PY 415, Dr. Chung $\qquad$ James Cook

20/20

11.2 Equation 11.14 can be expressed in coordinate-free form by $p_0 \cos\theta = \vec{p}_0 \cdot \hat{r}$

11.14
$$V(r,\theta,t) = \frac{-p_0 \omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\overbrace{\omega(t - r/c)}^{\delta}] = \boxed{\frac{-\omega(\vec{p}_0 \cdot \hat{r})}{4\pi\epsilon_0 c r}\sin[\omega(t - r/c)] = V(r,\theta,t)}$$

11.17
$$\vec{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin[\omega(t - r/c)]\hat{z} = \frac{-\mu_0 \omega}{4\pi r}\sin[\delta]\{p_0(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})\}$$

$$\vec{A}(r,\theta,t) = \frac{-\mu_0 \omega}{4\pi r}\sin\delta\{(\vec{p}_0 \cdot \hat{r})\hat{r} - p_0(\sin\theta)\hat{\theta}\}$$

$$\vec{p}_0 = p_0\cos\theta\,\hat{r} - p_0\sin\theta\,\hat{\theta} \Rightarrow -p_0\sin\theta\,\hat{\theta} = \vec{p}_0 - p_0\cos\theta\,\hat{r} = \vec{p}_0 - (\vec{p}_0 \cdot \hat{r})\hat{r}$$

$$\therefore \vec{A}(\vec{r},t) = \frac{-\mu_0 \omega}{4\pi r}\sin\delta[(\vec{p}_0 \cdot \hat{r})\hat{r} + \vec{p}_0 - (\vec{p}_0 \cdot \hat{r})\hat{r}]$$

$$\therefore \boxed{\vec{A}(\vec{r},t) = \frac{-\mu_0 \omega}{4\pi r}\sin[\omega(t - r/c)]\,\vec{p}_0}$$

11.18
$$\vec{E} = -\frac{\mu_0 \omega^2}{4\pi r}\cos\delta[p_0\sin\theta\,\hat{\theta}] = \boxed{\frac{\mu_0 \omega^2}{4\pi r}\cos[\omega(t - r/c)]\{\vec{p}_0 - (\vec{p}_0 \cdot \hat{r})\hat{r}\} = \vec{E}}$$

Also note $\hat{r} \times p_0\hat{z} = -p_0\sin\theta\,\hat{\phi}$ and $-p_0\sin\theta\,\hat{\phi} \times \hat{r} = p_0\sin\theta\,\hat{\theta}$

11.19
$$\vec{B} = -\frac{\mu_0 \omega^2}{4\pi c r}\cos\delta[p_0\sin\theta\,\hat{\phi}] \qquad \hat{r} \times \hat{\theta} = \hat{\phi}$$

r

$\phi \quad \theta$

$$p_0\hat{z} \times \hat{r} = p_0(\cancel{\cos\theta\,\hat{r}} - \sin\theta\,\hat{\theta}) \times \hat{r} = p_0\sin\theta\,\hat{\phi} = -\hat{r} \times \vec{p}_0$$

$$\boxed{\vec{B} = \frac{\mu_0 \omega^2}{4\pi c}\frac{1}{r}\cos[\omega(t - r/c)]\{\hat{r} \times \vec{p}_0\}}$$

(**11.18** continued)

thus $-p_0\sin\theta\,\hat{\theta} = (\hat{r} \times \vec{p}_0) \times \hat{r}$ so then

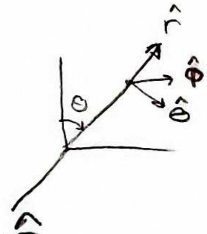
$$\boxed{\vec{E} = \frac{\mu_0 \omega^2}{4\pi}\frac{1}{r}\cos[\omega(t - r/c)]\{(\hat{r} \times \vec{p}_0) \times \hat{r}\}}$$

the { } expressions are equivalent the BAC-CAB Rule (triple product rule # 2, check it out.

