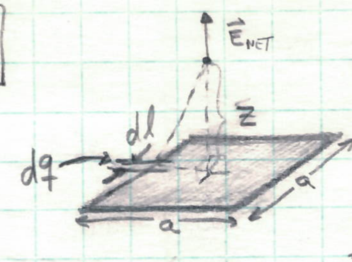
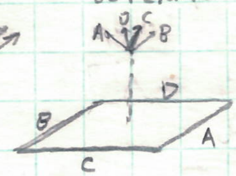
PY 414, DR. CHUNG

JAMES COOK

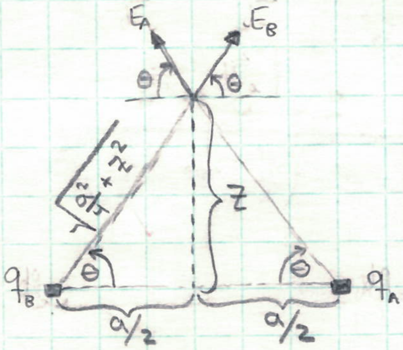
20/20

2-4





SEPERATE THE E_{NET} INTO 4 PARTS, ONE FROM EACH SIDE. IT'S HARD TO DRAW BUT THE FIELD FROM THE OPPOSING SIDE CANCELS THE (X, Y) PART ONLY THE Z PARTS CONTRIBUTE TO E_{NET}



$$E_A = \frac{\lambda}{2\pi\epsilon_0}\frac{\frac{a}{2}}{\sqrt{\frac{a^2}{4}+z^2}\left(\frac{a^2}{4}+z^2+\frac{a^2}{4}\right)^{1/2}} = E_B = E_C = E_D$$

I KNEW E_A HAS THIS FORM BECAUSE LAST CLASS WE DERIVED THE $\vec{E}$ THAT RESULTS AT A POINT ABOVE THE MIDDLE OF A LINE OF CHARGE. NOW $\vec{E}_A$ POINTS AWAY FROM Z AXIS SO I WILL ISOLATE Z COMPONENT USING

$$\sin\theta = \frac{z}{\sqrt{\frac{a^2}{4}+z^2}} \qquad E_{zA} = E_A\sin\theta$$

$$E_{NET} = E_{zA}+E_{zB}+E_{zC}+E_{zD} = \sin\theta\left(E_A+E_B+E_C+E_D\right) = 4\sin\theta E_A$$

(SOME MORE DETAIL ON BACK)

$$E_{NET} = \left(\frac{4\ z}{\left(\frac{a^2}{4}+z^2\right)^{1/2}}\right)\frac{\frac{a}{2}\ \frac{\lambda}{2\pi\epsilon_0}}{\sqrt{\frac{a^2}{4}+z^2}\ \left(\frac{a^2}{2}+z^2\right)^{1/2}} \Rightarrow \boxed{\vec{E} = \frac{\lambda a z\left(\frac{a^2}{2}+z^2\right)^{-1/2}}{\pi\epsilon_0\left(\frac{a^2}{4}+z^2\right)}\hat{k}}$$

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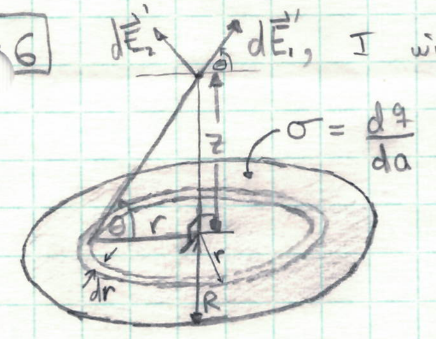
2-6



I will consider $d\vec{E}$ from a ring at r of width dr. note that $d\vec{E}_1'$ the contribution of a little bit of the ring has a countering $d\vec{E}_2$ on other side of ring. The two contributions cancel each others horizontal (x,y) components so the field is purely in z. This argument holds for all parts of the ring so we may write, dE_z for infitesimal ring at r.

$$da_{ring} = 2\pi r\,dr$$
$$dq_{ring} = \sigma\, da_{ring} = \sigma 2\pi r\,dr$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi r\,dr}{r^2+z^2}(\sin\theta) = \frac{\sigma}{2\epsilon_0}\frac{r\,dr}{r^2+z^2}\frac{z}{(r^2+z^2)^{1/2}} = \frac{\sigma z}{2\epsilon_0}\frac{r\,dr}{(r^2+z^2)^{3/2}}$$

$$E_z = \int dE_z = \frac{\sigma z}{2\epsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}} = \frac{\sigma z}{2\epsilon_0}\left(\frac{-1}{(r^2+z^2)^{1/2}}\Bigg|_0^R\right) = \frac{\sigma z}{2\epsilon_0}\left(\frac{-1}{(R^2+z^2)^{1/2}} - \frac{-1}{z}\right)$$

$$\boxed{E_z = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{(R^2+z^2)^{1/2}}\right)} \qquad E_z \longrightarrow \frac{\sigma}{2\epsilon_0} \text{ if } z \ll R$$

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$$\frac{1}{(R^2+z^2)^{1/2}} = \frac{1}{z(1+R^2/z^2)^{1/2}} \cong \frac{1}{z}\left(1-\frac{1}{2}\frac{R^2}{z^2}\right) \quad \text{for } z \gg R$$

Then $E_z \approx \frac{\sigma}{2\epsilon_0}\left(1-1+\frac{1}{2}\frac{R^2}{z^2}\right) = \frac{1}{4\pi\epsilon_0}\frac{\pi R^2\sigma}{z^2} = \frac{1}{4\pi\epsilon_0}\frac{Q_{DISK}}{z^2}$.

