0}}_{\text{By Biot-Savart} \oplus \text{symmetry.}} = \mu_0 \underbrace{\left(\begin{matrix}\text{wires per}\\ \text{unit length}\end{matrix}\right)}_{n = \frac{N}{L_{TOTAL}}} L$$

$$BL = \mu_0 n I$$

$$\therefore \boxed{B = \mu_0 n I = \frac{\mu_0 N I}{L_{TOTAL}}}$$

$\boxed{E73}$ toroidal a.k.a. donut solenoid

see my undergrad notes for pictures ...