$\boxed{11.2}$

$\boxed{11.21}$ $$\langle S \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{r}$$

$r \to \theta \to \varphi \to r$



$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} \qquad \vec{p}_0 = p_0\hat{z} \qquad \hat{\theta}\times\hat{\varphi} = \hat{r}$$

$$\vec{p}_0 = p_0(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) \Rightarrow p_0\sin\theta\,\hat{\theta} = p_0\cos\theta\,\hat{r} - \vec{p}_0$$

$$\begin{aligned} p_0^2\sin^2\theta &= (p_0\cos\theta\,\hat{r} - \vec{p}_0)\cdot(p\cos\theta\,\hat{r} - \vec{p}_0) \\ &= p_0^2\cos^2\theta - 2p_0^2\cos\theta\,\hat{r}\cdot\hat{z} + p_0^2 \\ &= p_0^2\cos^2\theta - 2p_0^2\cos^2\theta + p_0^2 \\ &= p_0^2 - p_0^2\cos^2\theta \end{aligned}$$

$$\begin{aligned} p_0^2\sin^2\theta\,\hat{r} = (p_0^2 - p_0^2\cos^2\theta)\hat{r} &= (p_0 - p_0\cos\theta)(p_0 + p_0\cos\theta)\hat{r} \\ &= (p_0 - \vec{p}_0\cdot\hat{r})(p_0 + \vec{p}_0\cdot\hat{r})\hat{r} \\ &= (p_0^2 - (\vec{p}_0\cdot\hat{r})^2)\hat{r} \end{aligned}$$

ignoring part work, not wrong but extranous

$$p_0^2\sin^2\theta\,\hat{r} = (p_0^2 - p_0^2\cos^2\theta)\hat{r} = (p_0^2 - (\vec{p}_0\cdot\hat{r})^2)\hat{r}$$

$$\therefore \boxed{\langle S \rangle = \frac{\mu_0\omega^4}{32\pi^2 c}\frac{1}{r^2}\left(p_0^2 - (\vec{p}_0\cdot\hat{r})^2\right)\hat{r}}$$

11.3

$$P = I^2 R \quad \therefore \quad R = \frac{P}{I^2}$$

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(q_0 \cos \omega t\right) = -\omega q_0 \sin \omega t$$

$$\langle I^2 \rangle = \langle \omega_0^2 q_0^2 \sin^2 \omega t \rangle = \omega_0^2 q_0^2 \langle \sin^2 \omega t \rangle = \frac{\omega_0^2 q_0^2}{2}$$

$$\langle P \rangle = \left\langle \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \right\rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} = \frac{\mu_0 q_0^2 d^2 \omega^4}{12\pi c}$$

Actually to find R I need time avg.... Fortunately I just calculated them i.e.,

$$R = \frac{\langle P \rangle}{\langle I^2 \rangle} = \frac{\frac{\mu_0 \cancel{q_0^2} d^2 \omega^{\cancel{4}\,2}}{12\pi c}}{\frac{\cancel{\omega^2 q_0^2}}{2}} = \frac{\mu_0 d^2 \omega^2}{6\pi c} =$$

$$\omega = 2\pi f = 2\pi \frac{c}{\lambda} \rightarrow \omega^2 = 4\pi^2 \frac{c^2}{\lambda^2}$$

$$\therefore R = \frac{\mu_0 d^2}{6\cancel{\pi c}}\left(\frac{4\pi^{\cancel{2}} c^{\cancel{2}}}{\lambda^2}\right) = \frac{\mu_0 4\pi c}{6}\left(\frac{d}{\lambda}\right)^2$$

$$\boxed{R \cong (790\,\Omega)\left(\frac{d}{\lambda}\right)^2}$$

I say Radio is def'd to be 1 kHz $\longrightarrow$ 1 GHz.

$$\lambda = \frac{c}{f} \Rightarrow \begin{array}{l} \lambda_L = \frac{3\times10^8}{1\times10^3}\,m = 3\times10^5\ m \\ \lambda_S = \frac{3\times10^8}{1\times10^9}\,m = 3\times10^{-1}\ m \end{array}$$

$$R_L = 790\left(\frac{0.05}{3\times10^5}\right)^2 \Omega = 1.32\times10^{-11}\ \Omega$$

$$R_S = 790\left(\frac{0.05}{3\times10^{-1}}\right)\Omega = 21.94\ \Omega$$

$R_{wire} < 1\,\Omega$ depending on exact dimension of wire

IS RADIATIVE RESISTANCE RELEVANT? No not for Low frequencies, However as f increases and λ shrinks it becomes much more relevant, like in Computer bread boards this factor is a significant limit to overcome. Basically it depends on who you are,