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2.9 GIVEN $\vec{E} = k r^3 \hat{r}$ in spherical coordinates with $k \in \mathbb{R}$.

(a) FIND charge density ρ.

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$\nabla \cdot \vec{E} = \dfrac{\rho}{\epsilon_0}$, $\nabla \cdot \vec{V}$ in spherical coordinates is given in front cover. ⇒

$$\rho = \epsilon_0 (\nabla \cdot k r^3 \hat{r}) = k\epsilon_0 \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(r^3)\right)\right] = k\epsilon_0\left(\frac{1}{r^2}(5r^4)\right) = \boxed{5k\epsilon_0 r^2}$$ ✓

(b)

(I) since the meaning of ρ is $\frac{dq}{d\tau}$ this implies that the sum of $\rho d\tau$ should give dq enclosed, (Q)

$$Q = \int_{\substack{\text{SPHERE}\\\text{RADIUS } R.}} (5k\epsilon_0 r^2) d\tau = \int_0^{\pi}\int_0^{2\pi}\int_0^{R} (5k\epsilon_0) r^2 (r^2 \sin\theta \, dr\, d\theta\, d\phi)$$

$$= 5k\epsilon_0 \int_0^R r^4 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = (5k\epsilon_0)\left(\frac{R^5}{5}\right)\left(-\cos\theta\Big|_0^{\pi}\right)(2\pi)$$

$$= (5k\epsilon_0)\left(\frac{R^5}{5}\right)(2)(2\pi) = \boxed{4\pi\epsilon_0 k R^5 = Q}$$ ✓

(II) GAUSS' LAW GIVES US $\epsilon_0 \oint_{\substack{\text{SPHERE}\\\text{RADIUS } (R)}} \vec{E} \cdot d\vec{a} = Q$. $d\vec{a}_{\text{SPHERE}} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$

$$Q = \epsilon_0 \oint_{\substack{\text{SPHERE}\\\text{RADIUS } (R).}} (k r^3 \hat{r}) \cdot (r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}) = k\epsilon_0 \oint_{\substack{\text{SPHERE}\\\text{RADIUS}\\(R).}} r^3 r^2 \sin\theta\, d\theta\, d\phi$$

r constant and equal to R over integral so we get,

$$Q = k\epsilon_0 R^5 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = k\epsilon_0 R^5 (2\pi)\left(-\cos\theta\Big|_0^{\pi}\right) = k\epsilon_0 R^5 (2\pi)(2)$$

10

$$\boxed{Q = 4\pi\epsilon_0 k R^5}$$ ✓

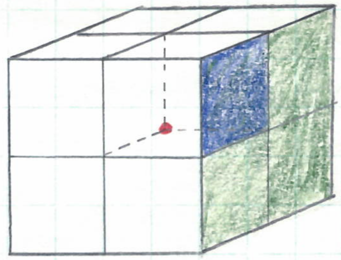
2-10

= q

= flux in?

= $\frac{1}{6}\Phi_E$

What is the flux through a side of a cube due to a charge in the back corner? I have constructed 7 other adjacent cubes as shown. The charge q is in the center of the composite cube made of 8 identical cubes like the cube given in this problem. Since the charge q is in the center of the cube and emits $\vec{E}$ in every direction radially outward no side of the composite cube is distinguishable from the other in the Φ_E that should pass through it. Because of this symetry and GAUSS LAW ($\Phi_E = \frac{Q_{enc}}{\epsilon_0}$) WE MAY CLAIM THAT THE FLUX Through just one side of the composite cube is $\frac{1}{6}\Phi_E = \frac{q}{6\epsilon_0}$. Now note that no quarter of the green side is prefered over another in the direction or magnitude of $\vec{E}$ produced by q. Thus by symetry the blue corner has $\frac{1}{4}$ of flux in green region so finally we get



$$\Phi_E(\text{blue}) = \frac{1}{4}\left(\frac{q}{6\epsilon_0}\right) = \boxed{\frac{q}{24\epsilon_0}}$$

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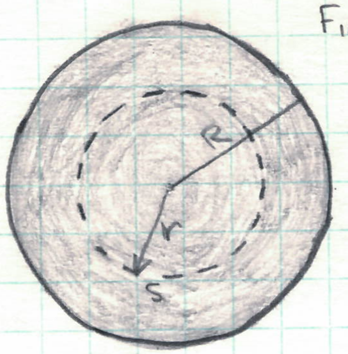

FIND $\vec{E}$ INSIDE SPHERE WITH CHARGE DENSITY,

$$\rho = kr = \frac{dq}{d\tau}$$

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GAUSS' LAW,

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint_S \vec{E} \cdot d\vec{a} = \oint_S (E(r))(da) = E(r)\, 4\pi r^2$$: $E(r)$ = E field strength for radius r.

$$\frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\oint_{\mathcal{V}} \rho\, d\tau = \frac{1}{\epsilon_0}\oint_{\mathcal{V}} kr\, d\tau$$: where $\mathcal{V}$ is sphere bounded by S.

now using sphericals,

$$\frac{Q_{enc}}{\epsilon_0} = \frac{k}{\epsilon_0}\oint_{\mathcal{V}} r(r^2 \sin\theta\, d\theta\, d\phi\, dr.$$

$$= \frac{k}{\epsilon_0}\int_0^r \bar{r}^3 d\bar{r} \int_S \sin\theta\, d\theta\, d\phi$$

$$= \frac{k}{\epsilon_0}\left(\frac{r^4}{4}\right)(4\pi) = \frac{kr^4\pi}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

