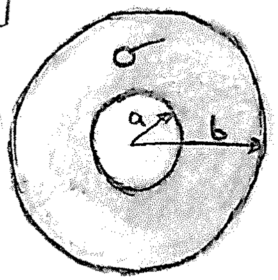
PY 415, Dr. Chung, Hwk # 4 James Cook

$\boxed{7.1}$



(a) if they have V between them what current I flows from one to the other?

$$I = \int \vec{J} \cdot d\vec{a} = \int \sigma \vec{E} \cdot d\vec{a} = \sigma \int \vec{E} \cdot d\vec{a}$$

$$E_r = \frac{Q_{in}}{4\pi\epsilon_0 r^2} = \frac{4\pi a^2 \alpha}{4\pi\epsilon_0 r^2} = \frac{a^2 \alpha}{\epsilon_0 r^2} \Rightarrow I = \frac{\sigma a^2 \alpha}{\epsilon_0} \int \frac{r^2 d\Omega}{r^2}$$

$$\Rightarrow I = \frac{\sigma a^2 \alpha}{\epsilon_0}(4\pi)$$

$$V = -\int_b^a \vec{E} \cdot d\vec{r} = \int_a^b \frac{\alpha a^2}{\epsilon_0 r^2} dr = \frac{\alpha a^2}{\epsilon_0}\left(\frac{-1}{r}\right)\Big|_a^b = \frac{\alpha a^2}{\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow \frac{\alpha a^2}{\epsilon_0} = V\left(\frac{1}{a} - \frac{1}{b}\right)^{-1}$$ but also from before $\frac{\alpha a^2}{\epsilon_0} = \frac{I}{4\pi\sigma}$

✓ $\therefore V\left(\frac{1}{a} - \frac{1}{b}\right)^{-1} = \frac{I}{4\pi\sigma} \Rightarrow \boxed{I = \frac{4\pi\sigma V}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$ {radially of course.}

✓(b) $V = \left\{\frac{\left(\frac{1}{a} - \frac{1}{b}\right)}{4\pi\sigma}\right\}(I) \Rightarrow \boxed{R = \left(\frac{1}{a} - \frac{1}{b}\right)\frac{1}{4\pi\sigma}}$

(c) if $a \ll b$ then the question has more meaning.

-2 $a \ll b \Rightarrow \frac{1}{a} \gg \frac{1}{b} \Rightarrow$ comparatively $\frac{1}{b} = 0$. So $\boxed{I = 4\pi\sigma V a}$

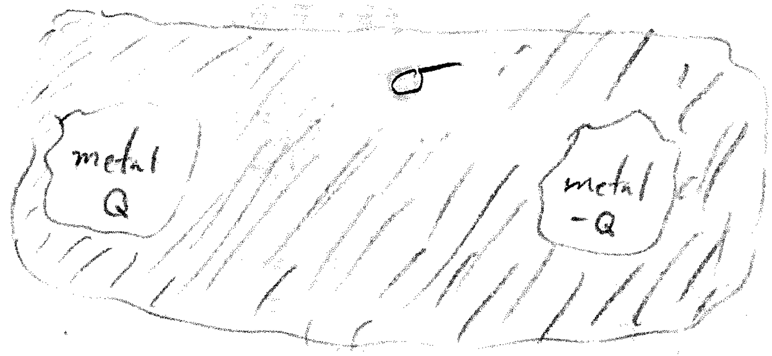
$\boxed{7.3}$ Two metal objects are in material with σ show that

(a) $R = \dfrac{\epsilon_0}{\sigma C}$ is the relation between their resistance and the capacitance of the arrangement.

$J \equiv \sigma E$

$V \equiv IR$

$C \equiv Q/V$



GAUSS' LAW, $\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q\,\hat{r}}{4\pi\epsilon_0 r^2} \Rightarrow I = \int \vec{J}\cdot d\vec{a} = \sigma \int E\, r^2\, d\Omega = \frac{\sigma Q}{\epsilon_0}$

✓ $V = IR = \frac{\sigma Q}{\epsilon_0} R$ also $V = \frac{Q}{C} \Rightarrow V = \frac{Q\sigma}{\epsilon_0} R = \frac{Q}{C}$ $\therefore \boxed{R = \frac{\epsilon_0}{\sigma C}}$