JAMES COOK

11.4

$$\vec{p} = P_o[\cos(\omega t)\,\hat{x} + \sin(\omega t)\,\hat{y}]$$, Define $S = \cos\omega(t - r/c)$ 16/20

$$\vec{E} = \frac{-\mu_o \omega^2}{4\pi} S \frac{1}{r}[(\vec{P}_o \cdot \hat{r})\hat{r} - \vec{P}_o]$$

$$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$

$$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$$

$$\vec{P}_o \cdot \hat{r} = P_o\Big[\big(\cos\omega t(\sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}\big) + \sin\omega t(\sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi})\Big] \cdot \hat{r}$$

$$(\vec{P}_o \cdot \hat{r})\hat{r} = P_o(\cos\omega t\sin\theta\cos\phi + \sin\omega t\sin\theta\sin\phi)\,\hat{r}$$

$$\vec{P}_o - (\vec{P}_o \cdot \hat{r})\hat{r} = P_o(\cos\omega t\cos\theta\cos\phi\,\hat{\theta} - \cos\omega t\sin\phi\,\hat{\phi} + \sin\omega t\cos\theta\sin\phi\,\hat{\theta} + \sin\omega t\cos\phi\,\hat{\phi})$$

$$\vec{E} = \frac{\mu_o \omega^2 P_o}{4\pi}\cos\omega(t - r/c)\Big\{\big[\cos\theta(\cos\omega t\cos\phi + \sin\omega t\sin\phi)\big]\hat{\theta} + \big[\sin\omega t\cos\phi - \cos\omega t\sin\phi\big]\hat{\phi}\Big\}$$

$$\vec{E} = \frac{\mu_o \omega^2 P_o}{4\pi}\cos\omega(t - r/c)\Big\{\cos\theta(\cos(\omega t - \phi))\,\hat{\theta} + \sin(\omega t - \phi)\,\hat{\phi}\Big\}$$

⋮

Sorry, Mechanics Test Today.

−4

$\boxed{11.6}$ Find $R_{rad.}$ for Magnetic Dipole.

$$\langle P \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3} = \frac{\mu_0 \pi^2 b^4 I_0^2 \omega^4}{12\pi c^3}$$

$$\omega = 2\pi f = \frac{2\pi c}{\lambda}$$

$$\langle I^2 \rangle = \langle I_0^2 \cos^2 \omega t \rangle = \frac{1}{2} I_0^2$$

$$R = \frac{\langle P \rangle}{\langle I^2 \rangle} = \frac{\frac{\mu_0 \pi^2 b^4 \cancel{I_0^2} \omega^4}{12\pi c^3}}{\frac{1}{2}\cancel{I_0^2}} = \frac{\mu_0 \pi b^4 \omega^4}{6 c^3} = \frac{\mu_0 \pi b^4}{6c^3}\left(\frac{2\pi c}{\lambda}\right)^4$$

$$R = \frac{\mu_0 \pi^5 b^4}{6 \cancel{c^3}} \frac{16 c^4}{\lambda^4} = \frac{\mu_0 \pi^5 c\, 8}{3}\left(\frac{b}{\lambda}\right)^4 \cong \boxed{(3\times 10^5\,\Omega)\left(\frac{b}{\lambda}\right)^4 = R}$$

as λ is typically larger than b this implies

$$\left(\frac{b}{\lambda}\right)^2 \gg \left(\frac{b}{\lambda}\right)^4 \quad (\text{I assume that } b \approx d.)$$

$$R_E = (790\,\Omega)\left(\frac{d}{\lambda}\right)^2 \quad \text{and} \quad R_B = (3\times 10^5\,\Omega)\left(\frac{d}{\lambda}\right)^4$$

So as long as $\underline{\lambda \gg d} \Rightarrow \boxed{R_E \gg R_B}$