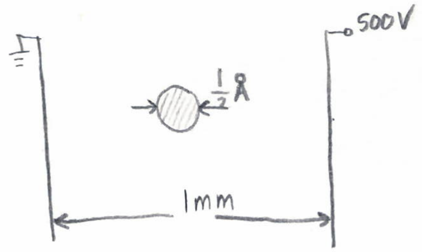
JAMES COOK

20/20

4-1

(a)



$1\,mm = 10^{-3}\,m$
$1\,Å = 10^{-10}\,m$

atomic radius $= r = \frac{1}{2}$ Å

$\alpha = 0.667(4\pi\epsilon_0)$
$q = e = 1.602\times10^{-19}\,C$

If the plates are quite large the the $\vec{E}$ is essential uniform between the plates and we remember that

$$V = Ex \Rightarrow E = \frac{V}{x} \text{ where } \begin{cases} V = 500\,V \\ x = 1\,mm \end{cases}$$

$$p = qd = \alpha E = \alpha\frac{V}{x} \Longrightarrow d = \frac{\alpha V}{qx}, \quad \alpha = (0.667)4\pi\epsilon_0$$

$$\frac{d}{\text{atomic radius}} = \frac{d}{\frac{1}{2}Å} = \frac{\frac{\alpha V}{qx}}{\frac{1}{2}Å} = \frac{(0.667)(4\pi\epsilon_0)(500V)(10)^{-30}}{(1.602\times10^{-19}\,C)(0.001m)(0.5\times10^{-10}m)} = 4.63\times10^{-6}$$

fraction $\approx \frac{1}{215,000}$ of an atomic radius ✓

$d = 2.3\times10^{-16}\,m$

(b) estimate potential necessary to ionize atom, to remove the electron from Bohr's model atom all we need is to seperate the electron by r from the center that is

$$d = r, \quad p = qd = \alpha E = \alpha\frac{V}{x} \Longrightarrow qr = \frac{\alpha V}{x}$$

$$V = \frac{xqr}{\alpha} = \frac{(0.001m)(1.602\times10^{-19}C)(10^{-10}m)\frac{1}{2}}{(4\pi\epsilon_0)(0.667)(10^{-30})} = 108\,MV$$ ✓

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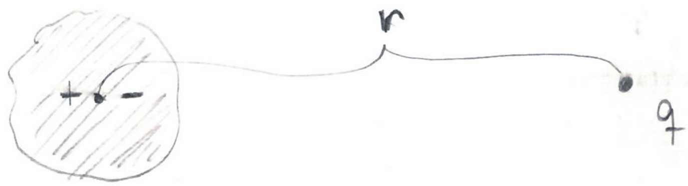
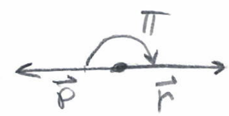
$1.1\times10^{8}\,V$

4-4 A point charge q is situated r away from a neutral atom of polarizeability α. Find force of attraction between them.

$$\vec{E}_q = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$



$$p = \alpha E_q = \frac{\alpha}{4\pi\epsilon_0}\frac{q}{r^2}$$



$$\vec{E}_{dip} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$
$$= \frac{p}{4\pi\epsilon_0 r^3}\left(-2\cos(\pi)\right)\hat{r}$$
$$= \frac{-2}{4\pi\epsilon_0 r^3}\left(\frac{\alpha}{4\pi\epsilon_0}\frac{q}{r^2}\right)\hat{r}$$
$$= \frac{-2\alpha q}{(4\pi\epsilon_0)^2 r^5}\hat{r}$$
$$= \frac{-\alpha q}{8\pi^2\epsilon_0^2 r^5}\hat{r}$$

See solution for alternate method.

$$\vec{F} = q\vec{E}_{dip} = \boxed{\frac{-\alpha q^2}{8\pi^2\epsilon_0^2 r^5}\hat{r} = \vec{F}}$$

10

$$F = -\frac{2\alpha q^2}{(4\pi\epsilon_0)^2 r^5}\hat{r}$$

4-5 PY 414, Dr. Chung James Cook

$$\vec{E}_{dip} = \frac{p}{4\pi\epsilon_0 r^3}\left(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right)$$

19/20

a) What is Torque on $\vec{P}_2$ from $\vec{P}_1$

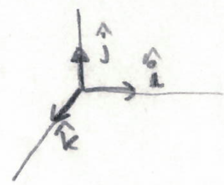
$$\vec{P}_1 = P_1\hat{j}, \quad E_1 = \frac{1}{4\pi\epsilon_0 r^3}\left(3(P_1\hat{j}\cdot\hat{i})\hat{i} - P_1\hat{j}\right) = \frac{1}{4\pi\epsilon_0 r^3}\left(-P_1\hat{j}\right) = \frac{-P_1}{4\pi\epsilon_0 r^3}\hat{j}$$

$$\vec{N}_{21} = \vec{P}_2 \times \vec{E}_1 = (P_2\hat{i}) \times \left(\frac{-P_1}{4\pi\epsilon_0 r^3}\hat{j}\right) = \frac{-P_1P_2}{4\pi\epsilon_0 r^3}(\hat{i}\times\hat{j}) = \boxed{\frac{-P_1P_2}{4\pi\epsilon_0 r^3}\hat{k}}$$

(b) What is Torque on $\vec{P}_1$ from $\vec{P}_2$

$$\vec{P}_2 = P_2 i \qquad \vec{E}_2 = \frac{1}{4\pi\epsilon_0 r^3}\left(3(P_2(i\cdot(-i))(-i) - P_2 i\right) = \frac{1}{4\pi\epsilon_0 r^3}\left(3P_2 i - P_2 i\right) = \frac{2P_2}{4\pi\epsilon_0 r^3} i$$

$$\vec{N}_{12} = \vec{P}_1 \times \vec{E}_2 = (P_1 j) \times \left(\frac{2P_2}{4\pi\epsilon_0 r^3} i\right) = \boxed{\frac{-2P_1P_2}{4\pi\epsilon_0 r^3} k}$$

note 

so my $\vec{N}$'s go into the page

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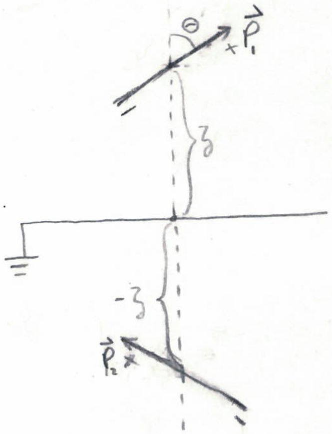
$\boxed{4-6}$



Using the method of images we situate the second pretend dipole below the plane to cancel real dipole.