✓(b) Suppose $V(t=0) = V_0$. As $V = IR$

$\Rightarrow V = I \frac{\epsilon_0}{\sigma C} = -\frac{dQ}{dt}\frac{\epsilon_0}{\sigma C} = -\frac{dQ}{dt}\frac{\epsilon_0}{\sigma}\frac{V}{Q}$

$\Rightarrow 1 = -\frac{dQ}{dt}\frac{\epsilon_0}{\sigma Q} \Rightarrow \frac{\sigma}{\epsilon_0} dt = -\frac{dQ}{Q}$

$\Rightarrow -\frac{\sigma t}{\epsilon_0} = \ln Q + \text{const} \Rightarrow k e^{-\frac{\sigma t}{\epsilon_0}} = Q(t)$

remember $V = \frac{Q}{C} \Rightarrow V(t) = \frac{k}{C} e^{-\frac{\sigma t}{\epsilon_0}}$

$V(0) \equiv V_0 = \frac{k}{C} e^0 = \frac{k}{C} \Rightarrow V(t) = V_0 e^{-\sigma t/\epsilon_0}$

thus $\boxed{V(t) = V_0 e^{-t/\tau}}$ where $\boxed{\tau \equiv \epsilon_0/\sigma}$

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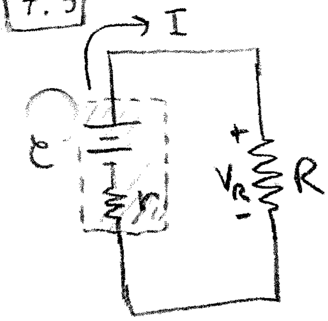
7.5



$r = R$ of course.

$$\mathcal{E} = IR + Ir \Rightarrow I = \frac{\mathcal{E}}{R+r}$$

$$IR = V_R = \frac{R}{R+r}\mathcal{E}$$

$$P_R = \frac{V_R^2}{R} = \frac{R^2\mathcal{E}^2}{(R+r)^2 R} = \frac{R\mathcal{E}^2}{(R+r)^2}$$

$$\frac{dP_R}{dR} = \frac{d}{dR}\left(\frac{R\mathcal{E}^2}{(R+r)^2}\right) = \mathcal{E}^2\frac{d}{dR}\left(R(R+r)^{-2}\right)$$

$$= \mathcal{E}^2\left\{(R+r)^{-2} - 2(R+r)^{-3}R\right\} = 0 \text{ at max.}$$

$$\mathcal{E}^2 \neq 0 \Rightarrow (R+r)^{-2} - 2(R+r)^{-3}R = 0$$

$$\Rightarrow (R+r)^{-2} = 2(R+r)^{-3}R$$

$$\Rightarrow R + r = 2R$$

$$\Rightarrow \boxed{r = R}$$

note
$$\frac{d}{dR}\left(\frac{dP_R}{dR}\right) = \mathcal{E}^2\left\{-2(R+r)^{-3} + 6(R+r)^{-4}R - 2(R+r)^{-3}\right\}$$

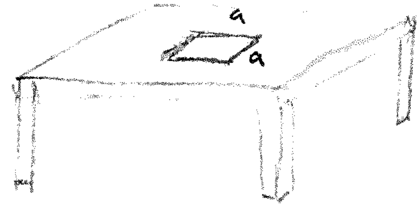
$$= \mathcal{E}^2\left\{6R(R+r)^{-4} - 4(R+r)^{-3}\right\}\Big|_{r=R}$$

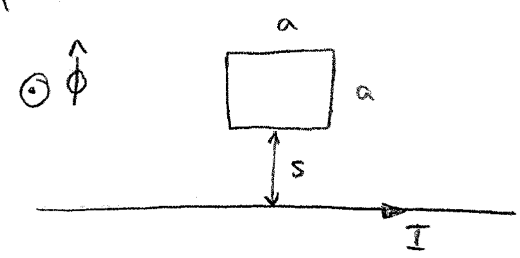
$$= \mathcal{E}^2\left\{6R(2R)^{-4} - 4(2R)^{-3}\right\}$$

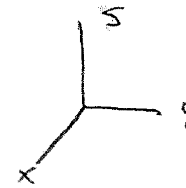
$$= \frac{\mathcal{E}^2}{R^3}\left\{\frac{6}{16} - \frac{4}{8}\right\} = \frac{\mathcal{E}^2}{R^3}\left\{\frac{-1}{8}\right\} < 0 \quad \therefore \text{ max. by } 2^{nd} \text{ der. test.}$$

