$\boxed{9.1}$

(iv) $$\frac{\partial^2 f_4}{\partial z^2} = \frac{\partial^2}{\partial z^2}\left(Ae^{-b(bz^2+vt)}\right) = A\frac{\partial}{\partial z}\left(e^{-b(bz^2+vt)}(-2b^2 z)\right)$$

$$= Ae^{-b(bz^2+vt)}\left[(-2b^2z)(-2b^2z) - 2b^2\right]$$

$$= Ae^{-b(bz^2+vt)}\left[(2b^2z)^2 - 2b^2\right]$$

$$\frac{1}{v^2}\frac{\partial^2 f^4}{\partial t^2} = \frac{A}{v^2}\frac{\partial}{\partial t}\left(e^{-b(bz^2+vt)}(-bv)\right) = \frac{A}{v^2}\left(e^{-b(bz^2+vt)}(-bv)(-bv)\right)$$

$$= Ab^2e^{-b(bz^2+vt)} \neq Ab^2e^{-b(bz^2+vt)}\left[4b^2z^2-2\right] = \frac{\partial^2 f_4}{\partial z^2}$$

unless $1 = 4b^2z^2-2 \iff z = \left(\frac{3}{4}\frac{1}{b^2}\right)^{1/2}$ certainly not $\forall\, t, z$ as we would like.

(v) $$\frac{\partial^2 f_5}{\partial z^2} = \frac{\partial^2}{\partial z^2}\left(A\sin(bz)\cos(bvt)^3\right) = A\frac{\partial}{\partial z}\left(b\cos(bz)\cos(bvt)^3 - 0\right)$$

$$= Ab\left(-(\sin bz)b\cos(bvt)^3 + \cancel{\cos(bz)(0)}^{\to 0}\right) = -Ab^2\sin(bz)\cos(bvt)^3$$

$$\frac{1}{v^2}\frac{\partial^2 f_5}{\partial t^2} = \frac{A}{v^2}\frac{\partial}{\partial t}\left(\left[\sin bz(-\sin(bvt)^3)3(bvt)^2bv\right] + \cancel{\left[0\cdot\cos(bvt)^3\right]}^{\to 0}\right)$$

$$= -\frac{A}{v^2}3bv\sin bz\,\frac{\partial}{\partial t}\left((bvt)^2\sin(bvt)^3\right)$$

$$= -\frac{A}{v^2}3bv\sin bz\left(2(bvt)bv\sin(bvt)^3 + (bvt)^2\,3(bvt)^2(bv)\cos(bvt)^3\right)$$

$$= -\frac{A}{v^2}3bv\sin bz\left(2b^2\cancel{v^2}t\sin(bvt)^3 + 3bv(bvt)^4\cos(bvt)^3\right)$$

$$= -3Ab^3v\sin bz\sin(bvt)^3 + 9Ab^2\frac{1}{v}\sin bz\,(bvt)^4\cos(bvt)^3$$

$$= -3Ab^3v\sin bz\sin(bvt)^3 + 9Av^3b^6t^4\sin(bz)\cos(bvt)^3$$

$\neq \frac{\partial^2 f_5}{\partial z^2}$ by the principle of gross disfigurement.

9.2

(a) $f(z,t) = A\sin(kz)\cos(kvt)$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(Ak\cos(kz)\cos(kvt)\right) = -\cos(kvt)\,Ak^2\sin(kz)$$

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{1}{v^2}A\sin kz\,(-kv)\frac{\partial}{\partial t}\left(\sin(kvt)\right) = \frac{-1}{v^2}Ak^2v^2\cos(kvt)\sin(kz)$$

$$= -\cos(kvt)\,Ak^2\sin kz \quad \therefore \quad \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \checkmark$$

(b) find g, h such that $f(z,t) = g(z-vt) + h(z+vt)$

the variables g, h depend on.

$f = A\sin(kz)\cos(kvt)$

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(A-B) = \sin A\cos B - \sin B\cos B$

$$\frac{\sin(A+B)+\sin(A-B)}{2} = \sin A\cos B$$

Let $A = kz$, $B = kvt$

then $\Rightarrow f = \frac{A}{2}\left(\sin(kz+kvt) + \sin(kz-kvt)\right)$

✓ thus $\boxed{f = \frac{A}{2}\sin(kz+kvt) + \frac{A}{2}\sin(kz-kvt)}$

Note/

$h(z+vt) = \frac{A}{2}\sin(kz+kvt)$

$g(z-vt) = \frac{A}{2}\sin(kz-kvt)$

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$\boxed{9.3}$

$$A_3 e^{i\delta_3} = A_2 e^{i\delta_2} + A_1 e^{i\delta_1} \qquad Eq.\ 9.19$$

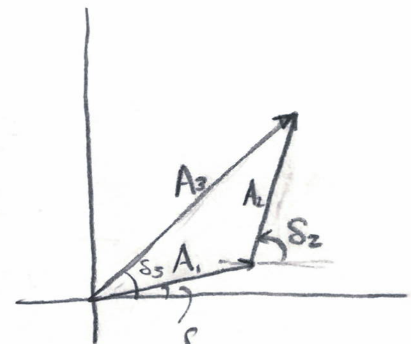
$$I \quad A_3(\cos\delta_3 + i\sin\delta_3) = A_2(\cos\delta_2 + i\sin\delta_2) + A_1(\cos\delta_1 + i\sin\delta_1)$$

$$Re(I) = A_3\cos\delta_3 = A_2\cos\delta_2 + A_1\cos\delta_1 \qquad (*)$$

$$Im(I) = A_3\sin\delta_3 = A_2\sin\delta_2 + A_1\sin\delta_1 \qquad (**)$$

$$\frac{A_3\sin\delta_3}{A_3\cos\delta_3} = \tan\delta_3 = \frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2}$$

$$\therefore \boxed{\delta_3 = \tan^{-1}\left\{\frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2}\right\}}$$



see just vector addition, break apart and add.

Complex congugate of eq 9.19 $\Rightarrow$

$$A_3 e^{i\delta_3} = A_2 e^{i\delta_2} + A_1 e^{i\delta_1}$$

$$A_3 e^{-i\delta_3} = A_2 e^{-i\delta_2} + A_1 e^{-i\delta_1}$$

$$(A_3 e^{i\delta_3})(A_3 e^{-i\delta_3}) = (A_2 e^{i\delta_2} + A_1 e^{i\delta_1})(A_2 e^{-i\delta_2} + A_1 e^{-i\delta_1})$$

$$= A_2^2 + A_1^2 + A_1A_2 e^{i(\delta_1-\delta_2)} + A_2A_1 e^{i(\delta_2-\delta_1)}$$

$$= A_2^2 + A_1^2 + A_1A_2\left(e^{i(\delta_1-\delta_2)} + e^{-i(\delta_1-\delta_2)}\right)\frac{1}{2}2$$

$$A_3^2 = A_2^2 + A_1^2 + 2A_1A_2\cos(\delta_1-\delta_2)$$

$$\therefore \boxed{A_3 = (A_2^2 + A_1^2 + 2A_1A_2\cos(\delta_1-\delta_2))^{1/2} = (A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_2-\delta_1))^{1/2}}$$

by PLA!

