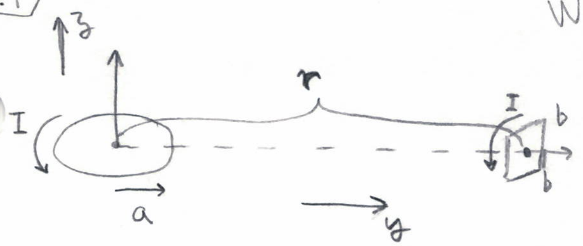
PY 415, DR. CHUNG HWK #1 JAMES COOK

6.1



What is the torque on the square loop due to the circular one.

$m_1 = \pi a^2 I \hat{z}$

20/20.

good

$$\vec{B} = \frac{\mu_0}{4\pi r^3}\left\{3(\vec{m}_1 \cdot \hat{r})\hat{r} - \vec{m}_1\right\}$$

$$= \frac{\mu_0}{4\pi r^3}\left\{3(\vec{0})\hat{r} - \vec{m}_1\right\}$$

$$= \frac{-\mu_0}{4\pi r^3}(\pi a^2 I \hat{z})$$

$m_2 = b^2 I \hat{y}$

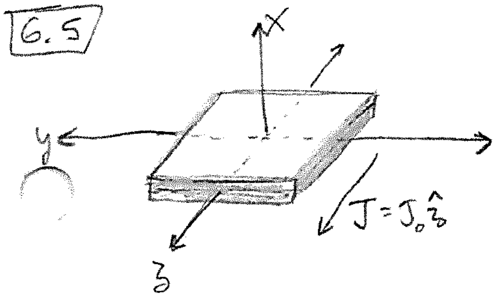
$$\vec{N} = \vec{m}_2 \times \vec{B} = (b^2 I \hat{y}) \times \left(\frac{-\mu_0}{4\pi r^3}\pi a^2 I \hat{z}\right)$$

$$\vec{N} = \frac{-b^2 I \mu_0 a^2 I \hat{x}}{4r^3}$$

$$= \boxed{-\frac{a^2 b^2 I^2 \mu_0 \hat{x}}{4r^3} = \vec{N}}$$

The equilibrium point where $\vec{N} = 0$ is where the loop lies x-y plane and m_2 is in the $\pm \hat{z}$ direction where $\vec{B}$ is $\perp$ to m_2.

6.5



B must depend only on x as a y dependence is prohibited by the symetry of the infinite plane. No z dependence is possible by the Biot-Savart law as the cross product of $\hat{z}$ with $\hat{z}$ is zero. The Biot Savart law also dictates that B is in the $\hat{j}$ direction.

$$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \int \mu_0 \vec{J}\cdot d\vec{a} = \mu_0 J_0 \int dx\,dy = \mu_0 J_0 x y = \int B\,dy = By$$
$$\Rightarrow \vec{B} = \mu_0 J_0 x\,\hat{y}$$

(a) $\vec{F} = \nabla(\vec{m}\cdot\vec{B}) = \nabla(m_0\hat{x}\cdot \mu_0 J_0 x\hat{y}) = \nabla(0) = \boxed{\vec{0} = \vec{F}}$

(b) $\vec{F} = \nabla(\vec{m}\cdot\vec{B}) = \nabla(m_0\hat{y}\cdot\mu_0 J_0 x\hat{y}) = \nabla(m_0\mu_0 J_0 x)$

$$\boxed{\vec{F} = m_0 \mu_0 J_0\,\hat{x}}$$

(c)

i. Prove $\nabla(\vec{p}\cdot\vec{E}) = (\vec{p}\cdot\nabla)\vec{E}$

By product rule 4 we may expand $\nabla(\vec{p}\cdot\vec{E})$ to

$\nabla(\vec{p}\cdot\vec{E}) = \vec{p}\times(\nabla\times\vec{E}) + \vec{E}\times(\nabla\times\vec{p}) + (p\cdot\nabla)\vec{E} + (\vec{E}\cdot\nabla)\vec{p}$

$\nabla\times\vec{E} = 0$ in electrostatics and $\nabla\times\vec{p} = 0$ as $\vec{p}$ is just a constant vector.

