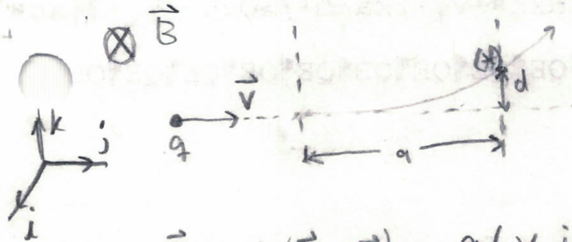
ELECTRODYNAMICS Chap 5 #'s 1, 3 JAMES COOK

20/20

$\boxed{5-1}$



is the charge negative or positive? in terms of a, d, B, and q find the momentum.

$$\vec{F} = q(\vec{V}\times\vec{B}) = q(V_x i + V_y j + V_z k)\times B(-i)$$
$$= q(-V_x B(i\times i) + V_y B(i\times j) + V_z B(i\times k))$$
$$= q(V_y B k - V_z B j) = mA = m(\ddot{x} i + \ddot{y} j + \ddot{z} k)$$

$$\ddot{x} = 0$$
$$\ddot{y} = \frac{-V_z B q}{m} = \frac{-Bq}{m}\dot{z}$$
$$\ddot{z} = \frac{V_y B q}{m} = \frac{qB}{m}\dot{y}$$

$\dot{x} = \text{const.}$, $\dot{x}(0) = 0 \Rightarrow \boxed{\dot{x} = 0.} \Rightarrow x = \text{const.}$, $x(0) = 0 \Rightarrow \boxed{x(t) = 0}$

$\ddot{y} = -\frac{qB}{m}\dot{z} \Rightarrow \dot{y} = -\frac{qB}{m}z + \text{const}$.

$\dot{y}(z=0) = V_0 \Rightarrow \text{const.} = V_0$

$$\boxed{\dot{y} = -\frac{qB}{m}z + V_0}$$

$\ddot{z} = \frac{qB}{m}\dot{y} \Longrightarrow \dot{z} = \frac{qB}{m}y + \text{const}$

$\dot{z}(y=0) = 0 \Rightarrow \text{const} = 0$

$$\Rightarrow \boxed{\dot{z} = \frac{qB}{m}y}$$

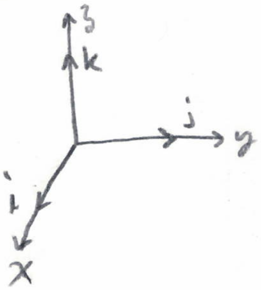
THE FORCE $q(\vec{V}\times\vec{B})$ is PURELY MAGNETIC THUS NO ΔW CAN HAPPEN SO T is CONSERVED AND $V(0,a,d) = V(0,0,0) = V_0$

Note that as $\dot{z} = \frac{qB}{m}y$ and $y, m, B > 0$ and $\dot{z} > 0$ this implies that

$$\boxed{q \text{ is positive}}$$

$\boxed{5-1}$

$V_0(a,d,B,q) = ?$ $\qquad P(q,B,m,v_0,a) = (-qBd + mv_0)\hat{j} - (Baq)k$



$$\dot{x} = 0$$
$$\dot{y} = -\frac{qB}{m}z + V_0$$
$$\dot{z} = \frac{qB}{m}y$$

Let $\left(\frac{qB}{m} = \alpha\right)$

$|\vec{v}| = v_0 = \sqrt{\dot{y}^2 + \dot{z}^2}$ {evaluated at $(0,a,d)$}

$$v_0 = \sqrt{\left(-\frac{qB}{m}d + v_0\right)^2 + \left(\frac{qB}{m}a\right)^2}$$
$$v_0 = \sqrt{\alpha^2 d^2 - 2\alpha d v_0 + v_0^2 + \alpha^2 a^2}$$
$$v_0^2 = \alpha^2 d^2 - 2\alpha d v_0 + \alpha^2 a^2 + v_0^2$$
$$v_0(2\alpha d) = \alpha^2 d^2 + \alpha^2 a^2$$
$$v_0 = \frac{\alpha(d^2 + a^2)}{2d}$$