7.8

Feb 21





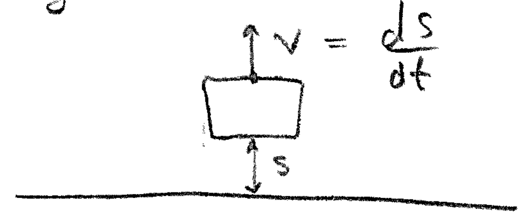


(a) Find Φ_B through loop

$$\Phi_B = \int \vec{B}\cdot d\vec{a} = \int \frac{\mu_0 I}{2\pi s}\hat{\phi}\cdot dy\,ds\,\hat{\phi} = \int \frac{\mu_0 I}{2\pi s}\,dy\,ds$$

$$= \frac{\mu_0 I}{2\pi} a \int_s^{s+a} \frac{d\bar{s}}{\bar{s}} = \frac{\mu_0 I a}{2\pi}\left(\ln \bar{s}\,\Big|_s^{s+a}\right) = \boxed{\frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right) = \Phi_B}$$

(b) If some one pulls directly away from wire at speed v what emf is generated?



$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{-d}{dt}\left(\frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right)\right) = -\frac{\mu_0 I a}{2\pi}\left(\frac{s}{s+a}\right)\frac{d}{dt}\left((s+a)s^{-1}\right)$$

$$= -\frac{\mu_0 I a}{2\pi}\left(\frac{s}{s+a}\right)\left((s+a)\left(\frac{-1}{s^2}\right) + (s^{-1})(1)\right)\frac{ds}{dt} = \frac{-\mu_0 I a}{2\pi}\left(\frac{-1}{s} + \frac{1}{s+a}\right)v$$

$$= \frac{\mu_0 I a v}{2\pi}\left(\frac{1}{s} - \frac{1}{s+a}\right) = \frac{\mu_0 I a v}{2\pi}\left(\frac{s+a-s}{s(s+a)}\right) = \boxed{\frac{\mu_0 I v a^2}{2\pi s(s+a)} = \mathcal{E}}$$

I flows counter clockwise to maintain the prexisting $\vec{B}$ field.

(c) What if the loop is pulled to the right at v

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{\mu_0 I a^2}{2\pi s(s+a)}\cancelto{0}{\frac{ds}{dt}} = 0$$ as s is not changing

$\mathcal{E} = 0$ as there is no change in flux.

the loop has no preference in $\vec{B}$ fields with identical characteristics

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$\boxed{7.15}$

A long solenoid carries $I(t)$. Find E induced from changing B in the quasistatic approximation.

$n = \frac{\text{turns}}{\text{length}}$

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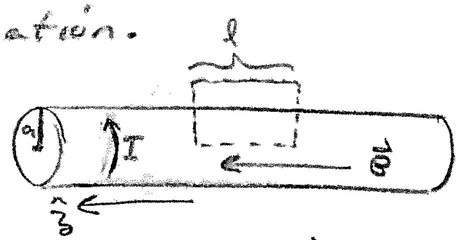
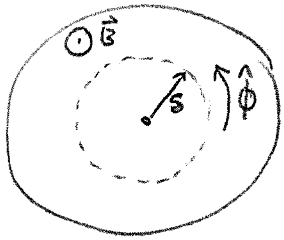


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow Bl = \mu_0 n l I \Rightarrow \vec{B} = \mu_0 n I \hat{z} \quad (\text{just inside})$$

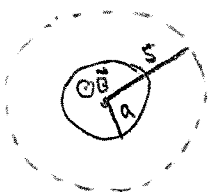
As $\rho = 0$ in solenoid $\Rightarrow \nabla \cdot \vec{E} = \vec{0} \Rightarrow \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

in integral form, $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$



$$\oint \vec{E} \cdot d\vec{l} = E(2\pi s) = -\frac{d}{dt}(BA) = -\mu_0 n \pi s^2 \frac{dI(t)}{dt}$$

$$\Rightarrow \boxed{\vec{E}_{in} = \frac{-\mu_0 n s}{2} \frac{dI(t)}{dt} \hat{\phi}}$$



$$\oint \vec{E} \cdot d\vec{l} = E(2\pi s) = \frac{-d\Phi}{dt} = \frac{-d}{dt}(B \pi a^2) = 0$$

$$\Rightarrow E = -\frac{\mu_0 n \dot{I} \pi a^2}{2\pi s} \hat{\phi} = \boxed{\frac{-\mu_0 n a^2}{2s} \frac{dI}{dt} \hat{\phi} = \vec{E}_{out}}$$