$$\vec{P}_1 = P\cos\theta\,\hat{z} + P\sin\theta\,\hat{x}$$

$$\vec{P}_2 = P\cos\theta\,\hat{z} - P\sin\theta\,\hat{x}$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0 (2z)^3}\left(3(\vec{P}_2\cdot\hat{z})\hat{z} - \vec{P}_2\right) = \frac{1}{4\pi\epsilon_0 (2z)^3}\left(3\left[(P\cos\theta\,\hat{z} - P\sin\theta\,\hat{x})\cdot\hat{z}\right]\hat{z} - \vec{P}_2\right)$$

$$= \frac{P}{32\pi\epsilon_0 z^3}\left(3\cos\theta\,\hat{z} - \cos\theta\,\hat{z} + \sin\theta\,\hat{x}\right)$$

$$= \frac{P}{32\pi\epsilon_0 z^3}\left(2\cos\theta\,\hat{z} + \sin\theta\,\hat{x}\right)$$

$$\vec{N} = \vec{P}_1 \times \vec{E}_2 = \left(\cos\theta\,\hat{z} + \sin\theta\,\hat{x}\right)\times\left(2\cos\theta\,\hat{z} + \sin\theta\,\hat{x}\right)\left[\frac{P^2}{32\pi\epsilon_0 z^3}\right]$$

$$= \left[\left(\cos\theta(2\cos\theta)\,\hat{z}\times\hat{z}\right) + \left(\cos\theta\sin\theta\,\hat{z}\times\hat{x}\right) + \left(\sin\theta(2\cos\theta)\,\hat{x}\times\hat{z}\right) + \left(\sin\theta\sin\theta\,\hat{x}\times\hat{x}\right)\right]\left[\frac{P^2}{32\pi\epsilon_0 z^3}\right]$$

$$= \left[-\sin\theta\cos\theta\,\hat{y} + 2\sin\theta\cos\theta\,\hat{y}\right]\left[\frac{P^2}{32\pi\epsilon_0 z^3}\right]$$

$$= \frac{P^2}{32\pi\epsilon_0 z^3}\left(\sin\theta\cos\theta\right)\hat{y}$$

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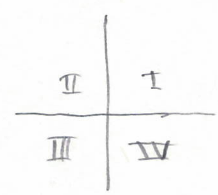
$$\boxed{\vec{N} = \frac{P^2\sin 2\theta}{4\cdot 16\pi\epsilon_0 z^3}\,\hat{y}}$$



$\vec{N} = 0$ for $\theta = 0, \frac{\pi}{2}, \pi$

Say $\vec{P}$ starts in quadrant I then $\vec{N}$ is $\hat{y}$ causing CCW rot once $\vec{N}$ goes to II then $\vec{N}$ is negative ($\theta = 0 \to -\pi/2$) so it causes a CW rotation back then a CCW then a CW ... It seems it will come to rest pointed upwards if it starts oriented as it drawn in the books diagram

N is largest at $\pi/4$

4.12 PY 414, DR. CHUNG. JAMES COOK

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$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}\cdot\hat{\imath}\,d\tau'}{\imath^2} = \frac{\vec{P}}{4\pi\epsilon_0}\cdot\int\frac{\hat{\imath}\,d\tau'}{\imath^2}$$

use trick to get

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q\vec{r}}{R^3}$$

(Uniformly charged sphere)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\hat{\imath}}{\imath^2}d\tau' = \frac{\rho}{4\pi\epsilon_0}\int\frac{\hat{\imath}}{\imath^2}d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\frac{4}{3}\pi R^3}\int\frac{\hat{\imath}}{\imath^2}d\tau'$$

(Coulomb's law)

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q\vec{r}}{R^3} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\frac{4}{3}\pi R^3}\int\frac{\hat{\imath}}{\imath^2}d\tau'$$

$$\Rightarrow \int\frac{\hat{\imath}}{\imath^2}d\tau' = \frac{Q\vec{r}}{R^3}\frac{\frac{4}{3}\pi R^3}{Q} = \vec{r}\,\frac{4}{3}\pi$$ the trick is complete

$$V = (\vec{P}\cdot\vec{r})\frac{4}{3}\pi\left(\frac{1}{4\pi\epsilon_0}\right) = \frac{\vec{P}\cdot\vec{r}}{3\epsilon_0} = \boxed{\frac{Pr\cos\theta}{3\epsilon_0} = V \quad \text{for } r \le R}$$

for $r \ge R$ entire Q enclosed

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\hat{\imath}\,d\tau'}{\imath^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\frac{4}{3}\pi R^3}\int\frac{\hat{\imath}}{\imath^2}d\tau'$$

$$\cancel{\frac{1}{4\pi\epsilon_0}}\frac{Q}{r^2}\hat{r} = \cancel{\frac{1}{4\pi\epsilon_0}}\frac{Q}{\frac{4}{3}\pi R^3}\int\frac{\hat{\imath}}{\imath^2}d\tau'$$

$$\Rightarrow \int\frac{\hat{\imath}}{\imath^2}d\tau' = \frac{Q}{r^2}\hat{r}\,\frac{\frac{4}{3}\pi R^3}{Q} = \frac{4}{3}\pi\frac{R^3}{r^2}\hat{r}$$

$$V = \frac{\vec{P}}{4\pi\epsilon_0}\cdot\int\frac{\hat{\imath}}{\imath^2}d\tau' = (\vec{P}\cdot\hat{r})\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{4}{3}\pi\frac{R^3}{r^2}\right) = \frac{R^3}{3\epsilon_0 r^2}(\vec{P}\cdot\hat{r})$$

$$\boxed{V = \frac{R^3}{3\epsilon_0 r^2}(P\cos\theta) \quad \text{for } r \ge R}$$

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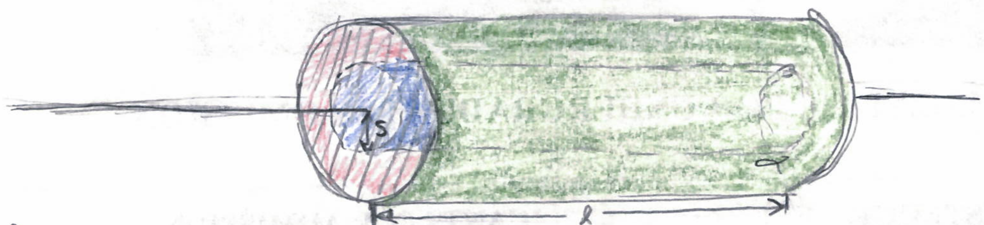
4.13 A VERY LONG CYLINDER, OF RADIUS a, CARRIES A UNIFORM POLARIZATION $\vec{P}$ PERPENDICULAR TO ITS AXIS. FIND THE ELECTRIC FIELD INSIDE THE CYLINDER. FIRST FIND $\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ FOR INSIDE CYLINDER TO SOLVE $V = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\mathscr{r}}\cdot\vec{P}}{\mathscr{r}^2}\,d\tau'$

IMAGINE UNIFORMLY CHARGED CYLINDER



$$\oint \vec{E}\cdot d\vec{a} = E(\ell 2\pi s) = \frac{Q_{enc}}{\epsilon_0} = \frac{\pi s^2 \ell \rho}{\epsilon_0} \Rightarrow \vec{E} = \frac{\pi s^2 \ell \rho \hat{s}}{\epsilon_0(\ell 2\pi s)} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\,\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau$$

$$\Rightarrow \int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\,d\tau' = (4\pi\epsilon_0)\left(\frac{\pi \vec{s}}{\epsilon_0 2\pi}\right) = 2\pi\vec{s}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' = \frac{\vec{P}}{4\pi\epsilon_0}\cdot\int\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' = \frac{\vec{P}}{4\pi\epsilon_0}\cdot 2\pi\vec{s} = \frac{1}{2\epsilon_0}Ps\cos\phi = \frac{1}{2\epsilon_0}Pz$$

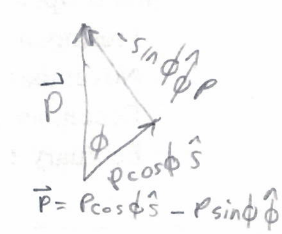
$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial z}\hat{z} = \boxed{\frac{-P}{2\epsilon_0}\hat{z} = E_{inside}} \checkmark \quad s \le a$$