$\vec{P}(0,a,d) = m(\dot{x}i + \dot{y}j + \dot{z}k)$ where we use $\dot{x},\dot{y},\dot{z}$ derived on last page.

$$= \left(-qBd + m\left(\frac{\alpha(d^2+a^2)}{2d}\right)\right)j - (Baq)k$$
$$= \left(-qBd + m\left(\frac{qB}{m}\right)\left(\frac{d^2+a^2}{2d}\right)\right)j - (Baq)k$$
$$= qB\left(-d + \frac{d^2+a^2}{2d}\right)j - qB(a)k$$
$$= qB\left\{\left(\frac{-2d^2 + d^2 + a^2}{2d}\right)j - ak\right\}$$

$\boxed{\vec{P} = qB\left\{\left(\frac{a^2 - d^2}{2d}\right)j - ak\right\}}$ at $(0,a,d)$.

$$|\vec{P}| = qB\sqrt{\left(\frac{a^2-d^2}{2d}\right)^2 + a^2} = qB\sqrt{\frac{a^4 - 2a^2d^2 + d^4 + 4a^2d^2}{4d^2}}$$
$$= qB\sqrt{\frac{a^4 + 2a^2d^2 + d^4}{4d^2}} = qB\sqrt{\frac{(a^2+d^2)^2}{4d^2}} = \boxed{\frac{\cancel{qB}(a^2+d^2)}{2d} = P}$$

10

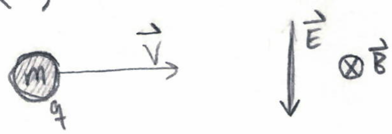
which is of course just $v_0 m = P$.

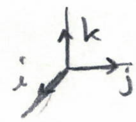
$\boxed{5.3}$

$\vec{E} = (bob)k \qquad bob < 0.$

$\vec{E} = -Ek.$

(a)





$$\vec{F} = q(\vec{E} + \vec{V}\times\vec{B}) = \vec{0}$$
$$= q(-Ek + \vec{V}\times\vec{B}) = \vec{0}$$
$$\Rightarrow (qE)k = -(qVB\sin\theta)k$$
$$\Rightarrow -E = -VB \quad \Rightarrow \boxed{V = +\frac{E}{B}}$$

✓

(b) $\vec{F} = q(\cancel{\vec{E}}^{\,0} + \vec{V}\times\vec{B}) = m\vec{A}$

force is always ⊥ to motion
$\Rightarrow$ circular motion where
$F = \frac{mv^2}{R}$

$$|q(\vec{V}\times\vec{B})| = \frac{mV^2}{R}$$

$$qVB = \frac{mV^2}{R} \Rightarrow \frac{q}{m} = \frac{V^2}{VBR} = \frac{V}{BR} = \boxed{\frac{E}{B^2R} = \frac{q}{m}}$$

✓

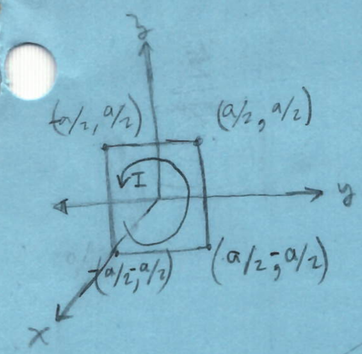
18

PY 414, Dr. Chung

James Cook

20/20

$\boxed{5.4}$ $\vec{B} = kz\hat{x}$ find Force on loop



$$d\vec{F} = dq(\vec{v}\times\vec{B}) = dq\frac{d\vec{l}}{dt}\times\vec{B} = i\,d\vec{l}\times\vec{B}$$

$$\vec{F} = \oint_{\text{loop}} d\vec{F} = \oint I(d\vec{l}\times\vec{B})$$ ✓

$(0,y,z) \longrightarrow (y,z)$

all $x = 0$.

