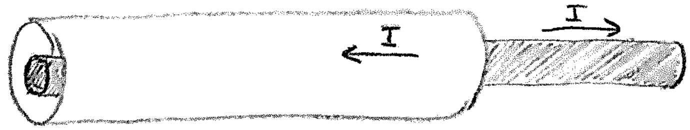
PY 415, Dr. Chung, James Cook

8.1

Calculate $\dot{E}$ transported by cables assuming the conductors are held at a constant potential and carry I in opposite directions along cable

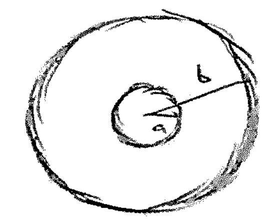


18/20 good

$$\vec{B}_{in} = \left(\frac{\mu_o I}{2\pi s}\hat{\phi}\right), \quad \vec{B}_{out} = \vec{0}$$ now as $B \neq B(t)$ we have $\frac{\partial \vec{B}}{\partial t} = 0$

and so $\nabla \times \vec{E} = 0$ thus we may use the electrostatic results. That is $\vec{E}_{in} = \left(\frac{\lambda}{2\pi\epsilon_o s}\right)\hat{s}$ where $\lambda = \frac{q}{\ell}$

$$S \equiv \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = \frac{1}{\mu_o}\left(\frac{\mu_o I}{2\pi s}\right)\left(\frac{\lambda}{2\pi\epsilon_o s}\right)\hat{s} \times \hat{\phi}$$



$$P = \frac{1}{\mu_o}\oint(\vec{E}\times\vec{B})\cdot d\vec{a} = \frac{I}{4\pi^2\epsilon_o}\oint\left(\frac{\lambda}{s^2}\hat{z}\cdot(2\pi s\, ds\, \hat{z})\right)$$

$$= \frac{I\lambda}{2\pi\epsilon_o}\int_a^b \frac{ds}{s}$$

$$= \frac{I\lambda}{2\pi\epsilon_o}\ln\left(\frac{b}{a}\right)$$

$$V = \oint_a^b \vec{E}\cdot d\vec{s} = \int_a^b \frac{\lambda}{2\pi\epsilon_o s}ds = \frac{\lambda}{2\pi\epsilon_o}\ln\left(\frac{b}{a}\right) \implies \lambda = \frac{V(2\pi\epsilon_o)}{\ln(b/a)}$$

$$P = \left(\frac{I\ln b/a}{2\pi\epsilon_o}\right)\left(\frac{V(2\pi\epsilon_o)}{\ln(b/a)}\right) = \boxed{IV = P} \; !$$

This is a general result for any element with current I that drops a voltage V the power is dissapated at the rate $P = \frac{dW}{dt} = \frac{dW}{dq}\frac{dq}{dt} = VI$.

$\therefore$ Ex. 7.58 no detail is needed. We have an element (the ribbon) which has the constant current I flow through it dropping the voltage V on it thus $\boxed{P = IV}$ regardless of the details!

(7.58)

8.2

Q PILING UP at gap.

$$I = \frac{dq}{dt} \Rightarrow Q = It. \qquad \text{Also } \vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} = \boxed{\frac{It}{\pi a^2 \epsilon_0}\hat{z} = \vec{E}}$$

$$\nabla \times \vec{B} = \cancel{\mu_0 \vec{J}}^{\,0} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(\frac{It}{\pi a^2 \epsilon_0}\right)\hat{z} = \frac{\mu_0 I}{\pi a^2}\hat{z}$$

$$\oint \vec{B} \cdot d\vec{l} = \oint \frac{\mu_0 I \hat{z}}{\pi a^2} \cdot (s\, d\phi\, ds\, \hat{z}) = \frac{\mu_0 I}{\pi a^2}(2\pi)\int s\, ds = \frac{\mu_0 I}{a^2} s^2 = \mu_0 I \frac{s^2}{a^2} = B_\phi (2\pi s)$$

$$\boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2}\hat{\phi}}$$

(b) $$u_{em} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left(\epsilon_0 \frac{I^2 t^2}{\pi^2 a^4 \epsilon_0^2} + \frac{1}{\mu_0}\frac{\mu_0^2 I^2 s^2}{4\pi^2 a^4}\right) = \boxed{\frac{I^2}{2\pi^2 a^4}\left(\frac{t^2}{\epsilon_0} + \frac{\mu_0 s^2}{4}\right) = u_{em}}$$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) = \frac{1}{\mu_0}\left(\frac{It}{\pi a^2 \epsilon_0}\right)\left(\frac{\mu_0 I s}{2\pi a^2}\right)\hat{z} \times \hat{\phi} = \boxed{\frac{-I^2 s t}{2\pi^2 a^4 \epsilon_0}\hat{s} = \vec{S}}$$