so $\nabla(\vec{p}\cdot\vec{E}) = \vec{p}\times\vec{0} + \vec{E}\times\vec{0} + (p\cdot\nabla)\vec{E} + (E\cdot\nabla)\vec{p} = (p\cdot\nabla)\vec{E} + (E\cdot\nabla)\vec{p}$

$E\cdot\nabla = E_x\frac{\partial}{\partial x} + E_y\frac{\partial}{\partial y} + E_z\frac{\partial}{\partial z}$ and when this operator acts on $\vec{p}$ a constant vector all the derivatives are zero

$\Rightarrow (E\cdot\nabla)\vec{p} = 0$ thus $\nabla(\vec{p}\cdot\vec{E}) = (\vec{p}\cdot\nabla)\vec{E} + \vec{0} = \boxed{(\vec{p}\cdot\nabla)\vec{E} = \nabla(p\cdot\vec{E})}$

ii. Why does $\nabla(\vec{m}\cdot\vec{B}) \neq (m\cdot\nabla)\vec{B}$, mathematically this does not hold because $\nabla\times\vec{B}\neq 0$ in general, the other two terms vanish because $\vec{m}$ is also just a constant vector

iii. Calculate $(m\cdot\nabla)\vec{B}$ for (a) and (b)

$(m_0\hat{x}\cdot\nabla)\vec{B} = m_0\frac{\partial}{\partial x}\vec{B} = m_0\frac{\partial}{\partial x}(\mu_0 J_0 x\hat{y}) = m_0\mu_0 J_0\hat{y} \neq \vec{0}$ ✓

$(m_0\hat{y}\cdot\nabla)\vec{B} = m_0\frac{\partial}{\partial y}(\vec{B}) = m_0\frac{\partial}{\partial y}(\mu_0 J_0 x\hat{y}) = \vec{0} \neq m_0\mu_0 I_0\hat{x}$ ✓

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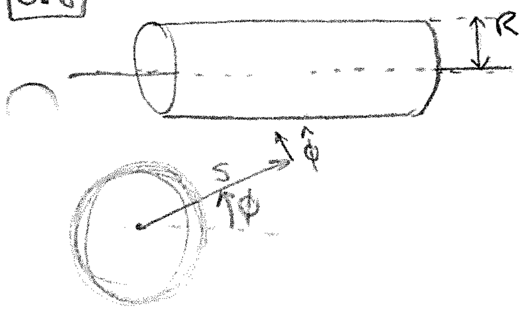
6.8 Find $\vec{B}$ from $\vec{M}$

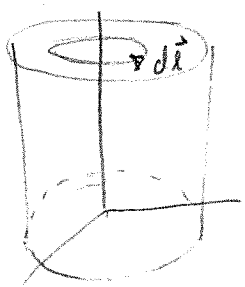


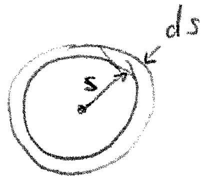
$$\vec{M} = ks^2\hat{\phi} \qquad V_\phi \equiv ks^2,\ V_s \equiv 0,\ V_z \equiv 0$$

$$\vec{J}_b = \nabla\times\vec{M}$$
$$= \left[\frac{1}{s}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right]\hat{s} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sV_\phi) - \frac{\partial V_s}{\partial\phi}\right]\hat{z}$$
$$= \frac{1}{s}\frac{\partial}{\partial s}(sks^2)\hat{z} = \frac{1}{s}k3s^2\hat{z} = \boxed{3ks\hat{z} = \vec{J}_b}$$

$$\vec{K}_b = \vec{M}\times\hat{n}$$
$$= ks^2\hat{\phi}\times\hat{s}$$
$$= -ks^2\hat{z}\Big|_{s=R} = \boxed{-kR^2\hat{z} = \vec{K}_b}$$

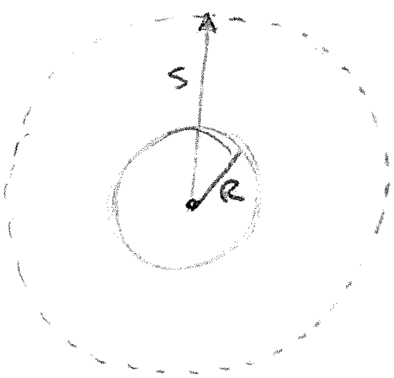
Amperes law is appropriate if we consider the following loop