OUTSIDE

$$\oint \vec{E}\cdot d\vec{a} = E(2\pi s\ell) = \frac{Q_{enc}}{\epsilon_0} = \frac{\pi a^2 \ell \rho}{\epsilon_0} \Rightarrow \vec{E} = \frac{\pi a^2 \ell \rho}{\epsilon_0(2\pi s\ell)}\hat{s} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$

$$\Rightarrow \int\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' = (4\pi\epsilon_0)\left(\frac{\pi a^2\hat{s}}{\epsilon_0(2\pi s)}\right) = \frac{2\pi a^2\hat{s}}{s}$$

$$V = \frac{\vec{P}}{4\pi\epsilon_0}\cdot\int\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' = \frac{\vec{P}}{4\pi\epsilon_0}\cdot\frac{2\pi a^2\hat{s}}{s} = \frac{a^2}{2\epsilon_0 s}\vec{P}\cdot\hat{s} = \frac{a^2 P\cos\phi}{2\epsilon_0 s}$$



$$\vec{E} = -\nabla V = -\hat{s}\frac{\partial}{\partial s}\left(\frac{a^2 P\cos\phi}{2\epsilon_0 s}\right) - \hat{\phi}\frac{\partial}{s\partial\phi}\left(\frac{a^2P\cos\phi}{2\epsilon_0 s}\right)$$

$$= -\hat{s}\left(\frac{a^2P\cos\phi}{2\epsilon_0}\right)\left(\frac{-1}{s^2}\right) - \hat{\phi}\left(\frac{a^2P}{2\epsilon_0 s^2}\right)(-\sin\phi)$$

$$= \frac{Pa^2\cos\phi}{2\epsilon_0 s^2}\hat{s} + \frac{Pa^2\sin\phi}{2\epsilon_0 s}\hat{\phi} = \frac{a^2}{2\epsilon_0 s^2}\left(P\cos\phi\,\hat{s} + P\sin\phi\,\hat{\phi}\right)$$

$$= \frac{a^2}{2\epsilon_0 s^2}\left(P\cos\phi\,\hat{s} \cancel{+ P\cos\phi\,\hat{s}} - \cancel{P\cos\phi\,\hat{s}} + P\sin\phi\,\hat{\phi}\right) \text{ ok}$$

$$\ne \frac{a^2}{2\epsilon_0 s^2}\left(2(P\cos\phi\,\hat{s}) - \vec{P}\right)$$

~~8~~ 10

$$\boxed{E_{out} = \frac{a^2}{2\epsilon_0 s^2}\left(2(\vec{P}\cdot\hat{s})\hat{s} - \vec{P}\right)}$$

$P = P\cos\phi\,\hat{s} - P\sin\phi\,\hat{\phi}$

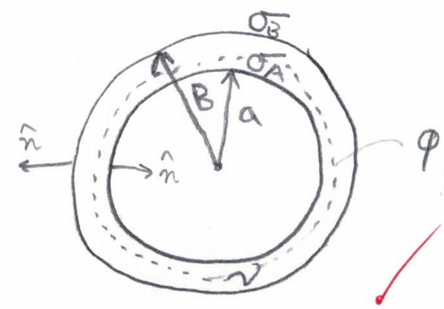
looks right, no s^2 in soln but check posted soln

$\boxed{4.15}$ A thick spherical shell is made of dielectric with an intrinsic Polarization

$P(\vec{r}) = \frac{k}{r}\hat{r}$ where k is a constant and r is the radial distance from center.

39/40

(a) locate all bound charge and then use GAUSS'S LAW TO FIND $\vec{E}$.



$$\sigma_B = \vec{P}\cdot\hat{n} = \frac{k}{r}\hat{r}\cdot\hat{r}\Big|_{r=B} = \frac{K}{B},$$

$$\sigma_A = \vec{P}\cdot\hat{n} = \frac{K}{r}\hat{r}\cdot(-\hat{r})\Big|_{r=a} = \frac{-K}{a}.$$

$$\rho_{SHELL} = -\nabla\cdot\vec{P} = \frac{-1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{K}{r}\right) = \frac{-1}{r^2}\frac{\partial}{\partial r}(kr) = \frac{-k}{r^2}\cdot \text{ for } a<r<b.$$

IN SHELL

$$\epsilon_0\oint\vec{E}\cdot d\vec{a} = \epsilon_0 E(4\pi r^2) = Q_{enc}$$

$$Q_{enc} = \int_\varphi \rho\, d\tau' + \sigma_A(4\pi a^2) = \int_\varphi \frac{-k}{r^2}r^2 d\Omega - \frac{K}{a}4\pi a^2 = -K(4\pi)(r-a) - 4\pi K a$$

$$\epsilon_0 E(4\pi r^2) = k(4\pi r + 4\pi a - 4\pi a) = -K4\pi r \Rightarrow \boxed{\vec{E} = \frac{-K}{\epsilon_0 r}\hat{r} \quad a<r<b}$$

OUTSIDE SHELL

$$\epsilon_0(4\pi r^2) = Q_{enc}, \qquad Q_{enc} = \int_{\mathcal{V}} \rho\, d\tau + \sigma_B(4\pi B^2) + \sigma_a(4\pi a^2)$$

$$= \int_{\mathcal{V}} -K\, d\Omega + 4\pi K B - 4\pi K a$$

$$= (a-B)(4\pi K) + 4\pi K B - 4\pi K a$$

$$= 0 \quad \Rightarrow \boxed{\vec{E} = \vec{0},\ r>b}$$

and of course $\boxed{\vec{E} = \vec{0} \quad \text{for } r<a}$ as $Q_{enc} = 0$ in that region

$\boxed{4.15}$

(b) Use Eq. 4.23 to find $\vec{D}$ and then get $\vec{E}$ from 4.21

$$\oint \vec{D} \cdot d\vec{a} = 4\pi r^2 D = 0 \implies \vec{D} = \vec{0}.$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

(i) $r < a \quad \vec{P} = \vec{0} \implies \vec{0} = \epsilon_0 \vec{E} + \vec{0} \implies \vec{E} = 0$

(ii) $a < r < b, \; \vec{P} = \frac{k}{r}\hat{r} \implies \vec{0} = \epsilon_0 \vec{E} + \frac{k}{r}\hat{r}$

$$\implies \vec{E} = \frac{-k}{\epsilon_0 r}\hat{r}$$

(iii) $r > b, \; \vec{P} = \vec{0} \implies \vec{0} = \epsilon_0 \vec{E} + \vec{0} \implies \vec{E} = 0$

$$\boxed{\vec{E} = \begin{cases} \vec{0} & r < a \\ \frac{-k}{\epsilon_0 r}\hat{r} & a < r < b \\ \vec{0} & r > b \end{cases}} = -P/\epsilon_0$$

10

$\boxed{4.16}$ SUPPOSE THE FIELD INSIDE A DIELECTRIC IS $\vec{E}_0$ such that $\vec{D}_0 = \epsilon_0 \vec{E}_0 + \vec{P}$

(a) a small spherical cavity is hollowed out of the material Find the field at the center in terms of $\vec{E}_0$ and $\vec{P}$

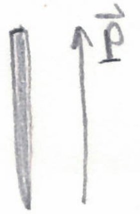
from the previous work if $\vec{P}$ points upward

then $\vec{E} = \frac{-1}{3\epsilon_0}\vec{P}$ so $\vec{E} = \frac{1}{3\epsilon_0}\vec{P}$ from Ex. 4.2.

$$\boxed{\vec{E}_{net} = \vec{E}_0 + \frac{1}{3\epsilon_0}\vec{P}}$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \cancelto{0}{\vec{P}} = \epsilon_0\left(\vec{E}_0 + \frac{1}{3\epsilon_0}\vec{P}\right) = \epsilon_0 \vec{E}_0 + \vec{P} - \frac{2}{3}\vec{P}$$

as $\vec{D}_0 = \epsilon_0 \vec{E}_0 + \vec{P} \Rightarrow \boxed{\vec{D} = \vec{D}_0 - \frac{2}{3}\vec{P}}$