Show the above relations satisfy Eq. 8.14

Pf// $$\frac{d}{dt}\left(u_{em} + u_{mech}\right) = -\nabla \cdot \vec{S} \qquad \text{Energy flow Equation}$$

$u_{mech} = 0$ no mass in motion in gap.

$$\frac{d}{dt}\left(u_{em}\right) = \frac{d}{dt}\left(\frac{I^2}{2\pi^2 a^4}\left(\frac{t^2}{\epsilon_0} + \frac{\mu_0 s^2}{4}\right)\right) = \frac{2t I^2}{2\pi^2 a^4 \epsilon_0} = \frac{t I^2}{\pi^2 a^4 \epsilon_0} = \dot{u}_{em}$$

$$-\nabla \cdot \vec{S} = \frac{-1}{s}\frac{\partial}{\partial s}\left(s\left[\frac{-I^2 s t}{2\pi^2 a^4 \epsilon_0}\right]\right) = \frac{1}{s}\frac{I^2 t}{2\pi^2 a^4 \epsilon_0}(2s) = \frac{t I^2}{\pi^2 a^4 \epsilon_0}$$

$$\therefore \frac{d}{dt}\left(u_{em} + u_{mech}\right) = -\nabla \cdot \vec{S} = \frac{t I^2}{\pi^2 a^4 \epsilon_0} \quad !$$

(c)

ENERGY IN GAP

Note sign

$$-\int \vec{S} \cdot d\vec{a} = \frac{I^2 t}{2\pi^2 a^4 \epsilon_0}\int s\,(s\, d\phi\, dz) = \frac{I^2 t\, a^2 w}{\pi a^4 \epsilon_0} = \frac{I^2 t}{\pi a^2 \epsilon_0} w \quad b^2$$

$$\frac{dU_{em}}{dt} = \frac{d}{dt}\int u_{em}\, d\tau = \frac{d}{dt}\int \frac{I^2}{2\pi^2 a^4}\left(\frac{t^2}{\epsilon_0} + \frac{\mu_0 s^2}{4}\right) ds\, dz\, s\, d\phi = \frac{d}{dt}\frac{w I^2}{2\pi^2 a^4} 2\pi \int_0^a \left(\frac{t^2}{\epsilon_0} + \frac{\mu_0 s^2}{4}\right) s\, ds$$

$$= \frac{d}{dt}\left(\frac{w I^2}{\pi a^4}\left(t^2 \frac{a^2}{2\epsilon_0} + \frac{\mu_0 a^4}{16}\right)\right) = \frac{w I^2 t}{\pi a^2 \epsilon_0}$$

$$\frac{dU_{em}}{dt} = \int \vec{S} \cdot d\vec{a} = \frac{w I^2 t}{\pi a^2 \epsilon_0}. \quad \Bigg\} \quad E_{in\ gap} = \boxed{\frac{w I^2 t^2 b^2}{2\pi a^4 \epsilon_0} + \frac{\mu_0 I^2 w b^4}{16\pi a^4} = E_{in\ gap}}$$

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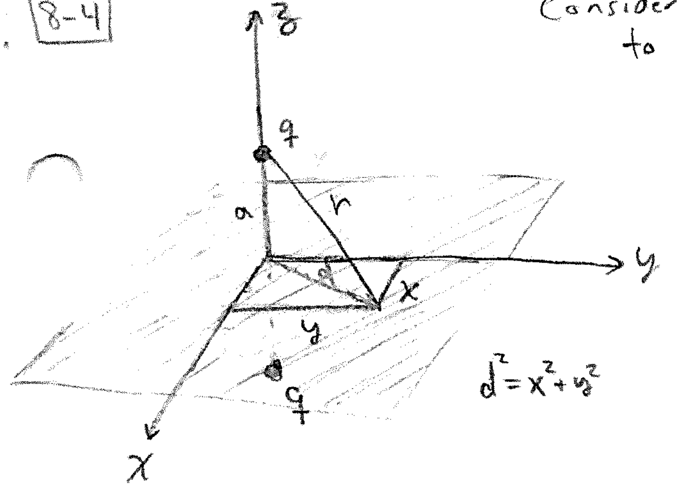
$\boxed{8\text{-}4}$

Consider charges on z axis for my convience.
to make picture easier.