(Principle of least astonishment)

or symetry or $\cos x = \cos(-x)$.

note the above is easily obtained by the application of the low of cosines

9.4

SUPPOSE WE MAY COMPOSE OUR SOLUTION TO THE WAVE Eq with THE SUM OF A PRODUCT OF $Z(z)$ and $y(t)$.

$$f(z,t) = Z(z)\,y(t) \qquad \text{Def}^{n} \quad Z' = \frac{\partial Z}{\partial z} \quad \text{and} \quad y' = \frac{\partial y}{\partial t}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \Rightarrow yZ'' = \frac{1}{v^2} Z y''$$

thus, $\frac{Z''}{Z} = \frac{1}{v^2}\frac{y''}{y} = \text{constant} = \frac{-\omega^2}{v^2}$, we know this must be constant as $v \neq v(t)$ and y, Z are independent.

$$Z'' = -\frac{\omega^2}{v^2} Z \Rightarrow Z = c_1 e^{i(\frac{\omega}{v}z + \delta)}$$, where c_1, δ are arbitrairy.

$$y'' = -\omega^2 y \Rightarrow y = c_2 e^{i\omega t} + c_3 e^{-i\omega t}$$, where c_2, c_3 are arbitrairy

Now since we are attempting to describe a <u>finite</u> wave packet $\Rightarrow c_2 = 0$. Thus,

$$f(z,t) = Zy = \left(c_1 e^{i(\frac{\omega}{v}z + \delta)}\right) c_3 e^{-i\omega t}$$
$$= c_1 c_3 e^{i\delta} e^{i(\frac{\omega}{v}z - \omega t)}$$

Note that $c_1 c_3 e^{i\delta} = \tilde{A}(k)$ since $\delta = \delta(k)$!
Then we compose some function "$\tilde{f}$" as the infinite sum of the multiplicative composition, Note $\frac{\omega}{v} \equiv k$.

$$\boxed{\tilde{f} = \int_{-\infty}^{\infty} \tilde{A}(k) e^{i(kz - \omega t)}\, dk}$$

✓

cute

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THIS STAPLE THROUGH BOTH

BUT EACH ASSIGNMENT STAPLED SEPERATELY, PULL APART IF YA WANT TO.

just this assign.

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9.6

(a) $\Delta F = m\frac{\partial^2 f}{\partial t^2}\Big|_{z=0} = T\left(\frac{\partial f}{\partial z}\Big|_{0^+} - \frac{\partial f}{\partial z}\Big|_{0^-}\right)$ Boundary condition derived from assumption θ small as in Book, we just know $\Delta F \neq 0$ rather $\Delta F = m\ddot{f}$

(b) $\tilde{f}(z,t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - wt)} + \tilde{A}_R e^{(-k_1 z - wt)} & \text{for } z < 0 \\ \tilde{A}_T e^{i(k_2 z - wt)} & \text{for } z > 0 \end{cases}$

$f(0^-,t) = f(0^+,t)$ for obvious reasons. Note $e^{ikz} \to 1$ for $z \to 0$.

$\Rightarrow f(0^-,t) = \tilde{A}_I e^{-iwt} + \tilde{A}_R e^{-iwt} = \tilde{A}_T e^{-iwt} = \tilde{f}(0^+,t)$

$\therefore \underline{\tilde{A}_I + \tilde{A}_R = \tilde{A}_T}$ (as before for $\Delta F = 0$)

$\frac{\partial f}{\partial z}\Big|_{0^-} = ik_1 \tilde{A}_I e^{-iwt} - ik_1 \tilde{A}_R e^{-iwt}$

$\frac{\partial f}{\partial z}\Big|_{0^+} = ik_2 \tilde{A}_T e^{-iwt}$

(again e^z type terms $\to 0$ as $z \to 0$)

$\frac{\partial^2}{\partial t^2}\left(\tilde{A}_T e^{i(kz - wt)}\right)\Big|_{z=0} = -\tilde{A}_T w^2 e^{-iwt}$

magnitude at $z = 0$ must match $\tilde{A}_T = \tilde{A}_I + \tilde{A}_R$

$\therefore -\frac{mw^2}{T}\tilde{A}_T = \underline{ik_2 \tilde{A}_T - ik_1 \tilde{A}_I + ik_1 \tilde{A}_R}$

$k_2 = \frac{w}{v_2}$ but $v_2 = \sqrt{\frac{T}{\mu_2}}$ however as $\mu_2 \to 0$ (massless) $\Rightarrow v_2 \to \infty \Rightarrow \underline{k_2 = 0}$

$\therefore -\frac{mw^2}{T}\tilde{A}_T = -ik_1 \tilde{A}_I + ik_1 \tilde{A}_R$

$= \frac{1}{5} 10^{-1} = .2$

10 cm =

$.01m = \frac{1}{2}at^2$ $10^{-2}\ 10^{4}$

$a = \frac{.02}{(.01)^2} = \frac{.02}{.0001} = 200\ m/s^2$

$V_f = at = 200(.01) = 20\ m/s$

$v = \sqrt{2gh}$

$h = \frac{v^2}{2g} =$

9.6

(b) $\tilde{A}_I + \tilde{A}_R = \tilde{A}_T$

$$-\frac{m\omega^2}{T}\tilde{A}_T = -ik_1\tilde{A}_I + ik_1\tilde{A}_R$$

$$\Rightarrow \quad -\frac{m\omega^2}{Tik}\tilde{A}_T = \tilde{A}_R - \tilde{A}_I = (\tilde{A}_T - \tilde{A}_I) - \tilde{A}_I$$

$$\therefore \quad \tilde{A}_T\left(\frac{-m\omega^2}{Tik} - 1\right) = -2\tilde{A}_I$$

$$\tilde{A}_T\left(\frac{-m\omega^2 i}{2Tk_1} + \frac{1}{2}\right) = \tilde{A}_I$$

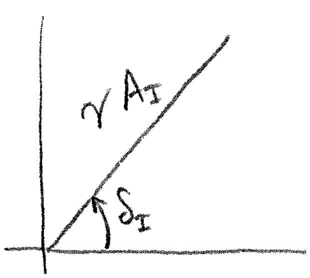
$$\tilde{A}_T = \left(\frac{1}{2} - \frac{m\omega^2 i}{2Tk_1}\right)^{-1}\tilde{A}_I$$

$$\frac{1}{2} - \frac{m\omega^2 i}{2Tk_1} = \frac{Tk_1 - m\omega^2 i}{2Tk_1}\left(\frac{Tk_1 + m\omega^2 i}{Tk_1 + m\omega^2 i}\right) = \frac{T^2k_1^2 + m^2\omega^4}{2Tk_1(Tk_1 + m\omega^2 i)}$$

$$\tilde{A}_T = \left[\frac{2T^2k_1^2}{T^2k_1^2 + m^2\omega^4} + i\,\frac{2Tk_1 m\omega^2}{T^2k_1^2 + m^2\omega^4}\right]\tilde{A}_I$$

take Re part because $\mu_2 < \mu_1$ and $v_2 > v_1$
$\Rightarrow$ phase $\delta_T = \delta_I$!
$\Rightarrow$ same phase
ignore Im term.

$$A_T e^{i\delta_T} = \underbrace{\frac{2T^2k_1^2}{T^2k_1^2 + m^2\omega^4}}_{\gamma} A_I e^{\delta_I}$$



$\boxed{A_T = \gamma A_I}$

$\boxed{\delta_T = \delta_I}$ by arguments of pg. 372 !