$$= I\left\{\int_{(-a/2,\,a/2)}^{(a/2,\,a/2)}(+d\vec{y})\times(kz\hat{x}) + \int_{(-a/2,\,-a/2)}^{(-a/2,\,a/2)}(+d\vec{z})\times(kz\hat{x}) + \int_{(a/2,\,-a/2)}^{(-a/2,\,-a/2)}(d\vec{y})\times(kz\hat{x}) + \int_{(a/2,\,a/2)}^{(a/2,\,-a/2)}(d\vec{z})\times(kz\hat{x})\right.$$

$$= kI\left\{\int +\hat{z}\,z\,dy + \int +\hat{y}\,z\,dz + \int +\hat{z}\,z\,dy + \int -\hat{y}\,z\,dz\right\}$$

$$= kI\left\{+\hat{z}\int_{-a/2}^{a/2}\frac{a}{2}dy + \hat{y}\int_{-a/2}^{a/2}z\,dz - \hat{z}\int_{a/2}^{-a/2}+\frac{a}{2}dy + \hat{y}\int_{a/2}^{-a/2}-z\,dz\right.$$

$$= kI\left\{+\hat{z}\left(\frac{a}{2}\left(\frac{a}{2}+\frac{a}{2}\right)\right) + \hat{y}\underbrace{\cancel{\left(\frac{1}{2}\left(\frac{a}{2}\right)^2 - \frac{1}{2}\left(\frac{-a}{2}\right)^2\right)}}_{0} + \hat{z}\left(\frac{-a}{2}\left(\frac{-a}{2}-\frac{a}{2}\right)\right) - \hat{y}\underbrace{\cancel{\left(\frac{1}{2}\left(\frac{-a}{2}\right)^2 - \frac{1}{2}\left(\frac{a}{2}\right)^2\right)}}_{0}\right.$$

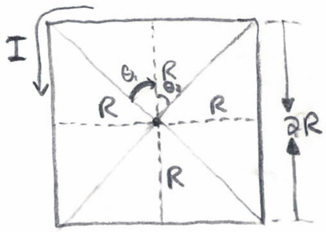
$$= kI\left\{+\hat{z}\left(\frac{a^2}{2}\right) + \hat{z}\left(\frac{a^2}{2}\right)\right\}$$

/0

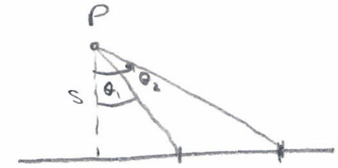
$$\boxed{\vec{F} = ka^2 I\,\hat{z}}$$ ✓

5.8

(a)



$$B = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1) \quad \text{for}$$



apply formula 4 times and we know that $\vec{B}$ points up by right hand rule and Bio Savart $\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

$$\begin{cases} \theta_1 = \pi/4 \\ \theta_2 = -\pi/4 \end{cases}$$

so $B_{TOTAL} = 4\left(\frac{\mu_0 I}{4\pi R}\sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)\right) = \boxed{\frac{\mu_0 I}{\pi R}(\sqrt{2}) \quad \underline{\text{out of page}}}$

(b) $B(n=4) = n\left(\frac{\mu_0 I}{4\pi R}\left(2\sin\left(\frac{\pi}{n}\right)\right)\right) = \frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)$ by inspection.

$\boxed{B_n = \frac{\mu_0 I}{2\pi R}\left(n\sin\left(\frac{\pi}{n}\right)\right)}$ which is out of page in direction if I flows counter clockwise

(c) $\lim_{n\to\infty} \frac{\mu_0 I}{2\pi R}\left(n\sin\left(\frac{\pi}{n}\right)\right) = \frac{\mu_0 I}{2\pi R}\lim_{n\to\infty} n\sin\left(\frac{\pi}{n}\right) = \frac{\mu_0 I}{2\pi R} = B_{circle}$

10

as $\sin\left(\frac{\pi}{n}\right) \longrightarrow n$ for $\frac{\pi}{n}$ small.