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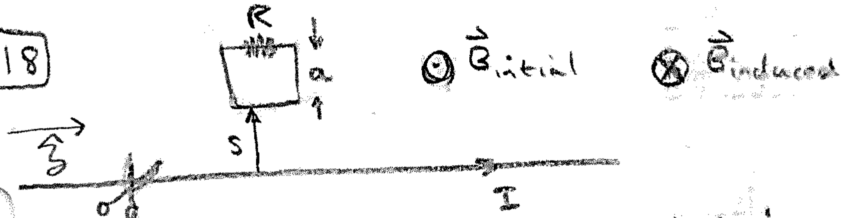


the current flows in the counter clockwise direction as to maintain the prexisting $\vec{B}$ field.

$$\mathcal{E} = -\frac{d\Phi}{dt} = IR$$

$$I = \frac{dq}{dt} \quad \Longrightarrow \quad -\frac{d\Phi}{dt} = \frac{dq}{dt}R \quad \Rightarrow \quad -\frac{d\Phi}{R} = dq$$

$$\int \frac{-d\Phi}{R} = \frac{-1}{R}\left(\Phi_f - \Phi_i\right) = \frac{\Phi_{intial}}{R}$$

$$\Phi_{intial} = \int \vec{B}\cdot d\vec{a} = \int \frac{\mu_0 I}{2\pi r}\, dr\, dz = \frac{\mu_0 a I}{2\pi}\int_s^{s+a} \frac{dr}{r} = \frac{\mu_0 a I}{2\pi}\ln\left(\frac{s+a}{s}\right)$$

$$\int dq = q = \frac{\Phi_{intial}}{R} = \boxed{\frac{\mu_0 I}{2\pi R}\ln\left(\frac{s+a}{s}\right) = q}$$

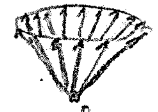
-1

$$\ln\left(\frac{s+a}{s}\right)^{\frac{\mu_0 I}{2\pi R}} = q$$

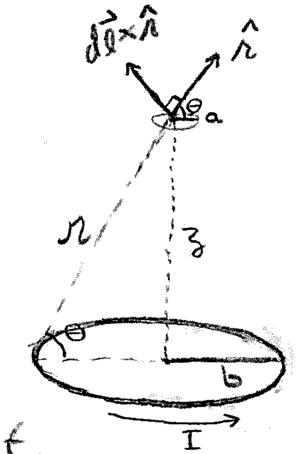
7.

(A)

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{\textit{r}}}{\textit{r}^2}$$ Biot Savart Law

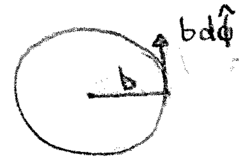


Cone of $d\vec{l} \times \hat{r}$



$$= \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{\textit{r}}}{z^2 + b^2} = \frac{\mu_0 I}{4\pi (z^2 + b^2)} \oint d\vec{l} \times \hat{\textit{r}}$$

$d\vec{l} \times \hat{\textit{r}} \Rightarrow$ a vector $\perp$ to $\hat{\textit{r}}$ for every $\hat{\textit{r}}$ around loop we see from symetry that only the verticle ($\hat{z}$) components survive $\oint$ as every horizontal part has a corresponding vector to cancel it.



$$\{d\vec{l} \times \hat{\textit{r}}\}_z = (b\, d\hat{\phi}) \times (\cos\theta)\hat{s} = (b\cos\theta)(-\hat{z})\, d\phi$$

$$\cos\theta = \frac{b}{(z^2 + b^2)^{1/2}}$$

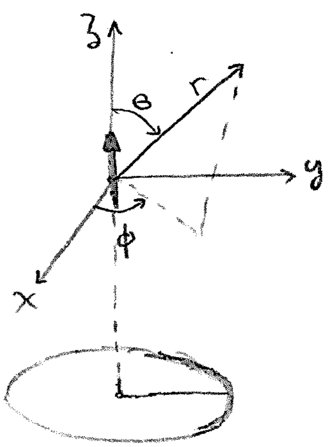
$$\vec{B} = \frac{\mu_0 I}{4\pi (z^2 + b^2)} \oint \frac{-b^2 \hat{z}}{(z^2 + b^2)^{1/2}} = \frac{-\mu_0 b^2 I \hat{z}}{2(z^2 + b^2)^{3/2}} = \vec{B} \text{(at center of loop of radius a)}$$