$$\tilde{A}_R = \tilde{A}_T - \tilde{A}_I = A_I(\gamma - 1)e^{i\delta_I}$$

$$\boxed{\begin{aligned} A_R &= A_I(\gamma - 1) \\ \delta_R &= \delta_I \end{aligned}}$$

9,7

(a) $T \frac{\partial^2 f}{\partial z^2} \Delta z - \gamma \Delta z \frac{\partial f}{\partial t} = \mu \Delta z \frac{\partial^2 f}{\partial t^2}$

$$\Rightarrow \boxed{\frac{\partial^2 f}{\partial z^2} - \frac{\gamma}{T}\frac{\partial f}{\partial t} - \frac{\mu}{T}\frac{\partial^2 f}{\partial t^2} = 0}$$

(b) $\tilde{f}(z,t) = e^{i\omega t}\tilde{F}(z)$

$$\cancel{e^{i\omega t}} \frac{\partial^2}{\partial z^2}\left(\tilde{F}(z)\right) - \frac{\gamma}{T} i\omega \cancel{e^{i\omega t}} \tilde{F}(z) + \frac{\mu}{T}\omega^2 \cancel{e^{i\omega t}} \tilde{F}(z) = 0$$

$$\frac{\partial^2}{\partial z^2}\left(\tilde{F}(z)\right) - i\frac{\gamma\omega}{T}\tilde{F}(z) + \frac{\mu\omega^2}{T}\tilde{F}(z) = 0$$

$$\frac{\partial^2}{\partial z^2}\left(\tilde{F}\right) = \left[i\frac{\gamma\omega}{T} + \frac{\mu\omega^2}{T}\right]\tilde{F} = -\beta^2 \tilde{F}(z)$$

$$\Rightarrow \tilde{F}(z) = ae^{i\beta z} + ce^{-i\beta z} = \tilde{\alpha}e^{i\beta z} = \tilde{F}(z)$$

thus $\tilde{f}(z,t) = e^{i\omega t}\tilde{\alpha}e^{i\beta z} = \boxed{\tilde{\alpha}e^{i(\omega t + \beta z)} = \tilde{f}(z,t)}$

$$-\beta^2 = i\frac{\gamma\omega}{T} + \frac{\mu\omega^2}{T} \Rightarrow \beta = \pm\left[-i\frac{\gamma\omega}{T} + \frac{\mu\omega^2}{T}\right]^{1/2}$$

(c) $\tilde{f}(z,t) = \frac{1}{e}\tilde{f}(0,t)$, what is this z?..

$$\cancel{e^{i\omega t}}\cancel{\tilde{\alpha}}e^{0-1} = \cancel{e^{i\omega t}}\cancel{\tilde{\alpha}}e^{i\beta z}$$

-2

$$e^{-1} = e^{i\beta z} \Longrightarrow i\beta z = -1 \Rightarrow \beta z = \frac{-1}{i}$$

$$z = \frac{i}{\left(\frac{m\omega^2}{T} - i\frac{\gamma\omega}{T}\right)^{1/2}} = Re\left(\frac{i\left(\frac{m\omega^2}{T} - i\frac{\gamma\omega}{T}\right)^{1/2}}{\left(\frac{\mu^2\omega^4}{T^2} + \frac{\gamma^2\omega^2}{T^2}\right)^{1/2}}\right)$$

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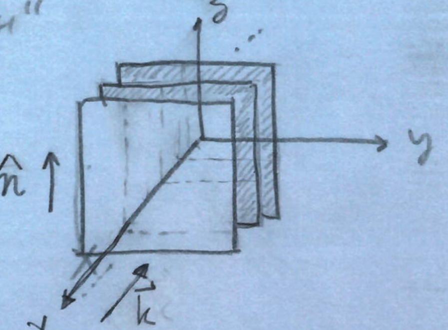
9.9

(a) Describe Plane wave's E, B if it has $\delta = 0$ and polarization in $\hat{z}$ and travels in x. By analogy to book,

$$\boxed{\begin{aligned} E(x,t) &= E_0 \cos(-kx - \omega t)\,\hat{z} \\ B(x,t) &= \frac{E_0}{c} \cos(-kx - \omega t)\,\hat{y} \end{aligned}}$$

"SKETCH"



these planes represent max's in waves 2π apart.

Directions Check easily by,

$$\vec{B}_0 = \frac{\vec{k} \times \vec{E}_0}{\omega}$$

$$\omega \vec{B}_0 = \vec{k} \times \vec{E}_0$$

$$\Downarrow$$

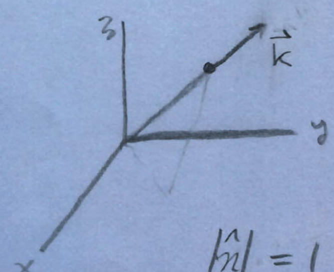
$$\frac{\omega E_0}{c}\hat{y} = \vec{k} \times E_0 \hat{z}$$

$$\Downarrow$$

$$\boxed{\vec{k} = -\frac{\omega}{c}\hat{x}}$$

$$\boxed{\hat{n} = \hat{z}}$$

(b) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$



$$\vec{k} = \frac{k}{\sqrt{3}}(i + j + k)$$

all $\parallel$ vectors to x-z plane $\neq \Delta y$!

$$\hat{k} \cdot \hat{n} = \frac{1}{\sqrt{3}}(i + j + k) \cdot (\alpha i + \gamma k)$$

$$= \frac{1}{\sqrt{3}}(\alpha + \gamma) = 0 \Rightarrow \alpha = -\gamma$$

$$|\hat{n}| = 1 \Rightarrow \boxed{\hat{n} = \frac{1}{\sqrt{2}}(\hat{i}) - \frac{1}{\sqrt{2}}(\hat{k})}$$

$$\left\{ \begin{matrix} k_x = k_y = k_z \\ k = \frac{\omega}{c} \end{matrix} \right\} \Rightarrow \boxed{\vec{k} = \frac{\omega}{\sqrt{3}\,c}(i + j + k)}$$

$\boxed{9.9}$

(b) Direction of $\vec{B}$?

$$\hat{k} \times \hat{n} = \frac{1}{\sqrt{3}}(i+j+k) \times \frac{1}{\sqrt{2}}(i-k)$$