$d^2 = x^2 + y^2$

20/20

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\vec{E}_{top} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{x}{d}\hat{x} + \frac{y}{d}\hat{y} - \frac{a}{r}\hat{z}\right)$$

$$\vec{E}_{bottom} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{x}{d}\hat{x} + \frac{y}{d}\hat{y} + \frac{a}{r}\hat{z}\right)$$

eval. at x-y plane.

$$\vec{E}_{total} = \frac{q}{4\pi\epsilon_0 r^2}\left(\frac{2x}{d}\hat{x} + \frac{2y}{d}\hat{y}\right)$$

$$E_x = \frac{q\,2x}{4\pi\epsilon_0 r^2 d} = \frac{q}{2\pi\epsilon_0}\frac{x}{r^2 d} \qquad \text{let } \beta \equiv \frac{q}{2\pi\epsilon_0}$$

$$E_y = \frac{q\,2y}{4\pi\epsilon_0 r^2 d} = \frac{q}{2\pi\epsilon_0}\frac{y}{r^2 d}$$

$$E_z = 0$$

$$F_j = \oint \sum_i T_{ij}\,da_i \quad , \quad d\vec{a} = da_z\hat{z} = dx\,dy\,\hat{z} \quad , \quad da_x = da_y = 0.$$

$$\Rightarrow F_j = T_{zj}\,da_z \quad , \quad F_x = T_{zx}\,da_z \quad , \quad F_y = T_{zy}\,da_z \quad , \quad F_z = T_{zz}\,da_z$$

$$T_{zx} = \epsilon_0 E_z E_x = 0 \quad , \quad T_{zy} = \epsilon_0 E_z E_y = 0$$

$$T_{zz} = \tfrac{1}{2}\epsilon_0\left(-E_x^2 - E_y^2\right) = \frac{-1}{2}\epsilon_0\left(\beta^2\left(\frac{x^2}{r^4 d^2} + \frac{y^2}{r^4 d^2}\right)\right) = \frac{-1}{2}\epsilon_0\beta^2\left(\frac{x^2+y^2}{r^4(x^2+y^2)}\right)$$

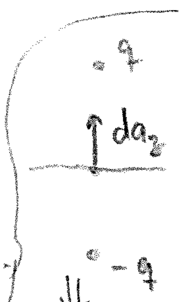
$$= \frac{-1}{2}\epsilon_0\beta^2\frac{1}{r^4} = \underbrace{\frac{-1}{2}\epsilon_0\left(\frac{q^2}{4\pi^2\epsilon_0^2}\right)}_{\delta}\frac{1}{r^4}$$

$$F_z = \int_{x\text{-}y \text{ plane}} T_{zz}\,da_z = \delta\int\frac{dx\,dy}{r^4} = \delta\int\frac{dx\,dy}{(x^2+y^2+a^2)^2} = \delta\left(\int\frac{dx}{(x^2+b^2)^2}\right)dy \quad , \quad y^2 + a^2 = b^2$$

$$= \delta\int\left(\frac{x}{2b^2(b^2+x^2)} + \frac{1}{2b^3}\tan^{-1}\left(\frac{x}{b}\right)\right)\Bigg|_{-\infty}^{\infty} dy$$

$$= \delta\int\lim_{x\to\infty}\left[\left(\frac{x}{2(y^2+a^2)(y^2+a^2+x^2)}\xrightarrow{}0\right) + \frac{1}{2(y^2+a^2)^{3/2}}\tan^{-1}\left(\frac{x}{y^2+a^2}\right)\right]\Bigg|_{-b}^{b} dy$$

$$= \delta\int\frac{\pi}{2(y^2+a^2)^{3/2}}\,dy = \frac{\delta\pi}{2}\left(\lim_{y\to\infty}\frac{y}{a^2}(y^2+a^2)^{1/2}\right)\Bigg|_{-\infty}^{\infty} = \frac{\delta\pi}{a^2}$$



$$F_z = \frac{-q^2}{8\pi^2\epsilon_0}\frac{\pi}{2a^2} = \frac{-q^2}{4\pi\epsilon_0 4a^2} \Rightarrow \boxed{\vec{F} = \frac{+q^2}{4\pi\epsilon_0}\frac{1}{(2a)^2}\hat{z}}$$

this is force on bottom charge! I took $da_z > 0$

✓

$\boxed{8.4}$

(b) ρ $q_{bottom} = -q$ where $q = q_{top}$ hence $E_z = \frac{2qa}{4\pi\epsilon_0 r^3}$

$T_{zz} = \frac{1}{2}\epsilon_0 \left(E_z^2\right)$ no E_x and E_y as E_x, E_y of q cancels E_x, E_y of $-q$!