Then $E(r)\, 4\pi r^2 = \frac{kr^4\pi}{\epsilon_0} \Rightarrow E(r) = \frac{kr^2}{4\epsilon_0}$

$$\boxed{\vec{E}(r) = \frac{k}{4\epsilon_0} r^2 \hat{r}}$$ where r is distance from origin of sphere of charge. And $r < R$. ✓

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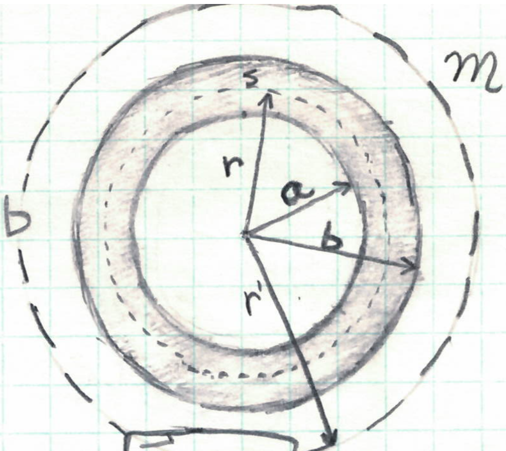
2-15



$\rho = \frac{k}{r^2}$ in the region $a \le r \le b$

(i) $r < a$.

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}, \quad Q_{enc}(r<a) = 0 \Rightarrow \boxed{\vec{E} = 0}$$

(ii) imagine s a GAUSSIAN SPHERICAL SHELL with radius r the $\vec{E}$ radiating from enclosed charge is parallel to $d\vec{a}$ for s everywhere on the shell "S" thus

$$\oint_s \vec{E} \cdot d\vec{a} = \oint_s E\, da = E \oint da = E\, 4\pi r^2$$

$$\frac{Q_{enc}}{\epsilon_0} = \oint_v \frac{\rho\, d\tau}{\epsilon_0} = \frac{k}{\epsilon_0} \oint_v \frac{r^2 \sin\theta}{r^2}\, dr\, d\theta\, d\phi = \frac{k}{\epsilon_0} \oint_v \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{k}{\epsilon_0} \int dr \int \sin\theta\, d\theta \int d\phi$$ note that v is shell of radius $a \to r$.

$$= \frac{k}{\epsilon_0} \int_a^r d\bar{r}\, (2)(2\pi) = \frac{4\pi k}{\epsilon_0}(r-a) = \frac{Q_{enc}}{\epsilon_0}.$$

then we have $\vec{E}(r) = \frac{4\pi k}{\epsilon_0}(r-a) \frac{1}{4\pi r^2} \hat{r}$

$$\boxed{\vec{E}(r) = \frac{k}{\epsilon_0}\left(\frac{r-a}{r^2}\right) \hat{r}} \quad a < r < b.$$

(iii) $r > b$

since $\vec{E} \parallel d\vec{a}$ for m (same arguments as (ii))

$$\oint_m \vec{E} \cdot d\vec{a} = E(4\pi r'^2) = \frac{Q_{enc}}{\epsilon_0}$$ using (ii) we have $Q_{enc} = \epsilon_0 \left(\frac{4\pi k}{\epsilon_0}\right)(b-a) = 4\pi k(b-a)$.

then $\vec{E}(r') = \frac{4\pi k(b-a)}{\epsilon_0} \frac{1}{4\pi r'^2} \hat{r}'$

$$\Downarrow$$

$$\boxed{\vec{E}(r) = \frac{k(b-a)}{\epsilon_0 r^2} \hat{r}} \quad \text{for } r > b$$

~~RB~~
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plot?

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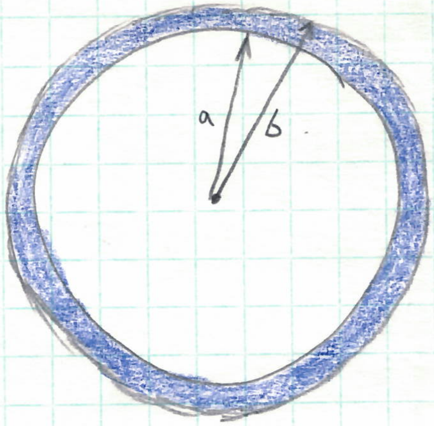
2.23 find potential at center of charge distribution of 2.15
use ∞ as reference point



from 2.15: PREVIOUS WORK YIELDED,

$$\begin{cases} \vec{E} = \dfrac{k(b-a)}{\epsilon_0 r^2} & \text{for } r > b \\ \vec{E} = \dfrac{k(r-a)}{r^2 \epsilon_0} & \text{for } a \le r \le b \\ \vec{E} = 0 & \text{fo } r < a \end{cases}$$

$$V(0) = -\int_\infty^0 \vec{E} \cdot d\vec{l} = \int_0^\infty \vec{E} \cdot d\vec{l} = \int_b^\infty \frac{k(b-a)}{\epsilon_0 r^2}\,dr + \int_a^b \frac{k(r-a)}{r^2 \epsilon_0}\,dr + \int 0.$$

$$= \left(\frac{k(b-a)}{\epsilon_0}\right) \frac{-1}{r}\Big|_b^\infty + \left(\frac{k}{\epsilon_0}\right) \int_a^b \left(\frac{r}{r^2} - \frac{a}{r^2}\right) dr$$

$$= \left(\frac{k(b-a)}{\epsilon_0}\right)\left(\frac{1}{b}\right) + \frac{k}{\epsilon_0}\left(\ln(r) + \frac{a}{r}\right)\Big|_a^b$$

$$= \frac{k}{\epsilon_0}\left(\frac{1(b-a)}{b} + \ln(b) + \frac{a}{b} - \ln(a) - \frac{a}{a}\right)$$

$$= \frac{k}{\epsilon_0}\left(1 - \frac{a}{b} + \ln(b) + \frac{a}{b} - \ln(a) - 1\right)$$

10

$$= \frac{k}{\epsilon_0}\left(\ln(b) - \ln(a)\right) = \boxed{\frac{k}{\epsilon_0} \ln\left(\frac{b}{a}\right) = V} \checkmark$$

$\boxed{2.20}$ To find if $\vec{E} = -\nabla V$ I will check $\nabla \times \vec{E} \stackrel{?}{=} \vec{0}$. If $\vec{0}$ then $\vec{E}$ can be written as gradiant of scalar function else $\vec{E}$ not an Electric Field.