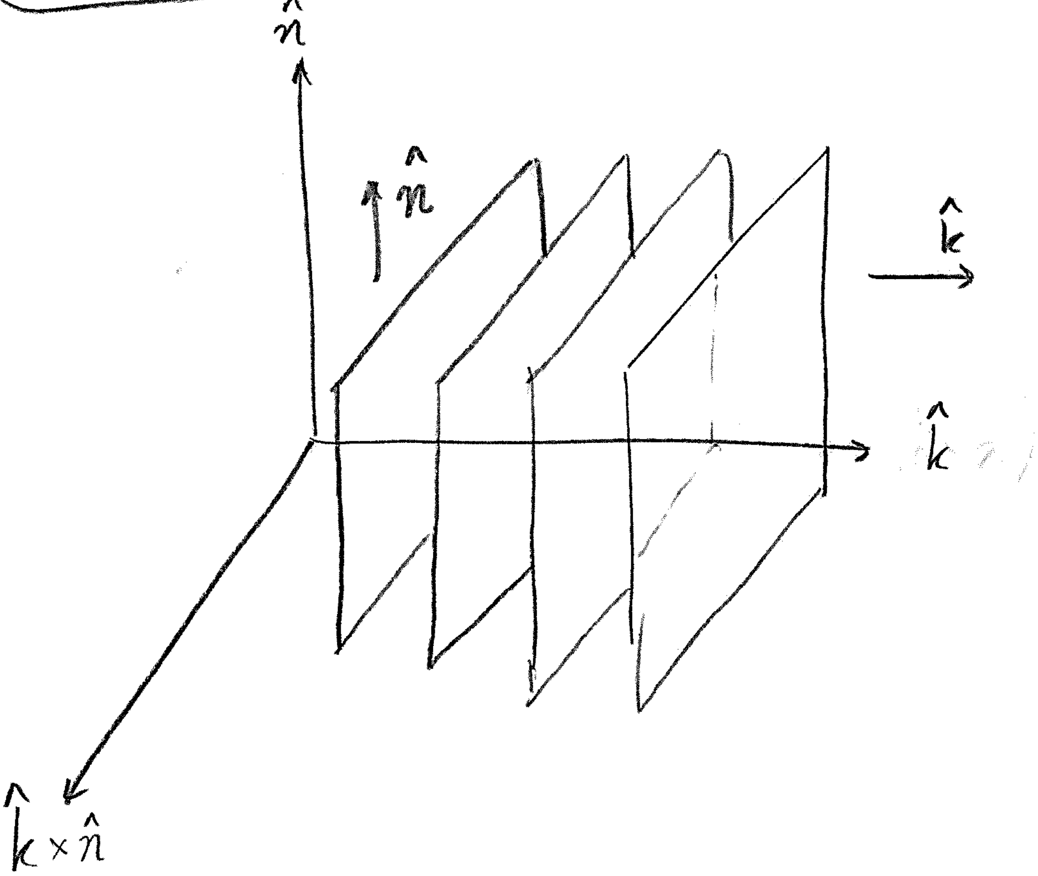
$$= \frac{1}{\sqrt{6}}(j-k-i+j) = \boxed{\frac{1}{\sqrt{6}}(2j-k-i) = \hat{k} \times \hat{n}}$$

$$\vec{k} \cdot \vec{r} = \frac{\omega}{\sqrt{3}c}(i+j+k) \cdot (xi+yj+zk)$$

$$= \frac{\omega}{\sqrt{3}c}(x+y+z)$$

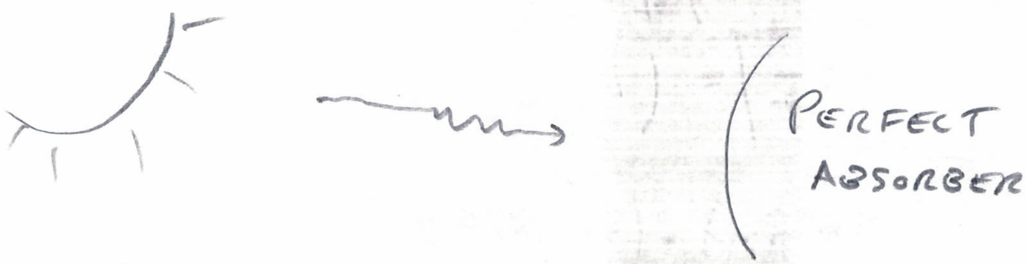
$\therefore$ BY ANALOGY TO BOOK

$$\boxed{\begin{aligned} \vec{E}(x,y,z,t) &= E_0 \cos\left(\frac{\omega}{c\sqrt{3}}(x+y+z) - \omega t\right)\hat{n} \\ \vec{B}(x,y,z,t) &= \frac{E_0}{c}\cos\left(\frac{\omega}{c\sqrt{3}}(x+y+z) - \omega t\right)(\hat{k} \times \hat{n}) \end{aligned}}$$



9.10

$I = 1300 \; W/m^2$



ON A PERFECT ABSORBER

$$P_A = \frac{I}{c} = \frac{1300 \; W/m^2}{3\times10^8 \; m/s} = \boxed{4.33\times10^{-6} \; PA = P_{ABS.}}$$

ON A PERFECT REFLECT

$$P_R = \frac{2I}{c} = \frac{2\times1300 \; W/m^2}{3\times10^8 \; m/s} = \boxed{8.67\times10^{-6} \; PA = P_{REFLECT}}$$

ATMOSPHERIC PRESSURE = 101,325 PA.

$$\frac{P_R}{P_{ATMOS}} = \frac{8.67\times10^{-6}}{1.01325\times10^{5}} = \boxed{8.55\times10^{-11} = \frac{P_R}{P_{ATMOS}}}$$

$\Rightarrow 8.55\times10^{-9}\,\%$ of P_{ATMOS}.

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$\boxed{9\text{-}16}$ Analyze polarization $\perp$ to plane of incidence (ie E in $\hat{y}$ for fig. 9.15)

IMPOSE 9.101.

(i) not much here no $E^{\perp}$, no E in $\hat{z}$

(ii) $(\tilde{B}_{0I} + \tilde{B}_{0R})_z = (\tilde{B}_{0T})_z$ where $\tilde{B}_0 = \frac{1}{v}\hat{k}\times\tilde{E}_0 \Rightarrow B_0 = \frac{k}{v}$

$$\tilde{B}_{0I} = \frac{1}{v_1}\hat{k}_I \times \tilde{E}_{0I} = \frac{1}{v_1} k_I \tilde{E}_{0I}\left(\cos\theta_I(\hat{z}\times\hat{y}) + \sin\theta_I(\hat{x}\times\hat{y})\right) = \frac{k_I}{v_1}\tilde{E}_{0I}\left(-\cos\theta_I\,\hat{x} + \sin\theta_I\,\hat{z}\right)$$

$$\tilde{B}_{0R} = \frac{1}{v_1}\hat{k}_R \times \tilde{E}_{0R} = \frac{1}{v_1} k_R \tilde{E}_{0R}\left(\cos\theta_R(-\hat{z}\times\hat{y}) + \sin\theta_R(+\hat{x}\times\hat{y})\right) = \frac{k_R}{v_1}\tilde{E}_{0R}\left(\cos\theta_R\,\hat{x} + \sin\theta_R\,\hat{z}\right)$$

$$\tilde{B}_{0T} = \frac{1}{v_2}\hat{k}_T \times \tilde{E}_{0T} = \frac{k_T}{v_2}\tilde{E}_{0T}\left(\cos\theta_T(\hat{z}\times\hat{y}) + \sin\theta_T(\hat{x}\times\hat{y})\right) = \frac{k_T}{v_2}\tilde{E}_{0T}\left(-\cos\theta_T\,\hat{x} + \sin\theta_T\,\hat{z}\right)$$

now that I have found the B's lets apply ii. note $\vec{k} \neq \hat{k}$!

$$\underline{\frac{1}{v_1}\tilde{E}_{0I}\sin\theta_I - \frac{1}{v_1}\tilde{E}_{0R}\sin\theta_R = \frac{1}{v_2}\tilde{E}_{0T}\sin\theta_T}$$

(iii) $(\tilde{E}_{0I} + \tilde{E}_{0R})_{x,y} = (\tilde{E}_{0T})_{x,y}$

by assumption $\tilde{E}_{0I}$ in y direction, from last hwk $\Rightarrow \tilde{E}_{0R}$ also in y $\therefore \tilde{E}_{0T}$ in y !
that is $\underline{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}}$ where all directions but y are zero for E.