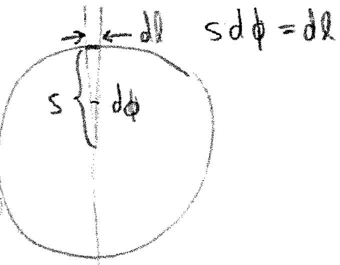


$$d\vec{a} = 2\pi s\, ds\,\hat{z}$$

$$\oint\vec{B}\cdot d\vec{l} = B(2\pi s) = \mu_0 I_{enc} = \mu_0\int\vec{J}\cdot d\vec{a}$$
$$= \mu_0\int 3ks\hat{z}\cdot 2\pi s\,ds\,\hat{z}$$
$$= \mu_0\int 6\pi ks^2\,ds$$
$$B(2\pi s) = 2\mu_0\pi ks^3$$
$$\boxed{\vec{B}_{in} = \mu_0 ks^2\hat{\phi}}$$

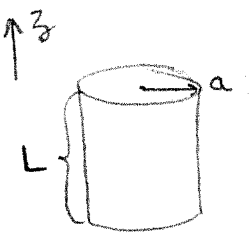


$$I_{enc} = \int_0^R\vec{J}\cdot d\vec{a} + \int_{0,\,s=R}^{2\pi} -kR^2 s\,d\phi = 2\pi ks^3\Big|_0^R + -2\pi R^2 Rk$$
$$= 2\pi kR^3 - 2\pi kR^3$$
$$= 0$$



$$\Rightarrow \boxed{\vec{B}_{out} = \vec{0}}$$

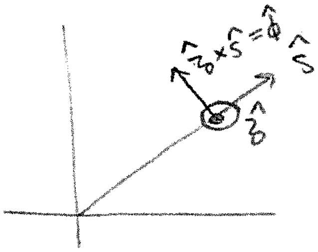
6.9



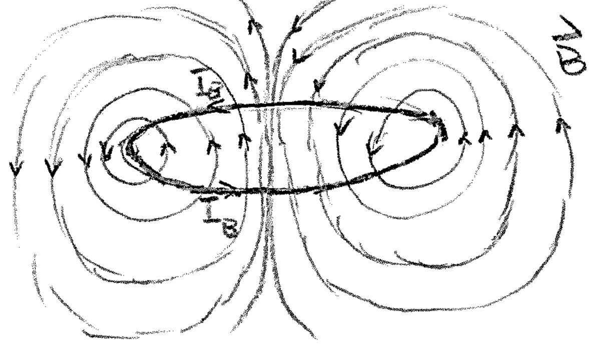
$\vec{M} = M\hat{z}$ where $M \in \mathbb{R}$

$\vec{J}_B = \nabla \times \vec{M} = 0$ as M is constant.

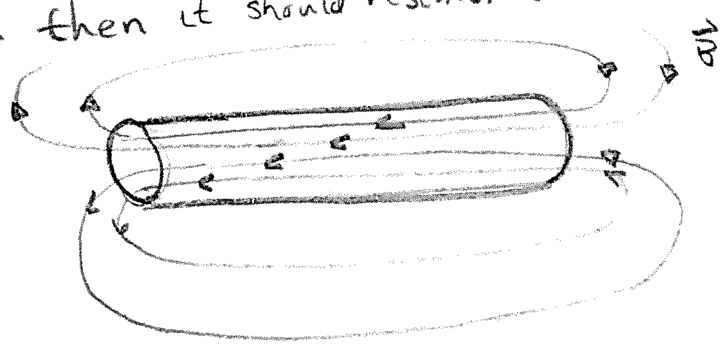
$\vec{K}_B = \vec{M} \times \hat{n} = \vec{M} \times \hat{s} = \boxed{M\hat{\phi} = \vec{K}_B \text{ Bound Current}}$



$L \ll a$ then it is just like a wire in the limit so it has a donut like field

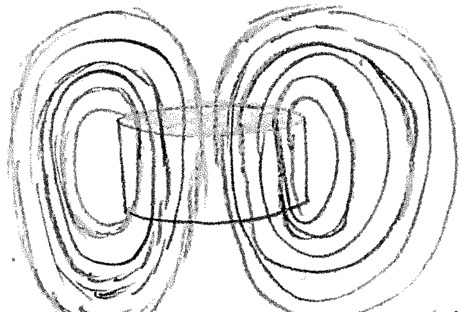


$L \gg a$ then it should resemble a solenoid.



for an ideal solenoid $\vec{B} = 0$ outside. I assumed less than perfection

$L \approx a$



a compromise between the 2 extremes of $a \ll L$ and $L \gg a$

from the problem 4.11 I would note this is the magnetic analogue to the bar electret.