(a)

$$\vec{E}_a = (kxy)\hat{i} + (2kyz)\hat{j} + (3kxz)\hat{k}$$

$$\nabla \times \vec{E}_a = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \nabla_1 & \nabla_2 & \nabla_3 \\ kxy & 2kyz & 3kxz \end{vmatrix} = \hat{i}\begin{vmatrix} \nabla_2 & \nabla_3 \\ 2kyz & 3kxz \end{vmatrix} - \hat{j}\begin{vmatrix} \nabla_1 & \nabla_3 \\ kxy & 3kxz \end{vmatrix} + \hat{k}\begin{vmatrix} \nabla_1 & \nabla_2 \\ kxy & 2kyz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(3kxz) - \frac{\partial}{\partial z}(2kyz)\right) - \hat{j}\left(\frac{\partial}{\partial x}(3kxz) - \frac{\partial}{\partial z}(kxy)\right) + \hat{k}\left(\frac{\partial}{\partial x}(2kyz) - \frac{\partial}{\partial y}(2kxy)\right)$$

$$= \hat{i}(-2ky) - \hat{j}(3kz) + \hat{k}(-2kx)$$

$\nabla \times \vec{E}_a$ is not identically zero forall $x, y, z, k \in \mathbb{R}$ for example $x = y = z = k \Rightarrow -2\hat{i} - 3\hat{j} - 2\hat{k} \neq \vec{0}$ So $\vec{E}_a$ fails. ✓

(b) $\vec{E}_b = k\left[y^2\hat{i} + (2xy + z^2)\hat{j} + (2yz)\hat{k}\right]$

$$\nabla \times \vec{E}_b = \hat{i}\left(\frac{\partial}{\partial y}(2yzk) - \frac{\partial}{\partial z}(2xy + z^2)k\right) - \hat{j}\left(\frac{\partial}{\partial x}(2yzk) - \frac{\partial}{\partial z}(y^2k)\right) + \hat{k}\left(\frac{\partial}{\partial x}(2xyk + kz^2) - \frac{\partial}{\partial y}(y^2k)\right)$$

$$= \hat{i}(2kz - 2kz) - \hat{j}(0 - 0) + \hat{k}(2yk - 2yk) \quad ✓$$

$$= \hat{i}(0) + \hat{j}(0) + \hat{k}(0) = \vec{0}, \ \forall x, y, z, k \in \mathbb{R}.$$

$$\vec{E} = -\nabla V$$

$$E_x = -\frac{\partial}{\partial x}V = ky^2 \Rightarrow V = -kxy^2 + \ldots$$

$$E_y = -\frac{\partial}{\partial y}V = (2xy + z^2)k \Rightarrow V = -kxy^2 - kyz^2$$

$$E_z = -\frac{\partial}{\partial z}V = -\frac{\partial}{\partial z}(kxy^2 + kyz^2) = -2kyz \quad \Rightarrow \boxed{V = -k(xy^2 + yz^2)} ✓$$

10 $$k\int_{(0,0,0)}^{(1,1,1)} (y^2\hat{i} + (2xy + z^2)\hat{j} + (2yz)\hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}) = k\int_{0,0,0}^{1,1,1} y^2dx + (2xy + z^2)dy + (2yz)dz = I$$

from (1,1,1) to (0,0,0) we have $x = y = z$ as well as $dx = dy = dz$ thus

$$I = k\int_0^1 (x^2 + 2x^2 + x^2 + 2x^2)dx = \int_0^1 k6x^2 = k\frac{6x^3}{3}\Big|_0^1 = 2k \stackrel{?}{=} V(1,1,1) - V(0,0,0)$$

$$V(1,1,1) = -k(1(1)^2 + 1(1)^2) = -2k, \quad V(0,0,0) = k(0) = 0.$$

$$\text{So } I = \int_{(0,0,0)}^{(1,1,1)} \vec{E} \cdot d\vec{l} = -\int_{(0,0,0)}^{(1,1,1)} \nabla V \cdot dl = V(1,1,1) - V(0,0,0) = 2k.$$

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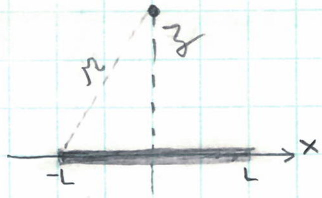
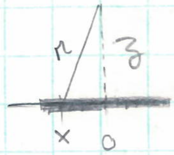
$\boxed{2\text{-}25\,b}$





$r = \sqrt{x^2 + z^2}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda}{r}\,dl = \frac{\lambda}{4\pi\epsilon_0}\int_{-L}^{L}\frac{dx}{\sqrt{x^2+z^2}} = \frac{\lambda}{4\pi\epsilon_0}\ln\left\{x+\sqrt{x^2+z^2}\right\}\Big|_{-L}^{L}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left\{\ln\left(L+\sqrt{L^2+z^2}\right) - \ln\left(-L+\sqrt{L^2+z^2}\right)\right\}$$