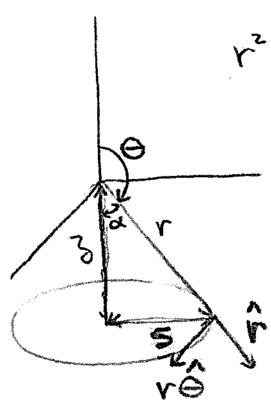
$\Phi = \oint \vec{B} \cdot d\vec{a} = B\pi a^2$, assuming $\vec{B}$ is constant and has value of $\vec{B}$ at center of loop, this is reasonable as a is small.

$$\boxed{\Phi = \frac{-\mu_0 \pi a^2 b^2 I}{2(z^2 + b^2)^{3/2}}}$$

(B) Conversely β the small loop to be a dipole and find Φ in big loop due to it.

$$\vec{m} = I(\text{AREA}) = (I\pi a^2)\hat{z} \qquad \vec{B}_{\text{DIPOLE}} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta})$$





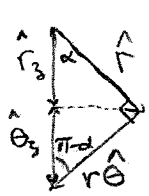
$$r^2 = s^2 + z^2$$

$$\cos(\alpha) = \cos(\pi - \theta) = \frac{z}{r}$$
$$\sin(\alpha) = \sin(\pi - \theta) = \frac{s}{r}$$

$$\cos(\pi - \theta) = \cos\pi\cos\theta + \sin\pi\sin\theta = -\cos\theta = \frac{z}{r}$$
$$\sin(\pi - \theta) = \sin\pi\cos\theta - \sin\theta\cos\pi = \sin\theta = \frac{s}{r}$$

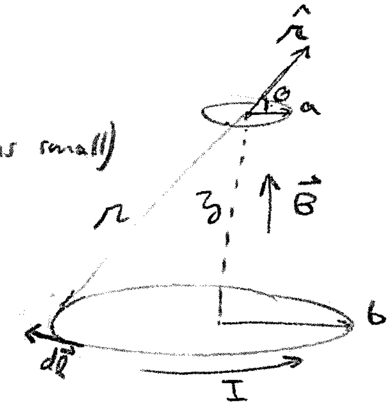
SYMETRY $\Rightarrow$ just $\hat{z}$ term survives in $\oint \vec{B} \cdot d\vec{a}$ over loop ($r = b$)

$$\{\hat{r}\}_z = (\cos\alpha)\hat{z} \qquad \{\hat{\theta}\}_z = \cos(\pi - \alpha)\, r\, \hat{z}$$



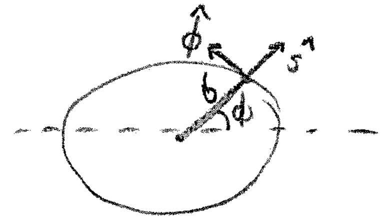
$\boxed{7.20}$

(a) If current I flows in big loop, Find Φ through small loop. (Assume field of big loop constant since little loop is small)



$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint \frac{d\vec{l}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\mathscr{r}^2 = z^2 + b^2$$
$$d\vec{l} = b\,d\hat{\phi}$$
$$\hat{\mathscr{r}} = (\cos\theta)\hat{s} + (\sin\theta)\hat{z}$$



$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint \frac{(b\,d\hat{\phi})\times(\cos\theta\,\hat{s} + \sin\theta\,\hat{z})}{z^2+b^2}$$

$$= \frac{\mu_0 bI}{4\pi(z^2+b^2)}\oint \cos\theta(d\hat{\phi}\times\hat{s}) + \sin\theta(d\hat{\phi}\times\hat{z})$$

$$= \frac{\mu_0 b I}{4\pi(z^2+b^2)}\oint\left(\cos\theta(-\hat{z}) + \sin\theta(\hat{s})\right)d\phi$$

$$= \frac{\mu_0 b I}{2\pi(z^2+b^2)}\left(z\hat{s} - b\hat{z}\right)$$

$$\tan\theta = \frac{z}{b} = \frac{\sin\theta}{\cos\theta}$$

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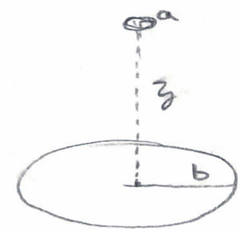
(a) from last semester

$$\vec{B} = \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}\hat{z} = B \text{ field at center of small loop}$$