$$F_z = \int T_{zz}\, da_z$$

$$= \underbrace{\frac{1}{2}\epsilon_0\left(\frac{2qa}{4\pi\epsilon_0}\right)^2}_{\varphi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dx\,dy}{(x^2+y^2+a^2)^3}$$

$$= \varphi \int_{-\infty}^{\infty} \left\{\frac{3\pi}{8(y^2+a^2)^{5/2}}\right\} dy \longleftarrow$$

(This is why I bought the TI-89)

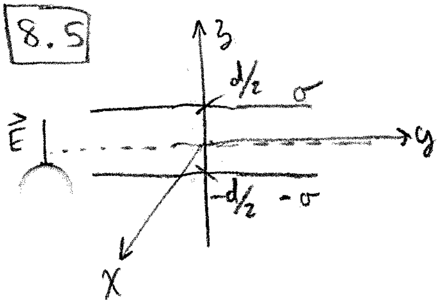
$$= \varphi \frac{\pi}{2a^4}$$

$$= \frac{1}{2}\epsilon_0 \frac{2^2 q^2 a^2}{4^2\pi^2\epsilon_0^2} \frac{\pi}{2a^4}$$

$$= \frac{4}{64}\frac{q^2}{\epsilon_0 a^2}$$

$$\checkmark\ \boxed{\vec{F} = \frac{-1}{4\pi\epsilon_0}\frac{q^2}{(2a)^2}\hat{z}}$$

$\boxed{8.5}$



$E_x = E_y = 0 \qquad B_x = B_y = B_z = 0$

$E_z = \frac{-\sigma}{\epsilon_0}$

$T_{xx} = \frac{1}{2}\epsilon_0(-E_z^2) = -\frac{1}{2}\frac{\sigma^2}{\epsilon_0} \qquad T_{xy} = T_{yx} = 0$

$T_{yy} = \frac{1}{2}\epsilon_0(-E_z^2) = -\frac{1}{2}\frac{\sigma^2}{\epsilon_0} \qquad T_{yz} = T_{zy} = 0$

$T_{zz} = \frac{1}{2}\epsilon_0(E_z^2) = \frac{1}{2}\frac{\sigma^2}{\epsilon_0} \qquad T_{zx} = T_{xz} = 0$

$$\checkmark \quad \overleftrightarrow{T} = \begin{pmatrix} -\frac{1}{2}\frac{\sigma^2}{\epsilon_0} & 0 & 0 \\ 0 & -\frac{1}{2}\frac{\sigma^2}{\epsilon_0} & 0 \\ 0 & 0 & \frac{1}{2}\frac{\sigma^2}{\epsilon_0} \end{pmatrix}$$

(b) Use eq. 8.22 to find F/A on top plate.

$$\vec{F} = \oint \overleftrightarrow{T}\cdot d\vec{a} - \cancel{\epsilon_0\mu_0\frac{d}{dt}\int \vec{S}\,d\tau}^{\,0} = \oint \overleftrightarrow{T}\cdot d\vec{a}$$

$$F_j = \cancel{\int T_{xj}\,da_x}^{\,0} + \cancel{\int T_{yj}\,da_y}^{\,0} + \int T_{zj}\,da_z$$

$$F_x = \int T_{zx}\,dx\,dy = 0$$

$$F_y = \int T_{zy}\,dx\,dy = 0$$

$$F_z = \int T_{zz}\,dx\,dy = \int \frac{1}{2}\frac{\sigma^2}{\epsilon_0}\,dx\,dy = \frac{\sigma^2}{2\epsilon_0}(xy) = \frac{\sigma^2}{2\epsilon_0}(\text{AREA}).$$

$$\checkmark \quad \Rightarrow \frac{\vec{F}}{\text{unit Area}} = \vec{\text{PRESSURE}} = \frac{\sigma^2}{2\epsilon_0}\hat{z} = -\frac{\sigma^2}{2\epsilon_0}\hat{n}$$ just like eq. 2.51 !

8.5

(c) What is momentum per unit area time crossing x-y plane between plates.