I would simplify further but, this is easy to take ∇ of.

$$-\vec{E} = \nabla V = i\frac{\partial}{\partial x}(V) + j\frac{\partial}{\partial y}(V) + k\frac{\partial}{\partial z}(V) = k\frac{\partial}{\partial z}(V)$$

, as V has just z dependence.

$$= k\frac{\partial}{\partial z}\left(\frac{\lambda}{4\pi\epsilon_0}\left[\ln\left(L+\sqrt{L^2+z^2}\right) - \ln\left(-L+\sqrt{L^2+z^2}\right)\right]\right)$$

$$= k\left(\frac{\lambda}{4\pi\epsilon_0}\left[\frac{\partial}{\partial z}\left(\ln\left(L+\sqrt{L^2+z^2}\right)\right) - \frac{\partial}{\partial z}\left(\ln\left(-L+\sqrt{L^2+z^2}\right)\right)\right]\right)$$

(sorry).

$$= \frac{k\lambda}{4\pi\epsilon_0}\left(\frac{1}{L+\sqrt{L^2+z^2}}\left(\frac{1}{2}\right)\left(\sqrt{L^2+z^2}\right)^{-1}(2z) - \frac{1}{-L+\sqrt{L^2+z^2}}\left(\frac{1}{2}\right)\left(\sqrt{L^2+z^2}\right)^{-1}(2z)\right)$$

$$= \frac{k\lambda}{4\pi\epsilon_0}\left(\frac{1}{z}\sqrt{L^2+z^2}\right)^{-1}\left(\frac{1}{L+\sqrt{L^2+z^2}} - \frac{1}{-L+\sqrt{L^2+z^2}}\right)$$

$$= \frac{k\lambda}{4\pi\epsilon_0}\left(\frac{1}{z}\sqrt{L^2+z^2}\right)^{-1}\left(\frac{-L+\sqrt{L^2+z^2} - L - \sqrt{L^2+z^2}}{\left(L+\sqrt{L^2+z^2}\right)\left(-L+\sqrt{L^2+z^2}\right)}\right)$$

$$= \frac{k\lambda}{4\pi\epsilon_0}\left(\frac{1}{z}\sqrt{L^2+z^2}\right)^{-1}\left(\frac{-2L}{-L^2 - L\sqrt{L^2+z^2} + L\sqrt{L^2+z^2} + \left(\sqrt{L^2+z^2}\right)^2}\right)$$

$$= \frac{-2k\lambda}{4\pi\epsilon_0}\left(\frac{1}{z}\sqrt{L^2+z^2}\right)^{-1}\left(\frac{L}{-L^2+L^2+z^2}\right)$$

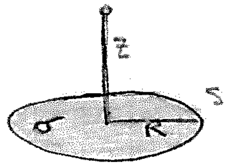
$$= \frac{-k\lambda}{2\pi\epsilon_0}\left(\frac{1}{z}\sqrt{L^2+z^2}\right)^{-1}\left(\frac{L}{z^2}\right)$$

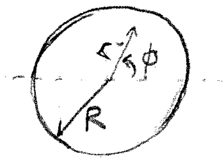
$$= \frac{-kL\lambda}{2\pi\epsilon_0}\left(z\frac{1}{\sqrt{L^2+z^2}}\right)\frac{1}{z^2} = \frac{-L\lambda}{2\pi\epsilon_0}\left(\frac{1}{z\sqrt{L^2+z^2}}\right)k = -\vec{E}$$

$$\Rightarrow \boxed{\vec{E} = \frac{L\lambda}{2\pi\epsilon_0}\left(\frac{1}{z\sqrt{L^2+z^2}}\right)k}$$

and this is what we derived for Ex. 2.1.

2.25 (c)





$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_s \frac{\sigma(\vec{r}\,')\,da'}{\mathscr{r}} = \frac{\sigma}{4\pi\epsilon_0}\int \frac{da}{\mathscr{r}}$$

$$da = dl_\phi\, dl_r = r\,d\phi\,dr \qquad \mathscr{r} = \sqrt{z^2 + r^2}$$

$$V(\vec{r}) = \frac{\sigma}{4\pi\epsilon_0}\oint_s \frac{r\,dr\,d\phi}{\sqrt{z^2+r^2}} = \frac{\sigma}{4\pi\epsilon_0}\left(\int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}}\int_0^{2\pi} d\phi\right) = \frac{\sigma}{4\pi\epsilon_0}(2\pi)\left[\frac{(z^2+r^2)^{1/2}}{1/2}\,\frac{1}{2}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[\left[z^2+R^2\right]^{1/2} - \left[z^2\right]^{1/2}\right] = \frac{\sigma}{2\epsilon_0}\left((z^2+R^2)^{1/2} - z\right) = V(z)$$

$-\vec{E} = \nabla V = \frac{\partial}{\partial z}(V)k$, as V has no ϕ or r dependence

$$= \frac{\partial}{\partial z}\left(\frac{\sigma}{2\epsilon_0}\left[(z^2+R^2)^{1/2} - z\right]\right) = \frac{\sigma}{2\epsilon_0}\left[\frac{\partial}{\partial z}\left((z^2+R^2)^{1/2}\right) - \frac{\partial}{\partial z}(z)\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left(\frac{2z(z^2+R^2)^{-1/2}}{2} - 1\right) = \frac{\sigma}{2\epsilon_0}\left(\frac{z}{(z^2+R^2)^{1/2}} - 1\right) = -\vec{E}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{(z^2+R^2)^{1/2}}\right)k}$$

and this is what was derived for problem 2.6.