(iv) $\frac{1}{\mu_1}(\tilde{B}_{0I} + \tilde{B}_{0R})_{x,y} = \frac{1}{\mu_2}(\tilde{B}_{0T})_{x,y}$, as E is in y $\Rightarrow$ B must be in x these are linear media $\therefore E \perp B$!
(also in z but who cares here)

$$\frac{1}{\mu_1}\left(-\frac{\tilde{E}_{0I}}{v_1}\cos\theta_I + \frac{\tilde{E}_{0R}}{v_1}\cos\theta_R\right) = \frac{1}{\mu_2}\left(\frac{-\tilde{E}_{0T}}{v_2}\cos\theta_T\right)$$

$$\underline{\frac{\cos\theta_I}{\mu_1 v_1}\left(\tilde{E}_{0I} - \tilde{E}_{0R}\right) = \frac{\cos\theta_T}{\mu_2 v_2}\left(\tilde{E}_{0T}\right)}$$

9-16

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} \quad \text{and} \quad \alpha \equiv \frac{\cos\theta_T}{\cos\theta_I}$$

from (iii) $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$

from (iv) $\frac{\cos\theta_I}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R}) = \frac{\cos\theta_T}{\mu_2 v_2}(\tilde{E}_{0T})$

$$\Rightarrow \frac{\cos\theta_T}{\cos\theta_I}(\tilde{E}_{0T}) = \frac{\mu_2 v_2}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R})$$

$$\Rightarrow \alpha \tilde{E}_{0T} = \frac{1}{\beta}(\tilde{E}_{0I} - \tilde{E}_{0R})$$

$$\therefore \alpha\beta(\tilde{E}_{0I} + \tilde{E}_{0R}) = (\tilde{E}_{0I} - \tilde{E}_{0R})$$

$$\Rightarrow \boxed{\tilde{E}_{0R} = \frac{(1-\alpha\beta)}{(1+\alpha\beta)}\tilde{E}_{0I}} =$$

OR $\alpha \tilde{E}_{0T} = \frac{1}{\beta}(\tilde{E}_{0I} - (\tilde{E}_{0T} - \tilde{E}_{0I}))$

$$= \frac{1}{\beta}(2\tilde{E}_{0I} - \tilde{E}_{0T})$$

$$(\alpha\beta + 1)\tilde{E}_{0T} = 2\tilde{E}_{0I} \quad \Rightarrow \quad \boxed{\tilde{E}_{0T} = \left(\frac{2}{\alpha\beta+1}\right)\tilde{E}_{0I}}$$

Brewsters $\angle$ / I tried 2 ways.

if $\tilde{E}_{0R} = 0 \Rightarrow (1-\alpha\beta)\tilde{E}_{0I} = 0 \Rightarrow$ as $\tilde{E}_{0I} \neq 0$, $1 = \alpha\beta$, $\alpha = \frac{1}{\beta}$

$\therefore \tilde{E}_{0T} = \left(\frac{2}{1+1}\right)\tilde{E}_{0I} \Rightarrow \tilde{E}_{0T} = \tilde{E}_{0I}$ (optically indistinguishable!)

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_I}}{\cos\theta_I} = \frac{1}{\beta} = \frac{\mu_2 n_1}{\mu_1 n_2} = \frac{n_1}{n_2} \quad (\text{if } \mu_1 = \mu_2)$$

$$\Rightarrow 1 - \left(\frac{n_1}{n_2}\right)^2\sin^2\theta_I = \left(\frac{n_1}{n_2}\right)^2\cos^2\theta_I = \cos^2\theta_I + \sin^2\theta_I - \sin^2\theta_I\left(\frac{n_1}{n_2}\right)^2 = \left(\frac{n_1}{n_2}\right)^2\cos^2\theta_I$$

$\Rightarrow \left(\frac{n_1}{n_2}\right)^2 \Rightarrow n_1^2 = n_2^2$ again the only case for no reflection is just the trivial case of not changing media.

9.-16

Confirming Proper forms for Fresnel Eq's.

$$\tilde{E}_{0R} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)\tilde{E}_{0I} \qquad \tilde{E}_{0T} = \left(\frac{2}{\alpha\beta+1}\right)\tilde{E}_{0I}$$

at normal incidence $\theta_I = \theta_R = \theta_T = 0$! $\therefore\ \alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{1}{1} = 1.$

thus $\boxed{\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}}$ and $\boxed{\tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}}$

R and T/

$$R = \frac{I_R}{I_I} = \frac{\frac{1}{2}\epsilon_1 v_1 E_{0R}^2}{\frac{1}{2}\epsilon_1 v_1 E_{0I}^2} = \boxed{\left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 = R}$$

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

$$v_i^2 = \frac{1}{\mu_i \epsilon_i} \Rightarrow \epsilon_i = \frac{1}{\mu_i v_i^2}$$

$$T = \frac{I_T}{I_I} = \frac{\frac{1}{2}\epsilon_2 v_2 \cos\theta_T \tilde{E}_{0T}^2}{\frac{1}{2}\epsilon_1 v_1 \cos\theta_I \tilde{E}_{0I}^2} \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{2}{1+\alpha\beta}\right)^2$$

$$= \alpha\frac{v_2\left(\frac{1}{\mu_2 v_2^2}\right)}{v_1\left(\frac{1}{\mu_1 v_1^2}\right)}\left(\frac{2}{1+\alpha\beta}\right)^2 = \frac{\mu_1 v_1}{\mu_2 v_2}\left(\frac{2}{1+\alpha\beta}\right)^2$$

$$= \boxed{\alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2 = T}$$

$$R + T = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 + \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$

$$= \frac{1 - 2\alpha\beta + (\alpha\beta)^2 + 4\alpha\beta}{(1+\alpha\beta)^2}$$

$$= \frac{1 + 2\alpha\beta + (\alpha\beta)^2}{(1+\alpha\beta)^2} = \frac{(1+\alpha\beta)^2}{(1+\alpha\beta)^2} = 1 \quad \therefore \boxed{R+T=1}$$

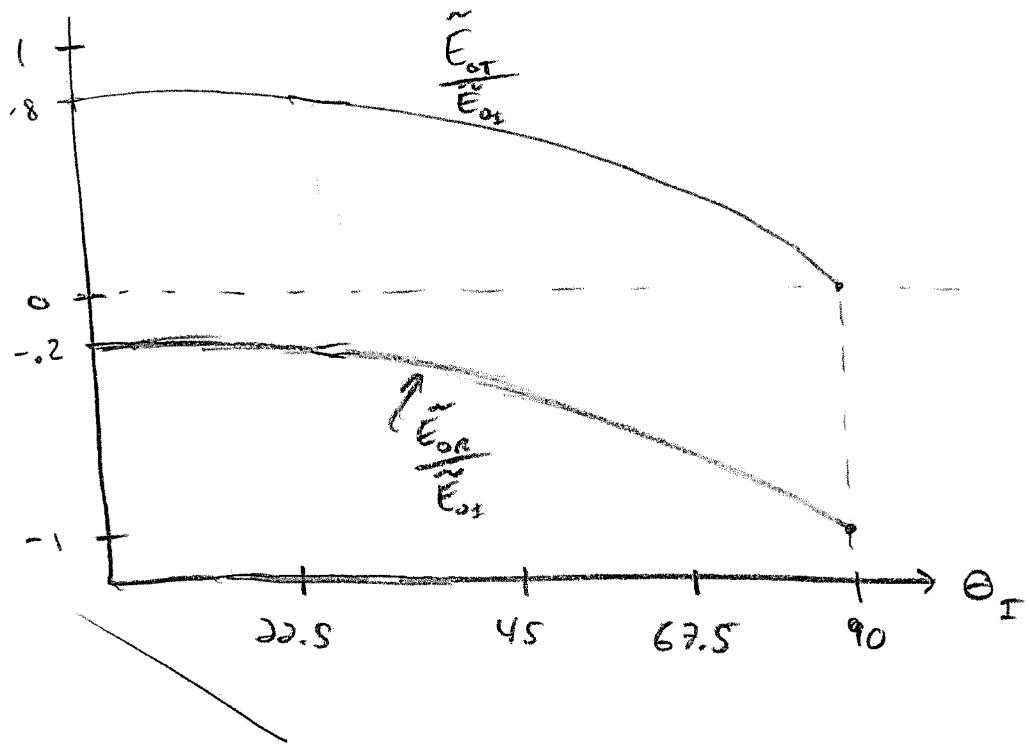
9-16 The "Sketch"