$-T_{ij}$ = the momentum in the i^{th} direction crossing a surface oriented in the j^{th} direction, per unit area time!

the x-y plane $\Longrightarrow$ $j = \hat{z}$.

$$T_{iz} = T_{xz},\ T_{yz},\ T_{zz} = 0, 0, T_{zz}$$

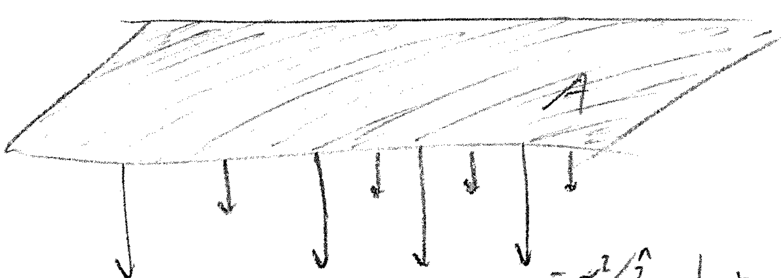
✓ $$\hat{z}(-T_{zz}) = \boxed{-\frac{1}{2}\frac{\sigma^2}{\epsilon_0}\hat{z} = \text{momentum per unit area, time}}$$

(d) find the recoil force per unit area on top plate.

$$\frac{\Delta P}{\Delta t \Delta A} = -\frac{1}{2}\frac{\sigma^2}{\epsilon_0} \quad \text{between plates}$$

then at plate field collides with plate which was at rest, if momentum is conserved as field is absorbed then plates recoil with $\frac{1}{2}\frac{\sigma^2}{\epsilon_0}$ = RECOIL FORCE/AREA

that is the plate pushes upward as the field pulls down with $\frac{-\sigma^2}{2\epsilon_0}$



$$-\frac{\sigma^2}{2\epsilon_0}\hat{z} = (-T_{zz})\hat{z} = \frac{dP}{dt}\left(\frac{1}{A}\right)$$

✓

$$\boxed{\vec{f}_{recoil} = \frac{-\sigma^2}{2\epsilon_0}\hat{z}}$$

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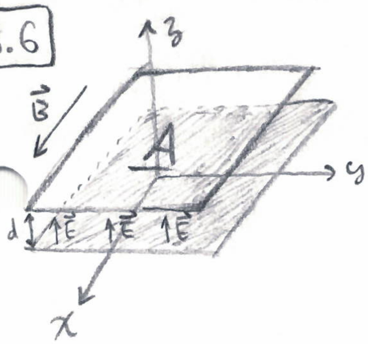
8.6



$\vec{E} = E\hat{z}$

$\vec{B} = B\hat{x}$

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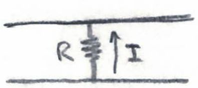
(a) $\vec{P}_{em} = \epsilon_0[\vec{E}\times\vec{B}] = \epsilon_0 E B(\hat{z}\times\hat{x}) = \underline{+\epsilon_0 E B\hat{y} = \vec{P}}$

Assumeably this is a <u>large</u> capacitor so we may ignore fring effects

thus $\vec{P}_{total} = (\text{Volume})(+\epsilon_0 E B\hat{y}) = \boxed{Ad\epsilon_0 E B\hat{y} = \vec{P}_{total}}$

(b)



$I = I_0 e^{-t/\tau}$ where $\tau = RC = R\frac{A\epsilon_0}{d}$

$V_0 = I_0 R$, $V_0 = Ed \Rightarrow I_0 = \frac{V_0}{R} = \frac{Ed}{R}$

$\therefore I = \frac{Ed}{R}\exp\left\{-t\frac{d}{RA\epsilon_0}\right\}$

$\vec{F} = I\vec{l}\times\vec{B}$

$= \frac{Ed}{R}e^{-t/\tau}\left(d\hat{z}\times B\hat{x}\right) = \frac{BEd^2}{R}e^{-t/\tau}\hat{y}$

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \int d\vec{P} = \int_0^\infty \frac{d\vec{P}}{dt}dt = \int_0^\infty \vec{F}dt = \frac{BEd^2}{R}\hat{y}\int_0^\infty e^{-t/\tau}dt$

$= \frac{\tau BEd^2\hat{y}}{R}\left(\cancel{-e^{-t/\tau}\Big|_0^\infty}\right)^{1} = \left(R\frac{A\epsilon_0}{d}\right)\left(\frac{BEd^2}{R}\right)\hat{y} = \boxed{BEAd\epsilon_0\hat{y} = \vec{P}_{tot}}$

(impulse) ΔP

(c) $\mathcal{E} = -\frac{d\Phi}{dt}$ and $\mathcal{E} = E_i l$

$\mathcal{E} = -\frac{d\Phi}{dt} = -dl\frac{dB}{dt} = E_i l \Rightarrow E_i = -d\frac{dB}{dt}$

$F_y = qE_i = CVE_i = \frac{A\epsilon_0}{d}Ed\left(-d\frac{dB}{dt}\right) = \frac{dP_y}{dt} \Rightarrow AE\epsilon_0 d(dB) = dP_y$

$\Rightarrow \boxed{\vec{P} = BEAd\epsilon_0\hat{y}} = \Delta P$ (impulse)

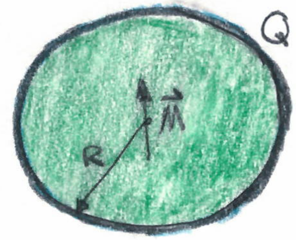
A better way to view parts b, c is to just consider $\vec{E}_{final}, \vec{B}_{final}$ and $\vec{E}_{initial}, \vec{B}_{initial}$ then it is trivial to find ΔP, however I gather the problem is too easy that way...