(b) 

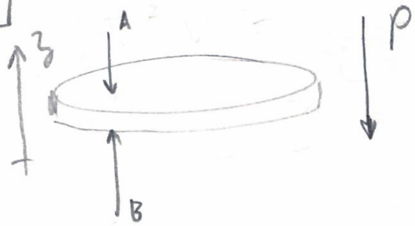
As the needle is soooo tiny no appreciable σ can accumulate at either end thus the E field between them is zero.

$$\vec{E}_{net} = \vec{E}_0 + \vec{0} = \boxed{\vec{E}_0 = \vec{E}}$$, also $\vec{P} = 0$ as we removed material

$$\vec{D} = \epsilon_0 \vec{E} + \cancelto{0}{\vec{P}} = \epsilon_0 \vec{E}_0 + \vec{P} - \vec{P} = \boxed{\vec{D}_0 - \vec{P} = \vec{D}}$$

4.16

(c)



$\sigma_A = \vec{P}\cdot(-\hat{z}) = -P$

$\sigma_B = \vec{P}\cdot\hat{z} = P$

$$\vec{E} = \frac{\sigma_A}{2\epsilon_0}(-\hat{z}) + \frac{\sigma_B}{2\epsilon_0}\hat{z} = \frac{P}{2\epsilon_0}\hat{z} + \frac{P}{2\epsilon_0}\hat{z} = \frac{P\hat{z}}{\epsilon_0} = \frac{\vec{P}}{\epsilon_0}$$

$\boxed{\vec{E} = E_0 + \frac{\vec{P}}{\epsilon_0}}$ then $\vec{D} = \epsilon_0\vec{E} + \cancel{\vec{P}} \to 0$ as we cut it out

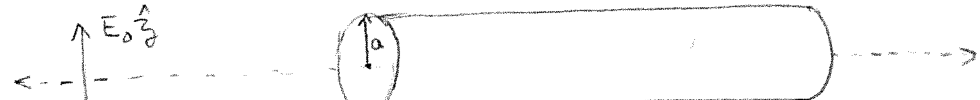
$$\vec{D} = \epsilon_0\vec{E} = \epsilon_0\left(\vec{E}_0 + \frac{\vec{P}}{\epsilon_0}\right) = \boxed{\vec{D}_0 = \vec{D}}$$

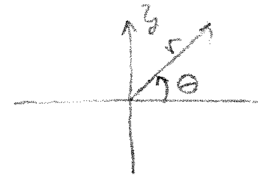
✓

10

$\boxed{4.22}$ A very long cylinder of linear dielectric material is placed in an otherwise uniform field $\vec{E}_0$. Find field inside cylinder with susceptibility χ_e

r = radial distance from axis





$r\sin\theta = z$.

BOUNDARY CONDITIONS

(i) $V_{in} = V_{out}$ at $r = a$

(ii) $\epsilon \frac{\partial V_{in}}{\partial r} = \epsilon_0 \frac{\partial V_{out}}{\partial r}$ at $r = a$

(iii) $V_{out} \longrightarrow -r\sin\theta\, E_0 = -z E_0$ as $\vec{E} = -\nabla V$ and $-\frac{\partial}{\partial z}(-E_0 z) = E_0$

GENERAL Solution for problems that possess cylindrical symetry.

$$V(r,\theta) = a_0 + b_0 \ln(r) + \sum_{n=1}^{\infty}\left\{r^n\left(a_n\cos(n\theta) + b_n\sin(n\theta)\right) + r^{-n}\left(c_n\cos n\theta + d_n \sin n\theta\right)\right\}$$

$$V_{in} = a_0 + \sum_{n=1}^{\infty} r^n\left(a_n\cos(n\theta) + b_n\sin(n\theta)\right)$$ as $r^{-n}, \ln(r) \longrightarrow \infty$ as $r \longrightarrow 0$.

$$V_{out} = -rE_0\sin\theta + \sum_{n=1}^{\infty} r^{-n}\left(c_n\cos n\theta + d_n\sin n\theta\right)$$ as all other terms would cause $V_{out} \neq -rE_0\sin\theta$ for $r \gg a$.

$$\left.\frac{\partial V_{in}}{\partial r}\right|_{r=a} = \sum_{n=1}^{\infty} n a^{n-1}\left(a_n\cos n\theta + b_n \sin n\theta\right)$$

$$\left.\frac{\partial V_{out}}{\partial r}\right|_{r=a} = -E_0\sin\theta + \sum_{n=1}^{\infty} -n a^{-n-1}\left(c_n\cos n\theta + d_n\sin n\theta\right)$$

$$\epsilon_0(1+\chi_e)\left\{\sum_{n=1}^{\infty} n a^{n-1}\left(a_n\cos n\theta + b_n\sin n\theta\right)\right\} = \epsilon_0\left\{-E_0\sin\theta + \sum_{n=1}^{\infty} -n a^{-n-1}\left(c_n\cos n\theta + d_n\sin n\theta\right)\right\}$$

the above implies that all terms with $n \neq 1$ must be zero or else a contradiction is reached as the $\sin\theta$ term cannot be canceled by any $n \neq 1$ sin or cos.

$$\epsilon_0(1+\chi_e)\left[a_1\cos\theta + b_1\sin\theta\right] = \epsilon_0\left[-E_0\sin\theta - a^{-2}(c_1\cos\theta + d_1\sin\theta)\right]$$

$$V_{in} = a_0 + a'(a_1\cos\theta + b_1\sin\theta) = -rE_0\sin\theta + a^{-1}(c_1\cos\theta + d_1\sin\theta)$$

$$a a_1 = a^{-1}c_1 \qquad \epsilon_0(1+\chi_e)a_1 = \epsilon_0(-a^{-2})c_1$$

$$a b_1 = -rE_0 + a^{-1}d_1 \qquad \epsilon_0(1+\chi_e)b_1 = \epsilon_0(-E_0 - a^{-2}d_1)$$

$$a^2 = \frac{c_1}{a_1} \quad \text{but} \quad a^2 = \frac{-c_1}{(1+\chi_e)a_1} \implies a_1 = c_1 = 0.$$

then $b_1 = \dfrac{-rE_0 + a^{-1}d_1}{a} \implies \epsilon_0(1+\chi_e)\left(\dfrac{-rE_0 + a^{-1}d_1}{a}\right) = \epsilon_0(-E_0 - a^{-2}d_1)$

$$(1+\chi_e)\left(\frac{-rE_0}{a}\right) + E_0 = -a^{-2}d_1 - (1+\chi_e)\frac{a^{-1}}{a}d_1 =$$

$\boxed{4.22}$

$$(1+\chi_e)\left(\frac{-rE_0}{a}\right) + E_0 = -a^{-2}\left(d_1 + (1+\chi_e)d_1\right)$$