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1-\left(\frac{1}{1.5}\right)^2 \sin^2\theta_I}}{\cos\theta_I}$$

$$\frac{E_{0R}}{E_{0I}} = \frac{1 - 1.5\left(\frac{\sqrt{1-\frac{\sin^2\theta_I}{2.25}}}{\cos\theta_I}\right)}{1 + 1.5\left(\frac{\sqrt{1-\frac{\sin^2\theta_I}{2.25}}}{\cos\theta_I}\right)}$$

$$\frac{E_{0T}}{E_{0I}} = \frac{2}{1 + 1.5\left(\frac{\sqrt{1+\frac{\sin^2\theta_I}{2.25}}}{\cos\theta_I}\right)}$$

note $(1.5)^2 = 2.25$



9-17

$n_{DIAMOND} = 2.42$. Construct analogous graph to 9.16 for E_{0T}/E_{0I} vs. E_{0R}/E_{0I} !
air / diamond interface. First calculate, (assume $\mu_0 = \mu_1 = \mu_2$)

(a) Amplitudes at normal incidence $\quad \alpha = \frac{\cos\theta_T}{\cos\theta_I} = 1. \quad \beta = \frac{\mu_1 n_2}{\mu_2 n_1} = \frac{n_2}{n_1} = 2.42$

$$\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} = \frac{1-\beta}{1+\beta}\tilde{E}_{0I} = \frac{1-2.42}{3.42}\tilde{E}_{0I} = \frac{-1.58}{3.42}\tilde{E}_{0I} = \boxed{-.415\,\tilde{E}_{0I} = \tilde{E}_{0R}}$$

$$\tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I} = \frac{2}{3.42}\tilde{E}_{0I} = \boxed{0.585\,\tilde{E}_{0I} = \tilde{E}_{0T}}$$

(b) $\tilde{E}_{0R} = 0$ for brewsters angle.

$$0 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} \Rightarrow (\alpha-\beta)E_{0I} = 0 \Rightarrow \alpha-\beta = 0 \Rightarrow \underline{\alpha = \beta}$$

and as the book nicely derives we know if $\mu_1 \approx \mu_2 \Rightarrow n_1 \approx n_2$
thus $\beta \approx n_1/n_2 \Rightarrow \sin^2\theta_B \approx \beta^2/(1+\beta^2)$ ∴

$$\tan\theta_B = \frac{n_2}{n_1} \Rightarrow \theta_B = \tan^{-1}(2.42) = \boxed{67.5^\circ = \theta_B}$$

(c) crossover angle, $\tilde{E}_{0R} = \tilde{E}_{0T}$

$$\Rightarrow \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I} = \tilde{E}_{0R}$$

$$\Rightarrow \frac{2}{\alpha+\beta} = \frac{\alpha-\beta}{\alpha+\beta} \Rightarrow 2 = \alpha - \beta = \alpha - 2.42$$

$$\therefore \alpha = 4.42 = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_I}}{\cos\theta_I} = \sqrt{1-\frac{1}{\beta^2}\sin^2\theta_I}$$

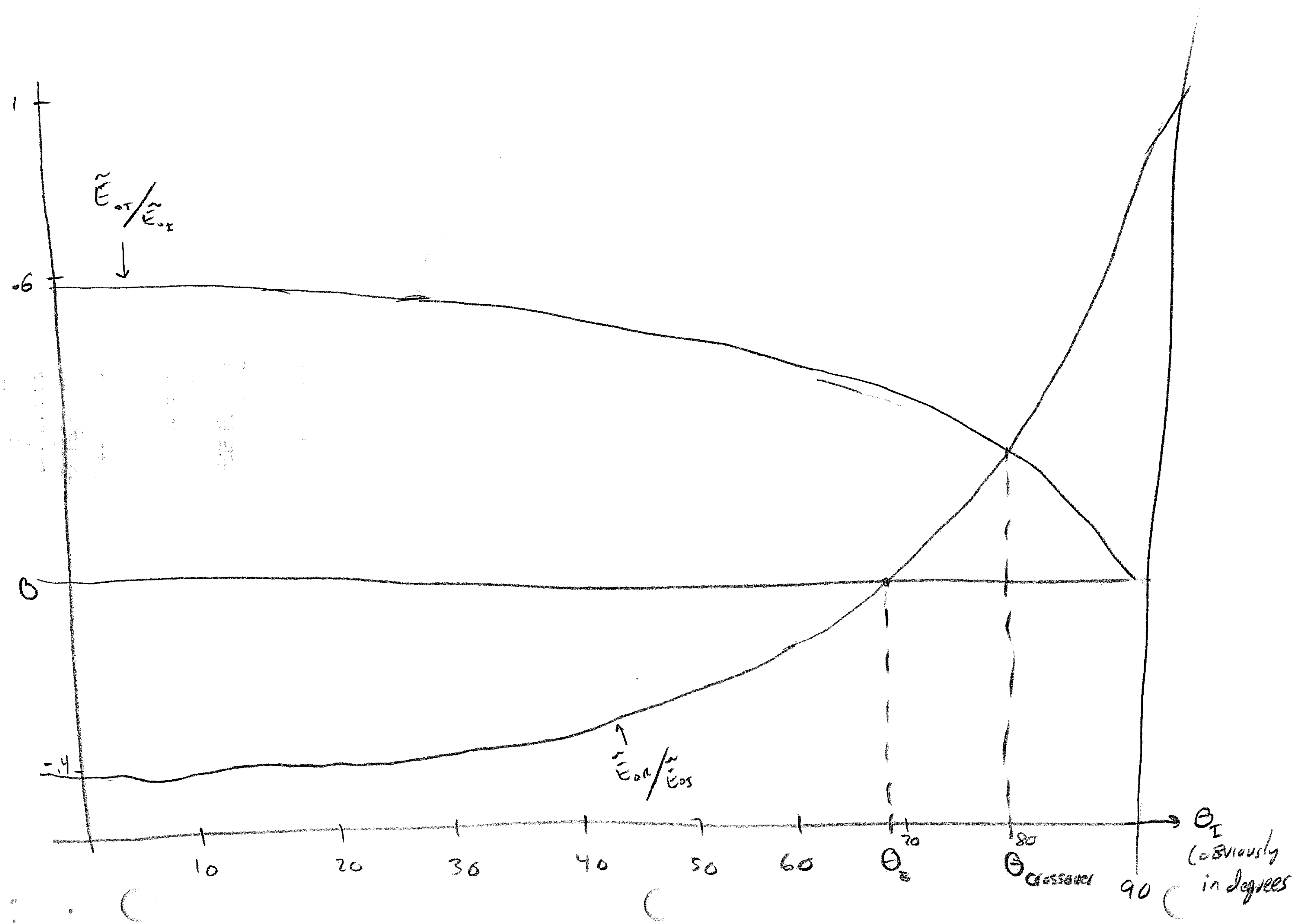
$$(2+\beta)^2\cos^2\theta_I = 1 - \frac{1}{\beta^2}\sin^2\theta_I$$
$$= 1 - \frac{1}{\beta^2}(1-\cos^2\theta_I)$$