Good

$\boxed{8.8}$

(a) As there is no charge on inside and $B \neq B(t)$ there is no induced or coulombic $\vec{E}$

$\therefore \vec{E} = \vec{0}$ for $r < R$.



$\therefore$ it is sufficient to consider only outside fields since $\vec{l}_{em} = \epsilon_0[\vec{r} \times (\vec{E} \times \vec{B})] = \epsilon_0[\vec{r} \times (\vec{0} \times \vec{B})] = \vec{0}$!

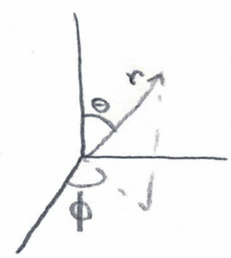
Outside, $r > R$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$: nothing revolutionary here.

$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\{(2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\}$ $\qquad m = M\frac{4}{3}\pi R^3$, as if there were just one dipole at origin

$$\vec{l}_{em} = \epsilon_0\left[\vec{r} \times \left\{\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)\hat{r} \times \left(\frac{\mu_0 m}{4\pi r^3}\right)\left[2\cos\theta\,\hat{r} + (\sin\theta)\hat{\theta}\right]\right\}\right]$$

$$= \frac{Q\,\mu_0 m}{(4\pi)^2 r^5}\left[\vec{r} \times \left(\hat{r} \times (\sin\theta)\hat{\theta}\right)\right]$$



$$= \frac{Q\,\mu_0 m}{(4\pi)^2 r^4}\left[\hat{r} \times \sin\theta(-\hat{\phi})\right]$$

$$\vec{l}_{em} = -\frac{\sin\theta\, Q\,\mu_0 m}{16\pi^2 r^4}\hat{\theta} \Longrightarrow \boxed{L = \frac{2}{9}MQ\mu_0 R^2\,\hat{z}}$$

we neglected the dependence of $\hat{\theta}$ on r, θ, ϕ.

how do you go from $l \to L$ and $\hat{\theta} \to \hat{z}$

Now integrate for $r > R$ for total,

$$\vec{L}_{em} = \int_{r>R} \frac{-\sin\theta\, Q\mu_0 m}{16\pi^2 r^4} r^2 \sin\theta\, d\theta\, dr\, d\phi\;\hat{\theta}$$

-3

show correct int. over $\hat{z}$ not $\hat{\theta}$.

$$= \frac{-Q\mu_0 m}{16\pi^2}\int_R^\infty \frac{dr}{r^2}\int_0^\pi \sin^2\theta\, d\theta \int_0^{2\pi} d\phi\;\hat{\theta}$$

Convert to $\hat{z}$

$$= \frac{-Q\mu_0 m}{16\pi^2}\left(\frac{1}{R}\right)\left(\frac{\pi}{2}\right)(2\pi)\hat{\theta}$$

$$= \frac{-Q\mu_0}{R}\frac{1}{16}M\frac{4}{3}\pi R^3\hat{\theta} = \boxed{\frac{-\mu_0 Q M R^2 \pi}{12}\hat{\theta} = \vec{l}_{em\ total}}$$

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$\boxed{9.13}$

Calculate the exact reflection and transmission coefficients. ($\mu_1 = \mu_2 = \mu_0$ not allowed)
Confirm $R + T = 1$.

$$\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I} \qquad \text{where } \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$\tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}$$

(Eq's 9.82, 9.81 from pg. 385)

$$I = \frac{1}{2}\epsilon v E_0^2$$

$$\boxed{\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}}$$

writing out β would be just the same answer!