$(r=a)$

$$(1+\chi_e)(-E_0) + E_0 = d_1\left(\frac{1+1+\chi_e}{a^2}\right)$$

$$-E_0 - \chi_e E_0 + E_0 = d_1\left(\frac{2+\chi_e)}{-a^2}\right)$$

$$d_1 = \frac{a^2 \chi_e E_0}{(2+\chi_e)}$$

$$b_1 = -E_0 + \frac{d_1}{a^2} = -E_0 + \frac{\chi_e E_0}{(2+\chi_e)}$$

$$V_{in} = \cancel{\alpha_0}^{\,0} + \sum_{n=1}^{\infty} r^n(a_n \cancel{\cos n\theta}^{\,0} + b_n \sin n\theta)^{\,0 \text{ if } n \neq 1}$$

thus $$\boxed{V_{in} = r\left(-E_0 + \frac{\chi_e E_0}{2+\chi_e}\right)\sin\theta}$$

$$\frac{-2\epsilon_0 E_0 z \cos\phi}{\epsilon+\epsilon_0}$$

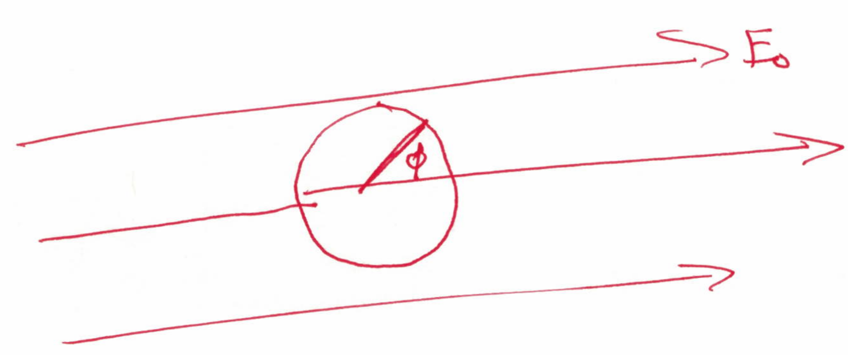
$$= rE_0\left(\frac{-2-\chi_e + \chi_e}{2+\chi_e}\right)\sin\theta$$

$$V_{in} = \frac{-2rE_0\sin\theta}{2+\chi_e}.$$

note this would differ if one set up θ differently.

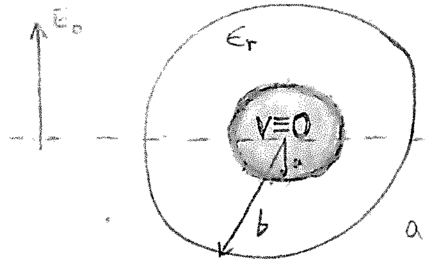
$$\vec{E} = -\nabla V = \left[\frac{-\partial}{\partial z}\left(\frac{-2E_0 z}{2+\chi_e}\right)\right]\hat{z} = \frac{-2E_0}{2+\chi_e}\hat{z} = \boxed{\frac{-2\vec{E}_0}{2+\chi_e} = \vec{E}_{in}}$$

10



$\boxed{4.24}$ A conducting sphere of radius a, is coated with a thick insulator (dielectric ϵ_r) out to radius b the object is placed in an otherwise uniform field $\vec{E}_0$. Find the $\vec{E}$ field in the insulator.

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + B_l/r^{l+1}\right)P_l(\cos\theta)$$



since $V_{out} \to -E_0 r\cos\theta$ for $r \gg R$

$$\Rightarrow \quad V_{out} = -E_0 r\cos\theta + \sum_{l=0}^{\infty}\frac{B_l}{r^{l+1}}P_l(\cos\theta)$$

$a<r<b \quad V_{in} = \sum_{l=0}^{\infty}\left(A_l r^l + C_l/r^{l+1}\right)P_l(\cos\theta)$

$$V_{in}(a,\theta) \equiv 0 = \sum_{l=0}^{\infty}\left(A_l a^l + C_l/a^{l+1}\right)P_l(\cos\theta) \quad \Rightarrow \quad A_l = -C_l/a^{2l+1}$$

$$= \sum_{l=0}^{\infty}\left(C_l\left(\frac{1}{r^{l+1}} - \frac{r^l}{a^{2l+1}}\right)\right)P_l(\cos\theta)$$

$$\epsilon\left.\frac{\partial V_{in}}{\partial r}\right|_b = \epsilon_0\left.\frac{\partial V_{out}}{\partial r}\right|_b \quad \Rightarrow \quad \frac{\epsilon}{\epsilon_0}\left.\frac{\partial V_{in}}{\partial r}\right|_b = \epsilon_r\left.\frac{\partial V_{in}}{\partial r}\right|_b = \left.\frac{\partial V_{out}}{\partial r}\right|_B$$

$$\frac{\partial}{\partial r}\left(\sum^{\infty}\left(C_l\left(\frac{1}{r^{l+1}} - \frac{r^l}{a^{2l+1}}\right)P_l(\cos\theta)\right)\right)\Bigg|_b = \sum^{\infty} C_l P_l(\cos\theta)\left[\frac{-l-1}{b^{l+2}} - \frac{l b^{l-1}}{a^{2l+1}}\right] = \left.\frac{\partial V_{in}}{\partial r}\right|_b$$

$$\frac{\partial}{\partial r}\left(-E_0 r\cos\theta + \sum^{\infty}\frac{B_l}{r^{l+1}}P_l(\cos\theta)\right)\Bigg|_b = -E_0\cos\theta + \sum^{\infty}\frac{-l-1}{b^{l+2}}P_l(\cos\theta)B_l$$

$$\epsilon_r\left(\sum^{\infty}C_l P_l(\cos\theta)\left[\frac{-l-1}{b^{l+2}} - \frac{l b^{l-1}}{a^{2l+1}}\right]\right) = -E_0\cos\theta + \sum^{\infty}\frac{-l-1}{b^{l+2}}P_l(\cos\theta)B_l$$

if $l \neq 1$ then $0 = -E_0\cos\theta \Rightarrow l$ must be just one for surviving terms

$$\epsilon_r C_1\cos\theta\left[\frac{-2}{b^3} - \frac{l}{a^3}\right] = -E_0\cos\theta - \frac{2B_1}{b^3}\cos\theta$$

$$C_1 = \frac{-E_0\cos\theta - \frac{2B_1}{b^3}\cos\theta}{\epsilon_r\cos\theta\left[\frac{-2}{b^3} - \frac{l}{a^3}\right]} \quad \Rightarrow \quad V_{in} = \frac{-E_0 - \frac{2B_1}{b^3}}{\epsilon_r\left[\frac{-2}{b^3} - \frac{l}{a^3}\right]}\left(\frac{1}{b^2} - \frac{b}{(a^3)}\right)P_1 =$$

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$$V_{in}\big|_{r=b} = V_{out}\big|_{r=b}$$

$$\frac{-\epsilon_0 - \frac{2B_1}{b^3}}{\epsilon_r\left[\frac{-2}{b^3} - \frac{l}{a^3}\right]}\left(\frac{1}{b^2} - \frac{b}{a^3}\right)P_1 = -E_0 b\cos\theta + \frac{B_1}{b^2}\cos\theta$$

$$-\frac{\left(\frac{1}{b^2} - \frac{b}{a^3}\right)}{\left(\frac{-2}{b^3} - \frac{l}{a^3}\right)}\epsilon_0 + E_0 b = B_1\left(\frac{1}{b^2} + \underbrace{\frac{\left(\frac{1}{b^2} - \frac{b}{a^3}\right)}{\left(\frac{-2}{b^3} - \frac{l}{a^3}\right)}}_{Q}\right)$$

$$B_1 = \frac{Q\epsilon_0 + E_0 b}{Q} = \epsilon_0 + \frac{E_0 b}{Q}$$

$$C_1 = \frac{-\epsilon_0 - \frac{2B_1}{b^2}}{\underbrace{\epsilon_r\left[\frac{-2}{b^3} - \frac{l}{a^3}\right]}_{\alpha}} = \frac{-\epsilon_0}{\alpha} - \frac{2}{\alpha b^2}\left(\epsilon_0 + \frac{E_0 b}{\left(\frac{\frac{1}{b^2} - \frac{b}{a^3}}{\frac{-2}{b^3} - \frac{l}{a^3}}\right)}\right)$$

$$C_1 = \frac{-\epsilon_0 - \frac{2}{b^2}\epsilon_0 - 2\epsilon_0 b\left(\frac{-2/b^3 - l/a^3}{1/b^2 - b/a^3}\right)}{\epsilon_r\left[\frac{-2}{b^3} - \frac{l}{a^3}\right]}$$