$$.171 = \frac{1}{\beta^2}$$

$$\cos^2\theta_I\left((2+\beta)^2 - \frac{1}{\beta^2}\right) = 1 - \frac{1}{\beta^2}$$

$$\cos\theta_I = \left(\frac{1-1/\beta^2}{(2+\beta)^2 - 1/\beta^2}\right)^{1/2}$$

$$\theta_I = \cos^{-1}\left[\left(\frac{1-1/\beta^2}{(2+\beta)^2-1/\beta^2}\right)^{1/2}\right] = \cos^{-1}(.2069) = \boxed{78.06^\circ = \theta_I}$$



1
.6
0
-.4
$\tilde{E}_{0T}/\tilde{E}_{0I}$
$\tilde{E}_{0R}/\tilde{E}_{0I}$
10
20
30
40
50
60
70
80
90
Θ_B
$\Theta_{crossover}$
Θ_I
(obviously in degrees)

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20/20

9-19

(a) Show that skin depth in poor conductor ($\sigma \ll \omega\epsilon$) is $\frac{2}{\sigma}\sqrt{\epsilon/\mu}$ independent of frequency. Find skin depth for water.

$$d = \frac{1}{K}, \quad K = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]^{1/2}$$

$$(1+x)^n = 1 + nx + \ldots \Rightarrow K = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[1 + \frac{1}{2}\left(\frac{\sigma}{\epsilon\omega}\right)^2 - 1\right]^{1/2}$$

$$\therefore K = \omega\sqrt{\frac{\epsilon\mu}{2}}\left(\sqrt{\frac{1}{2}}\frac{\sigma}{\epsilon\omega}\right) = \sqrt{\frac{\mu\sigma^2}{4\epsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\boxed{d = \frac{1}{K} = \frac{2}{\sigma}\sqrt{\epsilon/\mu}}$$

$$d = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}} \cong 2\rho\sqrt{\frac{\epsilon_0}{\mu_0}} = \left(2(2.5\times10^5)\sqrt{\frac{8.852\times10^{-12}\cdot80}{4\pi\times10^{-7}}}\right)m = \boxed{d = 11{,}869\ m}$$

(b) Show $d = \frac{\lambda}{2\pi}$ when $\sigma \gg \omega\epsilon$

$$K = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]^{1/2} \xrightarrow[\sigma \gg \omega\epsilon]{} K = \omega\sqrt{\frac{\epsilon\mu}{2}}\sqrt{\frac{\sigma}{\epsilon\omega}}$$

$$K = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}} \Rightarrow d = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1\right]^{1/2} \xrightarrow[\sigma \gg \omega\epsilon]{} k = \omega\sqrt{\frac{\epsilon\mu}{2}}\sqrt{\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$d = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{k} = \frac{1}{\frac{2\pi}{\lambda}} = \boxed{\frac{\lambda}{2\pi} = d}$$

if $\sigma \approx 10^7\ (\Omega m)^{-1}$, $\omega \approx 10^{15}/s$ and $\epsilon = \epsilon_0$, $\mu = \mu_0$

$$\Rightarrow d = \left(\frac{2}{10^{15}\cdot 4\pi\cdot10^{-7}\cdot10^7}\right)^{1/2} m = \boxed{12.6\times10^{-9}\ m \cong d}$$

the metal is opaque because light can not penetrate the metal very appreciably. The distance it would take for the light to drop to $\approx 98\%$ of its surface strength would be a mere $5d \approx 60\,nm$

9.21 Calculate the reflection coefficient for light at an air-to-silver interface ($\mu_1 = \mu_2 = \mu_0$, $\epsilon_1 = \epsilon_0$, $\sigma = 6\times10^7 (\Omega m)^{-1}$ with $\omega = 4\times10^{15}/s$.

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)^2 \ldots$$

$$\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2 \omega}\tilde{k}_2 = \frac{v_1}{\omega}\tilde{k}_2 \approx \underbrace{\frac{c}{\omega}}_{\alpha \equiv \frac{c}{\omega}}\tilde{k}_2$$

$$\tilde{k} = k + i\kappa \Rightarrow \tilde{\beta} = \alpha k + i\alpha\kappa$$

$$R = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)^*\left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right) = \left(\frac{1-\alpha k + i\alpha\kappa}{1+\alpha k - i\alpha\kappa}\right)\left(\frac{1-\alpha k - i\alpha\kappa}{1+\alpha k + i\alpha\kappa}\right)$$

$$= \left(\frac{(1-\alpha k - \cancel{i\alpha\kappa}) + (-\alpha k + (\alpha k)^2 + \cancel{i\alpha^2\kappa k}) + \cancel{i\alpha\kappa} - \cancel{i\alpha^2\kappa k} + (\alpha\kappa)^2}{1+\alpha k + \cancel{i\alpha\kappa} + \alpha k + (\alpha k)^2 + \cancel{i\alpha^2\kappa k} - \cancel{i\alpha\kappa} - i\alpha^2\kappa k + (\alpha\kappa)^2}\right)$$

$$= \frac{1-\alpha k - \alpha k + \alpha^2 k^2 + \alpha^2\kappa^2}{1+\alpha k + \alpha k + (\alpha k)^2 + (\alpha\kappa)^2}$$

$$= \frac{1 - 2\alpha k + \alpha^2 k^2 + \alpha^2\kappa^2}{1 + 2\alpha k + \alpha^2 k^2 + \alpha^2\kappa^2}$$

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2} \qquad \kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\overbrace{\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}}^{\gamma}-1\right]^{1/2}$$

note $\omega\sqrt{\frac{\epsilon\mu}{2}} = \frac{\omega}{c\sqrt{2}}$

$$R = \frac{1 - 2\frac{c}{\omega}\frac{\omega}{c\sqrt{2}}[\gamma+1]^{1/2} + \frac{c^2}{\omega^2}\frac{\omega^2}{c^2}\frac{1}{2}[\gamma+1] + \frac{c^2}{\omega^2}\frac{\omega^2}{c^2}\frac{1}{2}[\gamma-1]}{1 + 2\frac{c}{\omega}\frac{\omega}{c\sqrt{2}}[\gamma+1]^{1/2} + \frac{c^2}{\omega^2}\frac{\omega^2}{c^2}\frac{1}{2}[\gamma+1] + \frac{c^2}{\omega^2}\frac{\omega^2}{c^2}\frac{1}{2}[\gamma-1]}$$

$$R = \frac{1-\sqrt{2}[\gamma+1]^{1/2}+\gamma}{1+\sqrt{2}[\gamma+1]^{1/2}+\gamma} = \frac{1-\sqrt{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)^{1/2}+\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}}{1+\sqrt{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)^{1/2}+\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}} = \boxed{-.9989 = R}$$

James Cook

$\boxed{9.23}$

Take model Ex. 4.1, what natural frequency do you get?