PY414, ELECTROMAGNETICS 2-31, 2-32 JAMES COOK

2-31

(a) Calculate Work to place a charge q_D at (*), $q_D = q_B = -q_C = -q_A = q$.

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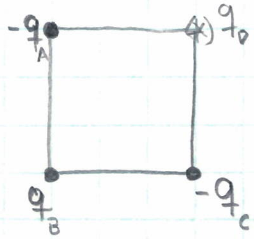


$$\Delta W_D = q_D\left(\frac{q_A}{4\pi\epsilon_0 r_{DA}} + \frac{q_B}{4\pi\epsilon_0 r_{DB}} + \frac{q_C}{4\pi\epsilon_0 r_{DC}}\right)$$
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{-q}{a} + \frac{q}{\sqrt{2}a} + \frac{-q}{a}\right)$$
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{-2q}{a} + \frac{q}{\sqrt{2}a}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{(\sqrt{2})(-2q)+q}{a\sqrt{2}}\right)$$
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{q-2\sqrt{2}q}{a\sqrt{2}}\right) = \boxed{\frac{q^2}{4\pi\epsilon_0 a}\left(1-2\sqrt{2}\right) = \text{work to put } q \text{ at } (*)}$$

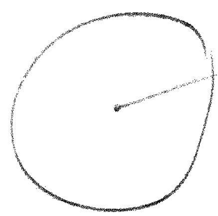
(b)

$$\Delta W_A = 0$$
$$\Delta W_B = q_B V(\vec{r}_B) = \frac{q_B q_A}{4\pi\epsilon_0 r_{BA}} = \frac{q}{4\pi\epsilon_0}\left(\frac{-q}{a}\right)$$
$$\Delta W_C = q_C V(\vec{r}_C) = \frac{q_C}{4\pi\epsilon_0}\left(\frac{q_A}{r_{CA}} + \frac{q_B}{r_{CB}}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{+q}{a\sqrt{2}} - \frac{q}{a}\right)$$
$$\Delta W_D = q_D V(\vec{r}_D) = \frac{q_D}{4\pi\epsilon_0}\left(\frac{q_A}{r_{DA}} + \frac{q_B}{r_{DB}} + \frac{q_C}{r_{DC}}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{-q}{a} + \frac{q}{a\sqrt{2}} + \frac{-q}{a}\right)$$

$$W_{TOTAL} = \Delta W_A + \Delta W_B + \Delta W_C + \Delta W_D = \frac{q}{4\pi\epsilon_0}\left(\frac{-q}{a} + \frac{q}{a\sqrt{2}} - \frac{q}{a} - \frac{q}{a} + \frac{q}{a\sqrt{2}} - \frac{q}{a}\right)$$
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{2q}{a\sqrt{2}} - \frac{4q}{a}\right)$$
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{2q - 4\sqrt{2}q}{a\sqrt{2}}\right) = \frac{q^2}{a4\pi\epsilon_0}\left(\frac{2-4\sqrt{2}}{\sqrt{2}}\right) = \frac{q^2}{4\pi\epsilon_0 a}\left(\sqrt{2}-4\right)$$

10

$\boxed{2-32}$



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}}\,d\tau = \frac{1}{4\pi\epsilon_0}\int \frac{q\,d\tau}{\frac{4}{3}\pi R^3 \mathscr{r}}$$

$$= \underbrace{\frac{3q}{4\pi\epsilon_0(4\pi R^3)}}_{\alpha}\int \frac{d\tau}{\mathscr{r}}$$

$$= \alpha \int \frac{r^2 \sin\theta\, d\theta\, d\phi\, dr}{r - r'} \qquad \vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{R} \frac{q\,dr}{4\pi\epsilon_0 r^2} - \int_{R}^{r} \frac{q\,r\,dr}{4\pi R^3 \epsilon_0} = \frac{q}{4\pi\epsilon_0}\left[\frac{+1}{r}\Big|_{\infty}^{R}\right] - \frac{q}{4\pi\epsilon_0 R^3}\left[\frac{r^2}{2}\Big|_{R}^{r}\right]$$

$$V_{inside} = \frac{q}{4\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right) = \boxed{\frac{q}{8\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 R^3}\left(\frac{r^2}{2}\right) = V_{in}}$$

$$\boxed{V_{out} = \frac{q}{4\pi\epsilon_0 r}}$$

(a) $$W = \frac{1}{2}\int \rho V\, d\tau = \frac{1}{2}\frac{3q}{4\pi R^3}\int\left(\frac{q}{8\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 R^3}\left(\frac{r^2}{2}\right)\right) r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$= \frac{3q}{8\pi R^3}\left\{\frac{q}{8\pi\epsilon_0 R}\int_{\mathcal{V}} r^2 \sin\theta\, d\theta\, d\phi\, dr - \frac{q}{8\pi\epsilon_0 R^3}\int_{\mathcal{V}} r^4 \sin\theta\, d\theta\, d\phi\, dr\right\}$$ where $\mathcal{V}$ = sphere of radius R.

$$= \frac{3q}{8\pi R^3}\left\{\frac{q}{8\pi\epsilon_0}\left(4\pi R^2\right) - \frac{q(4\pi)}{8\pi\epsilon_0 R^3}\frac{R^5}{5}\right\}$$

$$= \frac{3q}{8\pi R^3}\left\{\frac{qR^2}{2\epsilon_0} - \frac{qR^2}{2\epsilon_0(5)}\right\}$$

$$= \frac{3q^2R^2}{8\pi R^3(2\epsilon_0)}\left\{1 - \frac{1}{5}\right\}$$

$$= \frac{3q^2}{16\pi\epsilon_0 R}\left(\frac{4}{5}\right)$$

$$= \frac{q^2}{\pi\epsilon_0 R}\left(\frac{12}{80}\right)$$

$$= \frac{q^2}{\pi\epsilon_0 R}\left(\frac{(3)(4)}{(20)(4)}\right)$$

$$\boxed{W = \frac{3q^2}{20\pi\epsilon_0 R}}$$ by the method of Eq 2.43.