9

$$= \frac{-\epsilon_0\left(1 + \frac{2}{b^2}\right)}{\epsilon_r\left(\frac{-2}{b^3} - \frac{l}{a^3}\right)} - 2E_0 b\left(\frac{\frac{-2/b^3 - l/a^3}{1/b^2 - b/a^3}}{-2/b^3 - l/a^3}\right)$$

$$= \frac{-\epsilon_0\left(1 + 2/b^2\right)}{\epsilon_r\left(-2/b^3 - l/a^3\right)} - 2E_0 b\left(\frac{1}{1/b^2 - b/a^3}\right) = \frac{-\epsilon_0\left(1+2/b^2\right)\left(1/b^2 - b/a^3\right) - 2E_0 b\left(\epsilon_r\left(-2/b^3 - l/a^3\right)\right)}{\epsilon_r\left(-2/b^3 - l/a^3\right)\left(1/b^2 - b/a^3\right)}$$

$l=1$.

$$= \left(\frac{1}{b^2} - \frac{b}{a^3} + \frac{2}{b^4} - \frac{2}{ab}\right)(-\epsilon_0) - 2E_0\epsilon_r\left(\frac{-2}{b^2} - \frac{b}{a^3}\right) =$$

$$E_{in} = -\nabla V_{in} = \frac{-\partial}{\partial r}\left(C_1\left(\frac{1}{r^2} - \frac{r}{a^3}\right)\cos\theta\right)\hat{r} - \frac{\partial}{\partial\theta}\left(C_1\left(\frac{1}{r^2} - \frac{r}{a^3}\right)\cos\theta\right) = \overbrace{\boxed{C_1\left(\left(\frac{2}{r^3} + \frac{1}{a^3}\right)\cos\theta\,\hat{r} - \left(\frac{1}{r^2} - \frac{r}{a^3}\right)\sin\theta\,\hat{\theta}\right)}}^{\vec{E}}$$

almost

$$\text{where } C_1 = \frac{-\epsilon_0\left(1 + \frac{2}{b^2}\right) - 2E_0\left(\frac{-2/b^2 - b/a^3}{1/b^2 - b/a^3}\right)}{\epsilon_r\left[\frac{-2}{b^3} - \frac{1}{a^3}\right]}$$

check this

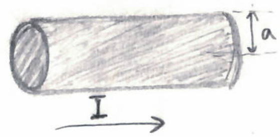
just could get C_1 to work out.

JAMES COOK

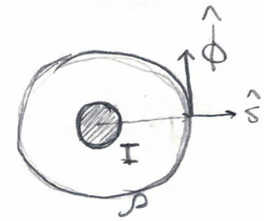
18/20

5.13 $\quad s \equiv r$

(a)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



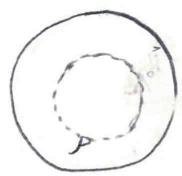
I_{enc} for $r<a$ is zero $\Rightarrow \vec{B}=0$

I_{enc} for $r>a$ is I thus $\oint_P \vec{B}\cdot d\vec{l} = \mu_0 I = B(2\pi r) \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}$

$$B = \begin{cases} 0 & \text{for } r<a \\ \frac{\mu_0 I}{2\pi r} & \text{for } r>a \end{cases}$$ which is in $\hat{\phi}$ direction

(b) The current density J is proportional to r, where r is the distance from axis

$$J = \frac{dI}{da_\perp} = kr \quad \Rightarrow \quad dI = kr\,da_\perp = kr(2\pi r\,dr)$$



$$\int dI = I_{enc} = \int_0^r 2\pi k \tilde{r}^2 d\tilde{r} = \frac{2\pi k}{3} r^3 = I$$

$$I_{enc}(a) \Rightarrow k = \frac{3I}{2\pi a^3}$$

$$\Rightarrow I_{enc} = \frac{2\pi}{3}\left(\frac{3I}{2\pi a^3}\right) r^3$$

$$\Rightarrow I_{enc} = \frac{I}{a^3} r^3$$

$$\oint_P \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = (2\pi r) B_{in} = \mu_0 \left(\frac{I}{a^3} r^3\right)$$

$$B_{in} = \frac{\mu_0 I}{2\pi a^3} r^2$$

$$B = \begin{cases} \frac{\mu_0 I}{2\pi a^3} r^2 & \text{if } r<a \\ \frac{\mu_0 I}{2\pi r} & \text{if } r>a \end{cases}$$ again in the $\hat{\phi}$ direction

10

$\boxed{5.19}$

(a) find ρ for electrons in copper wire, assume 2 free e^- for atoms.

D = density of copper $= 8900 \,kg/m^3 = 8.9\times10^6 \,g/m^3$

$63.59 \,g/mol$ of copper

$$\rho = \left(8.9\times10^6 \frac{g}{m^3}\right)\left(\frac{1}{63.59 \frac{g}{mol}}\right)\left(6.023\times10^{23}\frac{atoms}{mol}\right)\left(\frac{2e^-}{atom}\right) = \boxed{1.686\times10^{29}\frac{e^-}{m^3} = \rho}$$

$$= 2.7\times10^{10}\frac{C}{m^3} = \rho$$

(b) $J = \frac{dI}{da_\perp} = \rho v$

$$v = \frac{dI}{\rho\, da_\perp} = \frac{I}{\rho a} = \frac{I}{\rho(\pi(\frac{d}{2})^2)} = \frac{(1 \,C/s)\,4}{(2.7\times10^{10} \,C/m^3)\,\pi(0.001m)^2} = \boxed{4.716\times10^{-5}\frac{m}{s}}$$

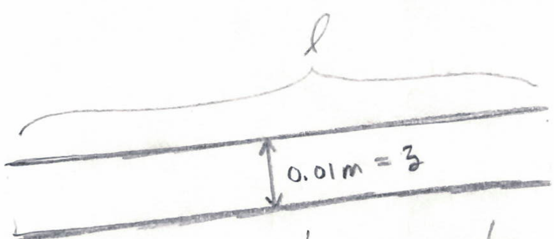
(c) The force of attraction must depend on the length of the wire involved thus the F will be in terms of then length of wire. l = length

$\vec{F} = i\vec{l}\times\vec{B}$ where $B = \frac{\mu_0 I}{2\pi s}$ where $s = 0.01 \,m$, $I = 1A$, $\mu_0 = 4\pi\times10^{-7} \,N/A^2$

$$F = ilB = (1A)\,l\left(\frac{4\pi\times10^{-7} N/A^2}{2\pi(0.01m)}\right)(1A) = \boxed{l\left(2\frac{N}{m}\times10^{-5}\right) = F_B}$$

if $l = 1m$ then $F_B = (2\times10^{-5})N$

(d)



$\rho = \frac{Q}{V} = \frac{Q}{lA}$

$Q = \rho l a$ by symetry

$$F = \frac{1}{4\pi\epsilon_0}\frac{QQ}{z^2}$$

8

$$F = \frac{1}{4\pi\epsilon_0}\frac{\rho^2 l^2 a^2}{z^2} = l^2\left(\frac{(2.7\times10^{10})^2(\pi(0.0005)^2)^2}{0.0001}\frac{N}{m^2}\right) = \boxed{l^2\left(4.043\times10^{22}\frac{N}{m^2}\right) = F_E}$$

20

$$F_E = \alpha F_B \Rightarrow \alpha = \frac{F_E}{F_B} = \frac{l^2(4.043\times10^{22}\frac{N}{m^2})}{l(2\frac{N}{m}\times10^{-5})} = \boxed{l\left(2.021\times10^{27}\right)}$$