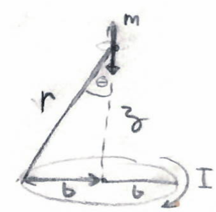


$$\Phi = \oint \vec{B}\cdot d\vec{a} = \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}\oint da = \boxed{\left(\frac{\mu_0 \pi a^2 b^2}{2(z^2+b^2)^{3/2}}\right) I = \Phi_a}$$

($\vec{B}$ is constant over ⓐ because the loop is small)

(b)

$$\Phi = \oint \vec{B}\cdot d\vec{a} = \oint (\nabla\times A)\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$$



$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m \sin\theta}{r^2}\hat{\phi} \qquad \vec{m} = \pi a^2 I$$

$$\Phi = \oint \vec{A}\cdot d\vec{l} = \frac{\mu_0 a^2 \pi I}{4\pi}\oint \frac{\sin\theta}{r^2}\hat{\phi}\cdot b\,d\hat{\phi} \qquad \sin\theta = \frac{b}{r} \quad : r = (z^2+b^2)^{1/2}$$

$$= \frac{\mu_0 a^2 I b}{4}\oint \frac{b}{r^3}d\phi = \frac{\mu_0 a^2 b^2}{4r^3}\oint d\phi = \boxed{\left(\frac{\mu_0 a^2 b^2 \pi}{2(z^2+b^2)^{3/2}}\right) I = \Phi_b}$$

(c)

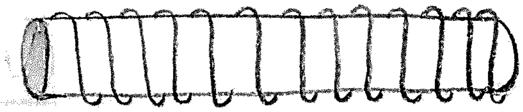
$$\Phi_a = M_{ab} I = \left(\frac{\mu_0 a^2 b^2 \pi}{2(z^2+b^2)^{3/2}}\right) I \implies M_{ab} = \frac{\mu_0 a^2 b^2 \pi}{2(z^2+b^2)^{3/2}}$$

$$\Phi_b = M_{ba} I = \left(\frac{\mu_0 a^2 b^2 \pi}{2(z^2+b^2)^{3/2}}\right) I \implies M_{ba} = \frac{\mu_0 a^2 b^2 \pi}{2(z^2+b^2)^{3/2}}$$

$\boxed{M_{ab} = M_{ba}}$ that is the mutual inductance calculated from $a \to b$ is the same as that found from $b \to a$.

7.22

find the self-inductance per length of a long solenoid. Radius R, n turns per length.



$$B = \mu_0 n I$$

$$\Phi_1 = \int \vec{B}\cdot d\vec{a} = BA = \pi R^2 \mu_0 n I$$ = flux through one loop.

$$\Phi_1 = L_1 I = (\pi R^2 \mu_0 n) I \Rightarrow L_1 = \pi R^2 \mu_0 n$$ = inductance due to one loop.

$$\Phi_\ell = \frac{N\Phi_1}{\ell} = \left(\frac{\text{\# of loops for length } \ell}{\ell}\right)\Phi_1 = \left(\frac{n\ell}{\ell}\right)\Phi_1 = (n^2 \pi R^2 \mu_0) I$$

$\therefore \boxed{L_\ell = n^2 \pi R^2 \mu_0}$ = inductance per unit length.