2-32

(b) $W = \frac{\epsilon_0}{2}\int_{\mathbb{R}^3} E^2 d\tau = \frac{\epsilon_0}{2}\underbrace{\int (E_{in})^2 d\tau}_{\text{inside sphere}} + \frac{\epsilon_0}{2}\underbrace{\int (E_{out})^2 d\tau}_{\text{outside sphere}}$

$E_{in} = \frac{\rho r}{3\epsilon_0} = \frac{q r}{4\pi R^3 \epsilon_0}$, from notes

$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$.

$$W = \frac{\epsilon_0}{2}\int \left(\frac{q}{4\pi R^3\epsilon_0}\right)^2 r^2 d\tau + \frac{\epsilon_0}{2}\int \left(\frac{q}{4\pi\epsilon_0}\right)^2 \frac{1}{r^4} d\tau$$; as $d\tau = r^2 \sin\theta \, d\theta \, d\phi \, dr$ we get,

$$= \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0 R^3}\right)^2 \int_0^R r^4 \sin\theta \, d\theta \, dr \, d\phi + \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2 \int_R^\infty \frac{1}{r^2} \sin\theta \, d\theta \, d\phi \, dr$$

$$= \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0 R^3}\right)^2 4\pi \left\{\frac{R^5}{5}\right\} + \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2 4\pi \left\{\frac{-1}{\infty} + \frac{1}{R}\right\}$$

$$= \frac{\epsilon_0}{2}\left(\frac{q^2 R^5}{4\pi\epsilon_0^2 R^6 5} + \frac{q^2}{4\pi\epsilon_0^2 R}\right)$$

$$= \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{5R} + \frac{1}{R}\right)$$

$$= \frac{q^2}{8\pi\epsilon_0}\left(\frac{6}{5R}\right) = \boxed{\frac{3q^2}{20\pi\epsilon_0 R} = W}$$ by method of 2.45.

(c)

$$W = \frac{\epsilon_0}{2}\left\{\int_v E^2 d\tau + \oint_s V\vec{E}\cdot d\vec{a}\right\}$$

for a spherical volume of radius A where v = sphere of raduis A at origin, s = surface of sphere v.

$$\frac{\epsilon_0}{2}\int_v E^2 d\tau = \frac{\epsilon_0}{2}\int_v (E_{in})^2 d\tau = \frac{q^2}{40\pi\epsilon_0 A}$$ from part (b) of this problem.

$$\frac{\epsilon_0}{2}\oint_s V\vec{E}\cdot d\vec{a} = \frac{\epsilon_0}{2}\oint_s \left(\frac{q}{4\pi\epsilon_0 A}\right)\left(\frac{q}{4\pi\epsilon_0 A^2}\hat{r}\right)\cdot\left(r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}\right)$$

$$= \frac{\epsilon_0}{2}\oint_s \frac{q^2}{(4\pi\epsilon_0)^2 A^3} r^2 \sin\theta \, d\theta \, d\phi$$ note that $r = A$ for sphere $(\forall \theta, \phi)$.

$$= \frac{\epsilon_0}{2}\frac{q^2 A^2}{(4\pi\epsilon_0)^2 A^3}\oint_s \sin\theta \, d\theta \, d\phi = \frac{q^2 (4\pi)}{32\pi^2\epsilon_0 A}$$

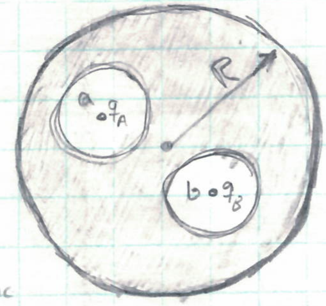
$$W = \frac{q^2}{40\pi\epsilon_0 A} + \frac{q^2}{8\pi\epsilon_0 A} = \frac{q^2}{\pi\epsilon_0}\left(\frac{1}{40A} + \frac{1}{8A}\right) = \frac{q^2}{\pi\epsilon_0}\left(\frac{8A + 40A}{320A^2}\right) = \frac{q^2}{\pi\epsilon_0}\left(\frac{48}{320A}\right)$$

$$= \frac{q^2}{\pi\epsilon_0}\left(\frac{(3)\cancel{(16)}}{(20)\cancel{(16)}A}\right) = \boxed{\frac{3q^2}{20\pi\epsilon_0 A} = W}$$ by the method of Eq. 2.44.

NOTE THAT $\lim_{A\to\infty}\frac{3q^2}{20\pi\epsilon_0 A} = \frac{3q^2}{20\pi\epsilon_0}\lim_{A\to\infty}\frac{1}{A} = 0$, So the work to distribute the charge into a really big sphere is zero. This seems logical as no 2 parts of q could have a finite r between them or else the sphere would be finite. So every part of q is only far away from another thus $V_{part} = \frac{kdq}{r} = 0$ AND thus $\sum V_{part} = 0 + 0 + \ldots = 0$ AS Before.