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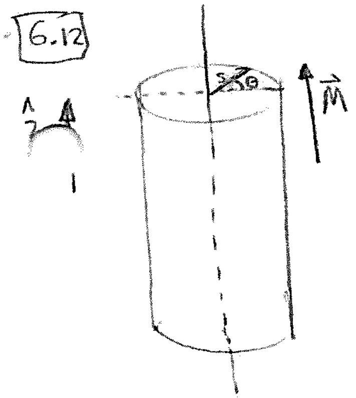
$\boxed{6.12}$

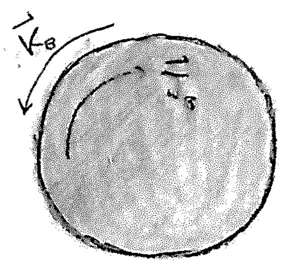


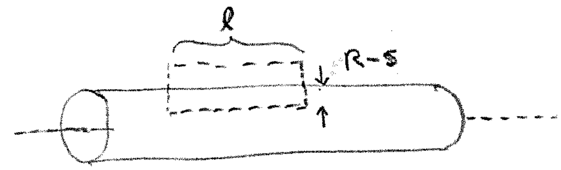
$\vec{M} = ks\,\hat{z}$ where k is a constant.

20/20 good.

(a) $\vec{K}_B = \vec{M}\times\hat{n} = ks(\hat{z}\times\hat{s}) = ks\,\hat{\theta} = kR\,\hat{\theta}$

$\vec{J}_B = \nabla\times\vec{M} = \frac{1}{s}\frac{\partial M_z}{\partial\theta}\hat{s} - \frac{\partial M_z}{\partial s}\hat{\theta} = \frac{1}{s}\cancel{\frac{\partial}{\partial\theta}(ks)}\,\hat{s} - \frac{\partial}{\partial s}(ks)\,\hat{\theta} = -k\hat{\theta}$

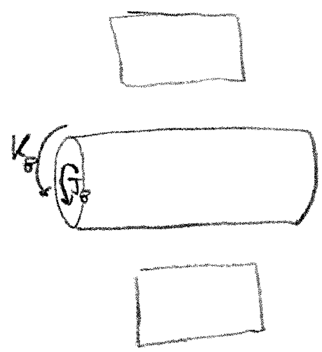




$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0(-k(R-s)\ell) + \mu_0 kR = B\ell$

$\Rightarrow B = \mu_0 sk - \mu_0 kR + \mu_0 kR$

$\Rightarrow \boxed{\vec{B}_{in} = \mu_0 sk\,\hat{z}}$



Both outside loops $\Rightarrow$ B is constant. As B comes from a finite current distribution B_{out} must be zero $\boxed{\vec{B}_{out} = \vec{0}}$

(b) $\oint \vec{H}\cdot d\vec{\ell} = I_{f\,enc}$ where $\vec{H} \equiv \frac{1}{\mu_0}(\vec{B} - \vec{M})$

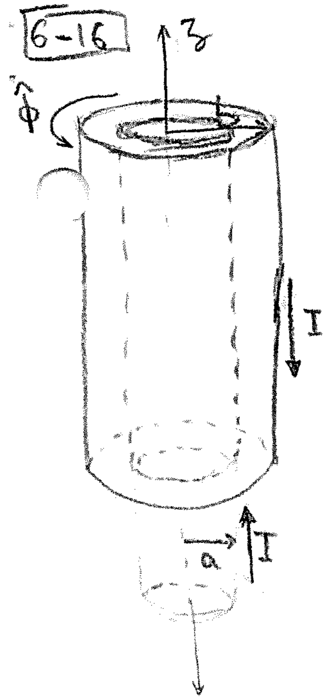
$I_{f\,enc} = 0 \Rightarrow \oint \vec{H}\cdot d\vec{\ell} = 0$ over all paths $\Rightarrow \vec{H} = \vec{0}$.