25

this many times greater

This question is poor. Assume $l = 1m$ then $F_E = (2.021\times10^{27})F_B$

See solution

$\boxed{5.24}$ If $\vec{B}$ is uniform show that $A(\vec{r}) = -\frac{1}{2}(\vec{r}\times\vec{B})$

$$\nabla\cdot A = \nabla\cdot\left(-\frac{1}{2}\vec{r}\times\vec{B}\right) = -\frac{1}{2}\left\{\vec{B}\cdot(\nabla\times\vec{r}) - \vec{r}\cdot(\nabla\times\vec{B})\right\}$$

$$\nabla\times\vec{r} = \begin{vmatrix} i & j & k \\ \nabla_x & \nabla_y & \nabla_z \\ x & y & z \end{vmatrix} = i\begin{vmatrix} \nabla_y & \nabla_z \\ y & z \end{vmatrix} - j\begin{vmatrix} \nabla_x & \nabla_z \\ x & z \end{vmatrix} + k\begin{vmatrix} \nabla_x & \nabla_y \\ x & y \end{vmatrix}$$

$$= i\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - j\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + k\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right) = 0.$$

$$\nabla\times\vec{B} = \begin{vmatrix} i & j & k \\ \nabla_x & \nabla_y & \nabla_z \\ B_x & B_y & B_z \end{vmatrix} = i\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) - j\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) + k\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

$= 0$ as B_x, B_y, B_z are just constants for a uniform $\vec{B}$ field.

thus

$$\nabla\cdot A = -\frac{1}{2}\left\{(\vec{B}\cdot\vec{0}) - (\vec{r}\cdot\vec{0})\right\} = 0.$$

$$\nabla\times A = \nabla\times\left(-\frac{1}{2}\vec{r}\times\vec{B}\right) = -\frac{1}{2}\left\{(\vec{B}\cdot\nabla)\vec{r} - (\vec{r}\cdot\nabla)\vec{B} + \vec{r}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{r})\right\}$$

$$(\vec{B}\cdot\nabla)\vec{r} = \left(B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z}\right)(xi + yj + zk) = B_x i + B_y j + B_z k = \vec{B}$$

$\vec{r}(\nabla\cdot\vec{B}) = \vec{r}\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) = 0$ as B_x, B_y, B_z are constants.

$(r\cdot\nabla)\vec{B} = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right)(B_x i + B_y j + B_z k) = 0$ as B_x, B_y, B_z are constant.

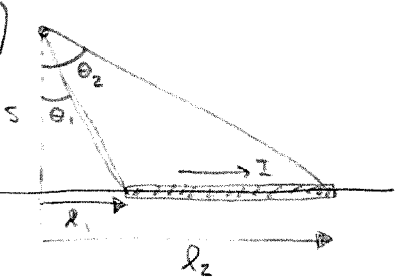
$$\vec{B}(\nabla\cdot\vec{r}) = (B_x i + B_y j + B_z k)\left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) = 3\vec{B}$$

thus $\nabla\times\vec{A} = -\frac{1}{2}\left\{\vec{B} - 3\vec{B}\right\} = -\frac{1}{2}\left\{-2\vec{B}\right\} = \vec{B}$ ✓

there are other functions for instance ~~let~~ $\vec{A}' \equiv -\frac{1}{2}(\vec{r}\times\vec{B}) + \vec{C} = \vec{A} + \vec{C}$. where $\vec{C}$ is a vector composed of constants. (no coordinate dependence) then $\nabla\cdot\vec{A}' = \nabla\cdot\left(\left(-\frac{1}{2}\vec{r}\times\vec{B}\right) + \vec{C}\right) = \nabla\cdot\left(-\frac{1}{2}\vec{r}\times\vec{B}\right) + \nabla\cdot\vec{C}$ where $\nabla\cdot\vec{C} = 0$ trivially and $\nabla\cdot\left(-\frac{1}{2}\vec{r}\times\vec{B}\right) = 0$ as shown above. furthermore

$\nabla\times\vec{A}' = \nabla\times(\vec{A} + \vec{C}) = \nabla\times\vec{A} + \nabla\times\vec{C}$ again $\nabla\times\vec{C} = 0$ and $\nabla\times\vec{A} = \vec{B}$ as shown above. There are many vector potentials that satisfy $\vec{B}$ being uniform.

5-22



$\ell' = s\tan\theta \qquad \theta = \tan^{-1}\left(\frac{\ell'}{s}\right)$

$d\ell' = \frac{s\,d\theta}{\cos^2\theta} \qquad \frac{1}{\mathscr{r}} = \frac{\cos\theta}{s}$

$\cos\theta_1 = \frac{s}{\sqrt{s^2 + z_1'^2}}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{\ell}'}{\mathscr{r}} = \frac{\mu_0 \vec{I}}{4\pi}\int \frac{d\ell'}{\mathscr{r}} = \frac{\mu_0 \vec{I}}{4\pi}\int \frac{s\,d\theta}{\cos^2\theta}\,\frac{\cos\theta}{s} = \frac{\mu_0 \vec{I}}{4\pi}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\cos\theta}$$

$$= \frac{\mu_0 \vec{I}}{4\pi}\ln\left\{\tan\left(\frac{\pi}{4} + \theta/2\right)\right\}\Bigg|_{\theta_1}^{\theta_2} = \frac{\mu_0 \vec{I}}{4\pi}\left\{\ln\left(\tan\left(\frac{\pi}{4} + \theta_2/2\right)\right) - \ln\left(\tan\left(\pi/4 + \theta_1/2\right)\right)\right\}$$

as $\theta_1 = \tan^{-1}\left(\frac{z_1'}{s}\right)$

$\theta_2 = \tan^{-1}\left(\frac{z_2'}{s}\right)$

$$= \frac{\mu_0 \vec{I}}{4\pi}\left\{\ln\left[\tan\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{z_2'}{s}\right)\right)\right] - \ln\left[\tan\left(\pi/4 + \tan^{-1}\left(\frac{z_2'}{s}\right)\right)\right]\right\}$$

$$\nabla \times \vec{A} = -\frac{\partial V_z}{\partial s}\hat{\phi} = \hat{\phi}\frac{\mu_0 I}{4\pi}\frac{\partial}{\partial s}\left\{\ln\left[\tan\left(\pi/4 + \tfrac{1}{2}\tan^{-1}\left(\frac{z_{12}'}{s}\right)\right)\right] - \ln\left[\tan\left(\pi/4 + \tfrac{1}{2}\tan^{-1}\frac{z_2'}{s}\right)\right]\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{\sec^2\left(\pi/4 + \frac{1}{2}\tan^{-1}(z_2'/s)\right)}{\tan\underbrace{\left(\pi/4 + \frac{1}{2}\tan^{-1}(z_2'/s)\right)}_{\beta_2}}\,\frac{1/2}{\left(1 + (z_2'/s)^2\right)}\,\frac{-z_2'}{s^2} - \frac{\sec^2\left(\pi/4 + \theta_1/2\right)(1/2)}{\tan\underbrace{\left(\pi/4 + \theta_1/2\right)}_{\beta_2}\left(1 + (z_1'/s)^2\right)}\,\frac{(-z_1')}{s^2}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{1}{\sin\beta_1\cos\beta_1}\left(\frac{1/2}{1 + (z_2'/s)^2}\right)\left(\frac{-z_2'}{s^2}\right) - \frac{1}{\sin\beta_2\cos\beta_2}\left(\frac{1/2}{1 + (z_1'/s)^2}\right)\left(\frac{-z_1'}{s^2}\right)\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{1}{\sin 2\beta_1}\,\frac{-z_2'}{s^2 + z_2'^2} - \frac{1}{\sin 2\beta_2}\,\frac{-z_1'}{s^2 + z_1'^2}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{1}{\sin\left(2\frac{\pi}{4} + \theta_2\right)}\,\frac{z_2'}{s^2 + z_2'^2} - \frac{1}{\sin\left(2\frac{\pi}{4} + \theta_1\right)}\,\frac{z_1'}{s^2 + z_1'^2}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{1}{\cos(\theta_2)}\,\frac{z_2'}{s^2 + z_2'^2} - \frac{1}{\cos\theta_1}\,\frac{z_1'}{s^2 + z_1'^2}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{\sqrt{s^2 + z_2'^2}}{s}\,\frac{z_2'}{s^2 + z_2'^2} - \frac{\sqrt{s^2 + z_1'^2}}{s}\,\frac{z_1'}{s^2 + z_1'^2}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi}\left\{\frac{z_2'}{\sqrt{s^2 + z_2'^2}}\,\frac{1}{s} - \frac{z_1'}{\sqrt{s^2 + z_1'^2}}\,\frac{1}{s}\right\}$$