$$R = \frac{I_R}{I_I} = \frac{\frac{1}{2}\epsilon_1 v_1 E_{0R}^2}{\frac{1}{2}\epsilon_1 v_1 E_{0I}^2} = \frac{\left(\frac{1-\beta}{1+\beta}\right)^2 E_{0I}^2}{E_{0I}^2} = \boxed{\left(\frac{1-\beta}{1+\beta}\right)^2 = R}$$

$$T = \frac{I_T}{I_I} = \frac{\frac{1}{2}\epsilon_2 v_2 E_{0T}^2}{\frac{1}{2}\epsilon_1 v_1 E_{0I}^2} = \frac{\epsilon_2 v_2 \left(\frac{2}{1+\beta}\right)^2 E_{0I}^2}{\epsilon_1 v_1 E_{0I}^2} = \boxed{\frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{2}{1+\beta}\right)^2 = T}$$

$$R + T = \left(\frac{1-\beta}{1+\beta}\right)^2 + \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{2}{1+\beta}\right)^2$$

$$= \frac{\epsilon_1 v_1 (1-\beta)^2 + 4(\epsilon_2 v_2)}{(1+\beta)^2 \epsilon_1 v_1}$$

$$= \frac{\frac{1}{\mu_1 v_1^2} v_1 (1-\beta)^2 + 4\left(\frac{1}{\mu_2 v_2^2} v_2\right)}{(1+\beta)^2 \frac{1}{\mu_1 v_1^2} v_1}$$

$$= \frac{\cancel{\frac{1}{\mu_1 v_1}}(1-\beta)^2 + 4\beta \cancel{\frac{1}{\mu_1 v_1}}}{\cancel{\frac{1}{\mu_1 v_1}}(1+\beta)^2}$$

$$= \frac{(1-\beta)^2 + 4\beta}{(1+\beta)^2}$$

$$= \frac{1 - 2\beta + \beta^2 + 4\beta}{1 + 2\beta + \beta^2}$$

$$= \frac{1 + 2\beta + \beta^2}{1 + 2\beta + \beta^2}$$

$$= 1 \quad \therefore \boxed{R + T = 1}$$

✓

$$c^2 = \frac{1}{\mu_0 \epsilon_0} \Rightarrow v^2 = \frac{1}{\mu \epsilon}$$

$$\therefore \epsilon = \frac{1}{\mu v^2}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} \Rightarrow \mu_2 v_2 = \frac{\mu_1 v_1}{\beta}$$

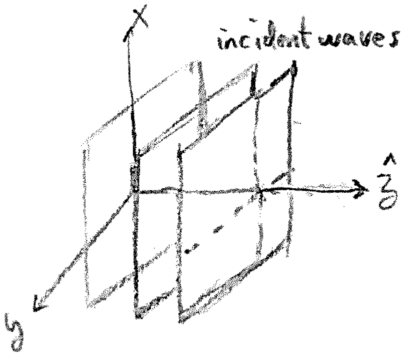
$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}$$

9.14

$$\tilde{\vec{E}}_R = \tilde{E}_{0R}\, e^{i(-k_1 z - \omega t)} \left[\cos\theta_R \hat{x} + \sin\theta_R \hat{y}\right]$$

$$\tilde{\vec{E}}_T = \tilde{E}_{0T}\, e^{i(k_2 z - \omega t)} \left[\cos\theta_T \hat{x} + \sin\theta_T \hat{y}\right]$$

$$\epsilon_1 E_R^{\parallel}\Big|_y = \epsilon_2 E_T^{\parallel}\Big|_y \Rightarrow \epsilon_1 \tilde{E}_{0R} e^{i(0-\omega t)} \sin\theta_R = \epsilon_2 \tilde{E}_{0T} e^{i(0-\omega t)} \sin\theta_T$$



$$\epsilon_1 \tilde{E}_{0R} \sin\theta_R = \epsilon_2 \tilde{E}_{0T} \sin\theta_T$$

DIRECTION OF B is $\hat{k} \times \hat{n}$,

$$\hat{k}_R \times \hat{n}_R = -\hat{z} \times \left[\cos\theta_R \hat{x} + \sin\theta_R \hat{y}\right] = -\cos\theta_R \hat{y} + \sin\theta_R \hat{x}$$

$$\hat{k}_T \times \hat{n}_T = \hat{z} \times \left[\cos\theta_T \hat{x} + \sin\theta_T \hat{y}\right] = \cos\theta_T \hat{y} - \sin\theta_T \hat{x}$$

$$\frac{1}{\mu_1} B_1^{\parallel}\Big|_x = \frac{1}{\mu_2} B_2^{\parallel}\Big|_x \Rightarrow \frac{1}{\mu_1}\frac{1}{v_1} \tilde{E}_{0R} \sin\theta_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0T} \sin\theta_T$$

$$\epsilon_1 = n_1^2 \qquad \epsilon_2 = n_2^2 \qquad n = \frac{c}{v} \qquad v^2 = \frac{1}{\mu\epsilon}$$

$$\Downarrow$$

$$n_1 \tilde{E}_{0R} \sin\theta_R = -n_2 \tilde{E}_{0T} \sin\theta_T$$

(i) $\epsilon_1 \tilde{E}_{0R} \sin\theta_R = \epsilon_2 \tilde{E}_{0T} \sin\theta_T$