inside $\vec{M} = ks\hat{z}$ and $\vec{H} = \frac{1}{\mu_0}(\vec{B} - ks\,\hat{z}) = \vec{0} \Rightarrow \boxed{\vec{B}_{in} = \mu_0 ks\,\hat{z}}$

outside $\vec{M} = 0$ and as $\vec{H} = \frac{1}{\mu_0}(\vec{B} - \vec{0}) = \frac{\vec{B}}{\mu_0} \Rightarrow \boxed{\vec{B}_{out} = \vec{0}}$

good

$\boxed{6-16}$



Find $\vec{B}$ between the tubes, a material of χ_m fills that space.

$$\oint \vec{H}\cdot d\vec{l} = I_{f\,enc} \Rightarrow H(2\pi s) = I \Rightarrow \vec{H} = \frac{I}{2\pi s}\hat{\phi}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} = \frac{1}{\mu_0}\vec{B} - \chi_m \vec{H} \Rightarrow \vec{B} = (\vec{H} + \chi_m \vec{H})\mu_0$$

$$\Rightarrow \vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi}$$

$$\Rightarrow \boxed{\vec{B} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi} = \frac{\mu I}{2\pi s}\hat{\phi}}$$

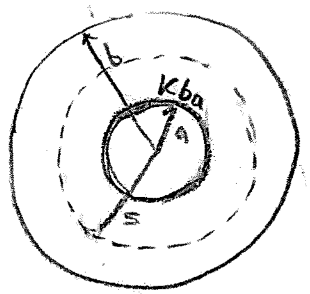
$$\vec{M} = \chi_m \vec{H} = \chi_m \frac{I}{2\pi s}\hat{\phi} = \vec{M}$$

"CHECK"

$$\vec{J}_B = \nabla\times\vec{M} = \nabla\times\chi_m\vec{H} = \frac{\chi_m I}{2\pi}\left(\nabla\times\frac{\hat{\phi}}{s}\right) = \frac{\chi_m I}{2\pi}\left(\frac{1}{s}\frac{\partial}{\partial s}\left(\frac{1}{s}s\right)\right)\hat{z} = \vec{0}$$

$$\vec{K}_{BA} = \vec{M}\times\hat{n}_A = \chi_m\vec{H}\times(-\hat{s}) = \frac{\chi_m I}{2\pi s}(\hat{\phi}\times(-\hat{s})) = +\frac{\chi_m I}{2\pi s}\hat{z}\Big|_{s=a} = +\frac{\chi_m I}{2\pi a}\hat{z} = \vec{K}_{BA}$$

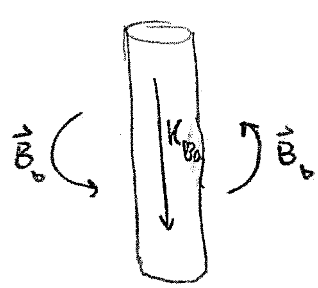
$$\oint \vec{B}_f\cdot d\vec{l} = \mu_0 I_f \Rightarrow \vec{B}_{free} = \frac{\mu_0 I}{2\pi s}$$



$$\oint \vec{B}_B\cdot d\vec{l} = \mu_0 I_{BOUND} = \mu_0(2\pi a)\frac{\chi_m I}{2\pi a}$$

$$\Rightarrow \vec{B}_B(2\pi s) = \mu_0(\chi_m I)$$

$$\Rightarrow \vec{B}_B = \frac{\mu_0\chi_m I}{2\pi s}\hat{\phi}$$



$$\vec{B}_{free} + \vec{B}_B = \left\{\frac{\mu_0 I}{2\pi s} + \frac{\mu_0\chi_m I}{2\pi s}\right\}\hat{\phi}$$

$$= \frac{\mu_0 I}{2\pi s}\left\{1+\chi_m\right\}\hat{\phi}$$

$$= \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi} = \boxed{\frac{\mu I}{2\pi s}\hat{\phi} = \vec{B}_{total}}$$