$$= \hat{\phi}\frac{\mu_0 I}{4\pi s}\left\{\sin\theta_2 - \sin\theta_1\right\}$$

happy day.

Electrodynamics PY 414, Chapter 4 #'s 26, 28 James Cook

$\boxed{4\text{-}26}$ A spherical conductor of radius a, carriers a charge Q. Its surrounded by linear dielectric material of χ_e out to radius b. find the energy of this configuration.

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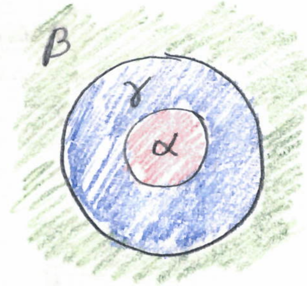
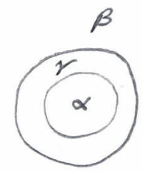
$$\oint \vec{D}\cdot d\vec{a} = Q_{enc} = D(4\pi r^2) \quad \text{for } r > a \qquad \vec{D} = \frac{Q}{4\pi r^2}\hat{r}$$

$$\vec{D} = 0 \quad \text{for } r < a$$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon r^2}\hat{r} & a < r < b \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r} & r > b \\ 0 & r < a \end{cases}$$





$$W = \frac{1}{2}\int_{\substack{\text{all}\\\text{space}}} \vec{D}\cdot\vec{E}\,d\tau = \frac{1}{2}\int_\alpha \vec{0}\,d\tau + \frac{1}{2}\int_\gamma \frac{Q\hat{r}}{4\pi r^2}\cdot\frac{Q\hat{r}}{4\pi\epsilon r^2}\,d\tau + \frac{1}{2}\int_\beta \frac{Q\hat{r}}{4\pi r^2}\cdot\frac{Q\hat{r}}{4\pi\epsilon_0 r^2}\,d\tau$$

$$= \frac{1}{2}\int_a^b \frac{Q^2}{(4\pi)^2\epsilon}\frac{r^2\,dr\,d\Omega}{r^4} + \frac{1}{2}\int_b^\infty \frac{Q^2}{(4\pi)^2\epsilon_0}\frac{r^2\,dr\,d\Omega}{r^4} = \frac{Q^2}{2(4\pi)^2\epsilon}\int_a^b \frac{dr}{r^2}\int d\Omega + \frac{Q^2}{2(4\pi)^2\epsilon_0}\int_b^\infty \frac{dr}{r^2}\int d\Omega$$

$$= \frac{Q^2}{2(4\pi)^2\epsilon}\left(\frac{-1}{r}\Big|_a^b\right)(4\pi) + \frac{Q^2}{2(4\pi)^2\epsilon_0}\left(\frac{-1}{r}\Big|_b^\infty\right)(4\pi)$$

$$= \frac{Q^2}{8\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{b}\right) = W$$

and $\epsilon = \epsilon_0(1 + \chi_e)$

so
$$\boxed{W = \frac{Q^2}{\epsilon_0(1+\chi_e)8\pi}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{b}\right)}$$

$$W = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{1+\chi_e}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{b}\right)$$

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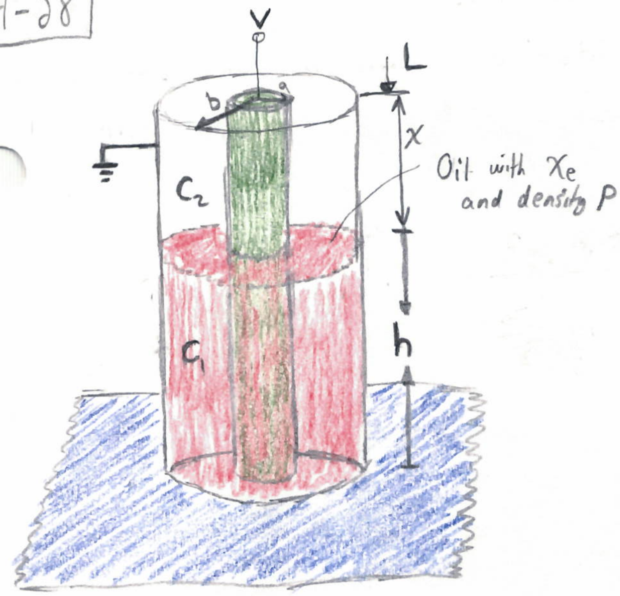
$$= \frac{Q^2}{8\pi\epsilon_0}\left(\frac{\frac{b}{a} - \frac{b}{b} + 1 + \chi_e}{b(1+\chi_e)}\right) = \boxed{\frac{Q^2}{8\pi\epsilon_0}\left(\frac{\frac{b}{a} + \chi_e}{b(1+\chi_e)}\right) = W}$$

$$\boxed{W = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a(1+\chi_e)} + \frac{\chi_e}{b(1+\chi_e)}\right)} \qquad \boxed{W = \frac{Q^2}{2(4\pi\epsilon_0)}\left(\frac{b + a\chi_e}{ab(1+\chi_e)}\right)}$$

$\boxed{4-28}$



To what point does the oil rise in the tube? The oil rises as far is energetically favorable.

THE CAPACITANCE OF THIS CYLINDER IS NOT JUST $\frac{\epsilon A}{d}$ as before with the flat plate cap. instead it is known from work in "208" that

$$C_i = \frac{2\pi}{\ln(b/a)} \epsilon_i l_i$$

where l is the length of the cylinder

thus $$C = C_1 + C_2 = \frac{2\pi}{\ln(b/a)}\left(\epsilon(L-x) + \epsilon_0 x\right) = \frac{2\pi}{\ln(b/a)}\left[\epsilon_0(1+\chi_e)[L-x] + \epsilon_0 x\right]$$

$$\frac{dC}{dx} = \frac{2\pi}{\ln(b/a)}\left[-\epsilon_0(1+\chi_e) + \epsilon_0\right] = \frac{-2\pi\epsilon_0\chi_e}{\ln(b/a)}$$

$\vec{F}$ (decreasing x)

mg (increasing x)

$F + mg = 0.$

$$F = \frac{1}{2}V^2\frac{dC}{dx} = \frac{-V^2\pi\epsilon_0\chi_e}{\ln(b/a)} = -mg = -\rho\pi(b^2-a^2)hg$$

$$\Rightarrow \boxed{h = \frac{V^2\chi_e\epsilon_0}{\ln(b/a)\rho(b^2-a^2)g}}$$ ✓

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