(ii) $n_1 \tilde{E}_{0R} \sin\theta_R = -n_2 \tilde{E}_{0T} \sin\theta_T$

(i) $\epsilon = n^2 \Rightarrow n_1^2 \tilde{E}_{0R} \sin\theta_R = n_2^2 \tilde{E}_{0T} \sin\theta_T$

now if $\sin\theta_T$ AND $\sin\theta_R \neq 0 \Rightarrow n_1 = n_1^2$ and $n_2 = -n_2^2$

as $n_2 \in \mathbb{R}$ $n_2 \neq -n_2^2$ unless $n_2 = 0$ (not possible here $\frac{v}{c} \neq 0$)

$\therefore \sin\theta_R = 0$ and $\sin\theta_T = 0$ $\therefore$ $\boxed{\theta_R = \theta_T = 0}$ ✓

PY 415 // Dr. Chang

JAMES COOK

20/20

9.1 Show f_1, f_2, f_3 do not satisfy wave eq. Show f_4, f_5 do

(i) $f_1 = Ae^{-b(z-vt)^2}$

SPACE

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{\partial}{\partial z}\left(Ae^{-b(z-vt)^2}\left(-b\frac{d}{dz}(z-vt)^2\right)\right) = \frac{\partial}{\partial z}\left(-Ab\,2(z-vt)e^{-b(z-vt)^2}\right)$$

$$= -2Abe^{-b(z-vt)^2}\left(1 + (z-vt)(-2b(z-vt))\right)$$

$$= -2Abe^{-b(z-vt)^2}\left(1 - 2b(z-vt)^2\right)$$

TIME

$$\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2} = \frac{1}{v^2}\frac{\partial}{\partial t}\left(-2Ab(z-vt)(-v)e^{-b(z-vt)^2}\right) = \frac{1}{v^2}\left(2Abv\,e^{-b(z-vt)^2}\left(-v + (z-vt)(2vb(z-vt))\right)\right)$$

$$= \frac{1}{v^2}\left(v^2\,2Abe^{-b(z-vt)^2}\left(-1 + 2b(z-vt)^2\right)\right)$$

✓ $$= -2Abe^{-b(z-vt)^2}\left(1 - 2b(z-vt)^2\right) \quad \therefore \frac{\partial^2 f_1}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2} \quad !$$

(ii) SPACE

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{\partial^2}{\partial z^2}\left(A\sin[b(z-vt)]\right) = \frac{\partial}{\partial z}\left(Ab\cos(b(z-vt))\right) = -Ab^2\sin(b(z-vt))$$

TIME

$$\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2} = \frac{\partial^2}{\partial t}\left(A\sin[b(z-vt)]\right) = \frac{1}{v^2}\frac{\partial}{\partial t}\left(-Abv\cos[b(z-vt)]\right) = \frac{1}{v^2}\left\{-Abv(-bv)(-\sin[b(z-vt)])\right\}$$

✓ $$= -Ab^2\sin[b(z-vt)] \quad \therefore \frac{\partial^2 f_2}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}$$

(iii) SPACE

$$\frac{\partial^2 f_3}{\partial z^2} = \frac{\partial^2}{\partial z^2}\left(\frac{A}{b(z-vt)^2+1}\right) = A\frac{\partial}{\partial z}\left(-[b(z-vt)^2+1]^{-2}(2b(z-vt))\right)$$

$$= A\left\{2[b(z-vt)^2+1]^{-3}(2b(z-vt))^2 - 2b[b(z-vt)^2+1]^{-2}\right\}$$

TIME

$$\frac{1}{v^2}\frac{\partial^2 f_3}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(\frac{A}{b(z-vt)^2+1}\right) = \frac{A}{v^2}\frac{\partial}{\partial t}\left\{-[b(z-vt)^2+1]^{-2}(-2bv(z-vt))\right\}$$

$$= \frac{A}{v^2}\left\{-2[b(z-vt)^2+1]^{-3}(-2bv(z-vt)\,2bv(z-vt)) - 2bv^2[b(z-vt)^2+1]^{-2}\right\}$$

$$= A\left\{2[b(z-vt)^2+1]^{-3}(2b(z-vt))^2 - 2b[b(z-vt)^2+1]^{-2}\right\}$$

✓ $$\therefore \frac{\partial^2 f_3}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$