$\boxed{2-36}$

19/20



(a) $\sigma_a = \frac{-q_A}{4\pi a^2}$ ✓ $\sigma_b = \frac{-q_B}{4\pi b^2}$ ✓ $\sigma_R = \frac{q_a + q_B}{4\pi R^2}$ ✓

imagine the guassian just sphere that surrounds a cavity since the field must be zero $\Phi_E = 0 = Q_{enc}$ so there must be $-q_A$ and $-q_B$ on surface of the respective cavities.

(b) $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{(q_A + q_B)}{r^2} \hat{r}$ ✓ again use guassian sphere just outside sphere, it contains q_A, q_B AND the field is symmetric as $q_A + q_B$ IS INDUCED ONTO the surface because $-q_A$ and $-q_A$ are on cavity surfaces and the whole sphere is itself nuetral. and $q_A + q_B$ is a sphereically symetric distribution so we may make $\Phi = EA = \frac{q}{\epsilon_0}$ ...

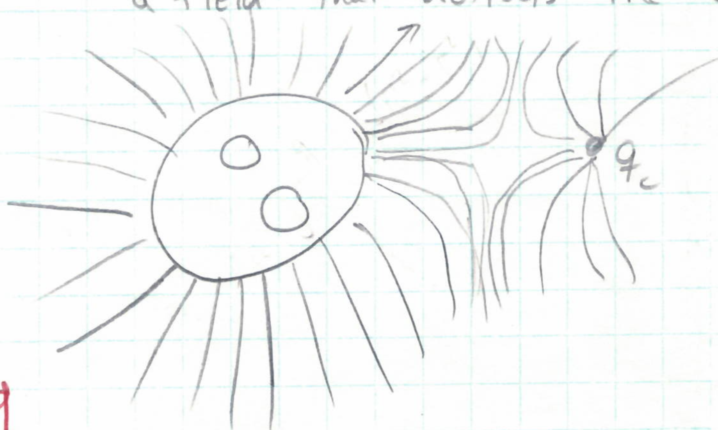
(c) $\vec{E}_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r_a^2} \hat{r}_a$ where r_a is the distance from q_a and $\hat{r}_a$ is radially out from q_a ✓

$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_B}{r_b^2} \hat{r}_b$ where r_b is the distance from q_B and $\hat{r}_b$ is radially out from q_B

(d) As $\vec{E} = \vec{0}$ inside a conducter in the static situation q_B CANNOT EFFECT q_A BECAUSE $-q_A$ IS ON THE SURFACE OF THE A CAVITY SO ALL q_A's FIELD LINES TERMINATE ON $-q_A$ ON THE SURFACE OF THE CAVITY

$F = q\vec{E} = q\vec{0} = \vec{0}$. ✓

(e) part (b) must change as q_c must produce a field that deflects the original field.



(very roughly)

assuming $(q_A + q_B)$, q_c have same magnitude of charge

σ_R would also change

9

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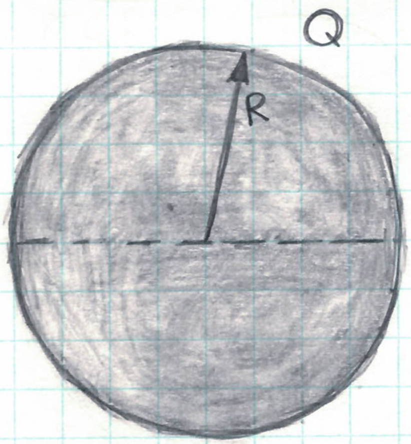

$$P = \frac{\epsilon_0}{2} E^2$$

$\cancel{P = \vec{f} \cdot \vec{A}}$ where $\vec{f} = \frac{1}{2\pi\epsilon_0} \sigma^2 \hat{r}$

I WANT TO FIND THE SUM OF ALL THE VERTICAL FORCES FOR BOTH HEMI-SPHERES (ONE AT A TIME)

$$F_{TOTAL} \text{ (vertical from top } 1/2) = \int \vec{f} \cdot \hat{z} = \int p \, d\vec{A} \cdot \hat{z}$$

$f = p\,d\vec{A}$ and $d\vec{A} \cdot \hat{z} = |dA||1| \cos(\theta)$

$p = \frac{\epsilon_0}{2} E^2$

$$\int \underbrace{\frac{\epsilon_0}{2} E^2}_{p} \underbrace{\cos\theta \, dA}_{d\vec{A}} = \frac{\epsilon_0}{2} E^2 \int \cos\theta \, r^2 \sin\theta \, d\phi \, d\theta$$

$$= \frac{\epsilon_0}{2} E^2 R^2 \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{\epsilon_0 E^2 R^2}{2} (2\pi) \int u \, du \qquad \text{where } u = \sin\theta, \; du = \cos\theta \, d\theta$$

$$= \epsilon_0 E^2 R^2 (\pi) \left. \frac{\sin^2\theta}{2} \right|_0^{\pi/2}$$

$$= \frac{\epsilon_0 E^2 R^2 (\pi)}{2}$$

As the of the sphere "feels" the top half pulling up and the bottom 1/2 pulling down so it must have a tension in it of $2F_{TOTAL} = \epsilon_0 E^2 R^2 (\pi)$ which is the repulsion between the two hemispheres.

just one

$$F_{repulsion} = \pi\epsilon_0 E^2 R^2 = \pi\epsilon_0 R^2 \left(\frac{Q}{4\pi\epsilon_0 R^2}\right)^2 = \frac{\pi\epsilon_0 R^2 Q^2}{(4\pi\epsilon_0)^2 R^4}$$

$$\boxed{F_{repulsion} = \frac{Q^2}{16\pi\epsilon_0 R^2}} \times \frac{1}{2}$$

~~4~~

right idea

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