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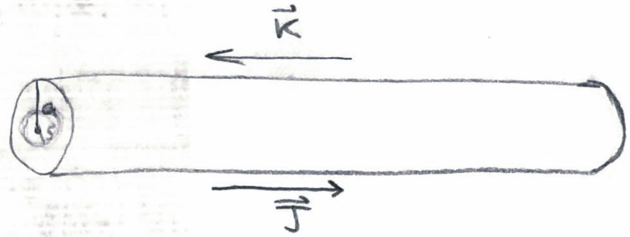
$\boxed{7.28}$

$\vec{J} = \text{constant} = \alpha$



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$\oint_S \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$I_{enc} = \int \vec{J}\cdot d\vec{a} = \alpha\pi s^2 = I_{enc}$ at s

$B(2\pi s) = \mu_0 \alpha \pi s^2 \Rightarrow B = \frac{\mu_0 \alpha s}{2}$ for $0 < s < a$

B for $s > a$ is zero since $I_{enc} = 0$ as $\vec{K}$ cancels $\vec{J}$.

$$B = \begin{Bmatrix} \frac{\mu_0 \alpha s}{2} & 0 < s < a \\ 0 & s > a \end{Bmatrix}$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

$$W = \frac{1}{2\mu_0}\int \vec{B}^2 d\tau = \frac{1}{2\mu_0}\int \left(\frac{\mu_0\alpha}{2}\right)^2 s^2 (s\,ds\,d\phi\,dz)$$

$$= \frac{\mu_0 \alpha^2}{8}\int s^3 ds\,d\phi\,dz$$

$$= \frac{\mu_0\alpha^2}{8}\left(\frac{a^4}{4}\right)(2\pi)(l) \quad \text{Let } l = \text{length of cable}$$

$$W = \frac{\mu_0 \alpha^2 a^4 \pi l}{16}$$

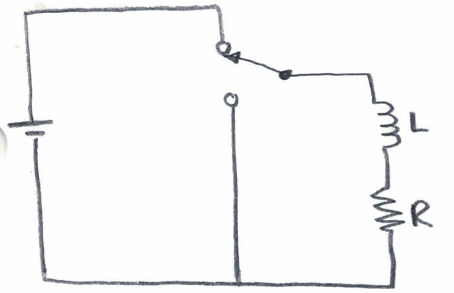
note $J = \alpha = \frac{I}{\pi a^2}$ $\therefore (\alpha\pi a^2)^2 = I^2$
$I^2 = \alpha^2\pi^2 a^4$

$$W = \frac{1}{2}LI^2 = \frac{\mu_0\alpha^2 a^4 \pi l}{16} = \frac{\mu_0 l I^2}{16\pi} = \frac{1}{2}\left(\frac{\mu_0 l}{8\pi}\right)I^2$$

$$\therefore L = \frac{\mu_0 l}{8\pi} \Rightarrow \boxed{L\,(\text{per length}) = \frac{\mu_0}{8\pi}}$$

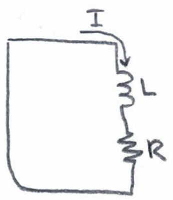
✓

7.29



Suppose this circuit has been connected as shown when at $t=0$ the battery is disconnected and the L/R combination is left to drain.

(a)



$$L\frac{dI}{dt} + IR = 0 \Rightarrow \frac{dI}{I} = -dt\left(\frac{R}{L}\right) \Rightarrow \ln I = -t\left(\frac{R}{L}\right) + \text{const.}$$

$$I_0 = I(t=0) = \frac{\mathcal{E}_0}{R} \qquad \text{note } I = e^{-t\frac{R}{L} + \text{const}} = k e^{-t\frac{R}{L}}$$

$$I(0) = I_0 = ke^0 \Rightarrow k = I_0$$

$$\therefore\ I(t) = I_0 e^{-t\frac{R}{L}} = \boxed{\frac{\mathcal{E}_0}{R} e^{-\frac{R}{L}t} = I(t)}$$

(b)

$$P = \frac{dW}{dt} = I^2 R = \left(\frac{\mathcal{E}_0}{R} e^{-\frac{R}{L}t}\right)^2 R = \frac{\mathcal{E}_0^2}{R} e^{-\frac{2R}{L}t}$$

$$W = \int dW = \int_0^\infty \frac{\mathcal{E}_0^2}{R} e^{-\frac{2R}{L}t}\, dt = \frac{\mathcal{E}_0^2}{R}\left(\frac{-L}{2R}\right) e^{-\frac{2R}{L}t}\Big|_0^\infty = \boxed{\frac{\mathcal{E}_0^2 L}{2R^2} = W}$$

(c)

$$W = \frac{1}{2} L I_0^2 = \frac{1}{2} L \left(\frac{\mathcal{E}_0}{R}\right)^2 = \boxed{\frac{L\mathcal{E}_0^2}{2R^2} = W}$$