PY 414, Chapter 5 #'s 10, 11

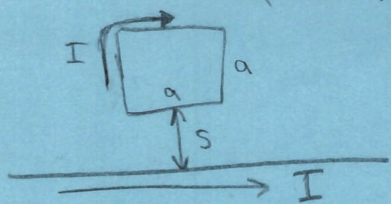
JAMES COOK

10/20

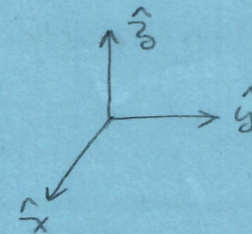
$\boxed{5-10}$

(a) $\vec{F} = I(\vec{l} \times \vec{B})$

$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{x}$



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$
$$= I\, d\vec{l} \times \frac{\mu_0 I}{2\pi s}\hat{x}$$
$$= I\, d\vec{z} \times \frac{\mu_0 I}{2\pi s}\hat{x}$$
$$= \frac{\mu_0 I^2 dz}{2\pi z}\hat{y}$$



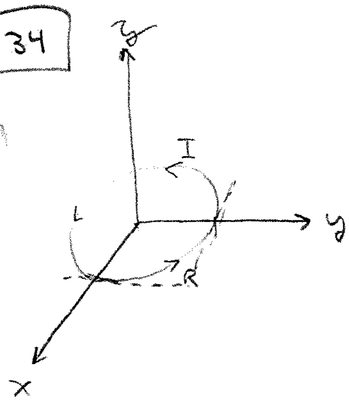
$$(\hat{y})\int_s^{s+a} \frac{\mu_0 I^2 dz}{2\pi z} + (-\hat{y})\int_s^{s+a} \frac{\mu_0 I^2 dz}{2\pi z} = 0$$ so the verticle sides cancel out

$$\vec{F}_{top} = I\,\vec{l}_t \times \vec{B} = I\left[(a\hat{y}) \times \left(\frac{\mu_0 I\,\hat{x}}{2\pi(s+a)}\right)\right] = \frac{\mu_0 I^2 a}{2\pi(s+a)}(-\hat{z})$$

$$\vec{F}_{below} = I\,\vec{l}_b \times \vec{B} = I\left[(-a\hat{y}) \times \left(\frac{\mu_0 I}{2\pi s}\hat{x}\right)\right] = \frac{\mu_0 I^2 a}{2\pi s}(\hat{z})$$

$$\vec{F}_{NET} = \frac{\mu_0 I^2}{2\pi}\hat{z}\left(\frac{a}{s} - \frac{a}{s+a}\right) = \frac{\mu_0 I^2}{2\pi}\hat{z}\left(\frac{as + a^2 - as}{s(s+a)}\right) = \frac{\mu_0 I^2}{2\pi}\hat{z}\left(\frac{a^2}{s(s+a)}\right)$$

5-34



(a) $\vec{m} \equiv I\int d\vec{a} = I\vec{a} = I(\pi R^2)\,\hat{z}$

(b) FAR FROM THE ORIGIN THE DIPOLE TERM OF THE MULTIPOLE EXPANSION DOMINATES AS THE OTHER TERMS ARE MINISCULE IN COMPARISON TOO THE DIPOLE.

$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{1}{r}\,d\vec{l}' = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\theta')\,d\vec{l}' \approx \vec{A}_{DIPOLE}$ as the monopole term is identically zero and all other terms die away for $r \gg R$.

now the text gives the $\vec{A}_{DIPOLE} = \frac{\mu_0 m}{4\pi r^2}\sin\theta\,\hat{\phi}$

take the curl and we get $\vec{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

as $m = \pi I R^2$ $\Rightarrow$ $\boxed{\vec{B} = \frac{\mu_0 R^2 I}{4 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$

c) first note that on z-axis $\theta = 0$ and $r = z$ thus

$$\vec{B} = \frac{\mu_0 I R^2}{2 z^3}\hat{z}$$

The exact $\vec{B}$ is given as $\frac{\mu_0 I R^2}{2}\frac{1}{(R^2+z^2)^{3/2}}\hat{z} = \vec{B}_{EXACT}$

to show that $\vec{B}_{EXACT} \longrightarrow \vec{B}$ for $R \ll z$ we need $\frac{1}{(R^2+z^2)^{3/2}} \longrightarrow \frac{1}{z^3}$.

$(R^2+z^2)^{-3/2} = z^{-3}\left(\frac{R^2}{z^2}+1\right)^{-3/2} \approx z^{-3}$ as $\frac{R^2}{z^2}$ is very small.

thus

$$\frac{\mu_0 I R^2}{2}\frac{1}{(R^2+z^2)^{3/2}}\hat{z} \longrightarrow \frac{\mu_0 I R^2}{2z^3}\hat{z} = \vec{B}$$

and all is well.