15/20

$$E_e = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3} = E \implies p = qd = (4\pi\epsilon_0 a^3)E$$

$$\therefore \alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0(\text{Volume})$$

$$F_{binding} = -k_{spring}\, x = -m\omega_0^2 x \qquad \omega_0 = \text{natural oscillation frequency}$$

$$F_{damping} = -m\gamma\frac{dx}{dt}$$

$$F_{driving} = qE = qE_0\cos(\omega t)$$

$$m\frac{d^2x}{dt^2} = F_{total} = F_{bind} + F_{damp.} + F_{driving}$$

electron as damped harm. osc. with massive nucleus assumed at rest

no damping here,

$$F_{bind} = F_{driv} \implies m\omega_0^2 d = qE_e = \frac{q^2 d}{4\pi\epsilon_0 a^3}$$

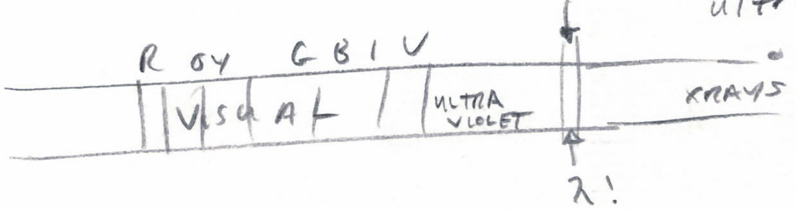
$$\omega_0^2 = \frac{q^2}{m\,4\pi\epsilon_0 a^3} \implies \boxed{\omega_0 = \sqrt{\frac{q^2}{4\pi\epsilon_0 m a^3}}}$$

$$\omega_0 = \left(\frac{(1.6\times10^{-19})^2}{(4\times\pi\times 8.852\times10^{-12})(9.11\times10^{-31})(0.5\times10^{-10})^3}\right)^{1/2} s^{-1} = \boxed{4.5\times10^{16}\ \frac{rad}{s} = \omega_0}$$

$$f_0 = \frac{\omega_0}{2\pi} = \boxed{7.15\times10^{15}\ Hz = f_0}$$

$$\Updownarrow \quad \lambda = \frac{f_0}{c}$$

$\lambda = 41.9\,nm \implies$ in very low x-ray or very high energy ultraviolet region depending on which way you like to look at it.



James Cook

9.23

Find Coefficients of ref. and disp., comp. to Hyd. $A = 1.36\times10^{-4}$, $B = 7.7\times10^{-15} m^2$

$$n = 1 + \left(\frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2}\right) + \omega^2\left(\frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^4}\right) \qquad \omega^2 = \frac{4\pi^2 c^2}{\lambda^2}$$

$$n = 1 + \frac{Nq^2}{2m\epsilon_0}\left(\frac{1}{\omega_0^2} + \frac{4\pi^2c^2}{\omega_0^2\lambda^2}\right) = 1 + A\left(1 + \frac{B}{\lambda^2}\right)$$

-1 $\times$ $A = \frac{Nq^2}{2m\epsilon_0\omega_0^2} = \frac{3q^2}{8m\epsilon_0\omega_0^2 r^3\pi} = \boxed{1.50 = A}$

$$B = \frac{4\pi^2c^2}{\omega_0^2} = \boxed{1.75\times10^{-15} = B}$$

$A_{got} \approx 10{,}000\ A_{HYD.}$

$B_{got} \approx 4.5\ B_{HYD.}$

$\boxed{9.24}$

Find the width of the anomalous dispersion region for case of single resonance freq. ω_0, assume $\gamma \ll \omega_0$

Show that the index of refrax assumes min/max values at points where absorption coeff. is at $\frac{1}{2}$ max.

$\alpha \equiv 2K$ = abs. coeff.

$$\alpha = 2K \cong \frac{Nq^2\omega^2}{m\epsilon_0 c}\sum_j \frac{f_j\gamma_j}{(\omega_j^2-\omega^2)^2+\gamma_j^2\omega^2} = \frac{Nq^2\omega^2}{m\epsilon_0 c}\frac{\gamma_0}{(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2}$$

$$n \cong 1 + \frac{Nq^2}{2m\epsilon_0}\frac{(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2} \equiv 1 + \delta\frac{\beta}{\beta^2+z^2} \qquad \beta = \omega_0^2-\omega^2, \quad z = \gamma_0\omega$$

$$\frac{dn}{d\omega_0} = \frac{dn}{d\beta}\frac{d\beta}{d\omega_0} = \delta\frac{d\beta}{d\omega_0}\frac{d}{d\beta}\left(\frac{\beta}{\beta^2+z^2}\right) \qquad \frac{d\beta}{d\omega_0} = 2\omega_0$$

$$= \delta 2\omega_0\left(\frac{1}{\beta^2+z^2} - \frac{\beta}{(\beta^2+z^2)^2}\frac{d}{d\beta}(\beta^2+z^2)\right) \qquad \frac{d}{d\omega_0}\beta^2 + \cancel{\frac{d}{d\omega_0}z^2} \to 0$$

$$\frac{d}{d\omega_0}\beta^2 = 2\beta 2\omega_0$$

$$\underset{(max,min)}{0} = \delta 2\omega_0\left(\frac{1}{\beta^2+z^2} - \frac{\beta}{(\beta^2+z^2)^2}2\beta\right)$$

$$\frac{1}{\beta^2+z^2} = \frac{2\beta^2}{(\beta^2+z^2)^2} \Rightarrow 1 - \frac{2\beta^2}{\beta^2+z^2} = 0$$

$$\beta^2 + z^2 - 2\beta^2 = 0 \Rightarrow \beta^2 = z^2$$

$$(\omega_0^2-\omega^2)^2 = (\gamma\omega)^2 \Rightarrow \omega_0 = \pm\sqrt{\omega^2+\gamma\omega}$$

$$\omega_2 = \sqrt{\omega^2+\gamma\omega} \qquad \omega_1 = -\sqrt{\omega^2+\gamma\omega}$$

$$\omega_2 - \omega_1 = \boxed{2\sqrt{\omega^2+\gamma\omega}}$$ = width between max and min where the anamolous disp happens

-4

PY 415 - Dr. Chang JAMES COOK

$\boxed{9.25}$ Show $V_g < C$ even when $V > C$.

10/10

Given $\gamma_j = 0 \Rightarrow n = 1 + \frac{Nq^2}{2m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2} = \frac{ck}{\omega}$

$$\therefore \; k = \left[\omega + \frac{Nq^2}{2m\epsilon_0} \sum_j \frac{f_j \omega}{\omega_j^2 - \omega^2}\right] \frac{1}{C} \qquad \text{Let } \delta \equiv \frac{Nq^2}{2m\epsilon_0}$$

$$\frac{1}{V_g} = \frac{dk}{d\omega} = \frac{1}{c}\frac{d}{d\omega}\left\{\omega + \delta \sum_j \frac{f_j \omega}{\omega_j^2 - \omega^2}\right\} \qquad \text{Let } \delta_j \equiv \frac{\delta}{c}\sum_j$$

$$= \frac{1}{c} + \delta_j \frac{d}{d\omega}\left(\omega(\omega_j^2 - \omega^2)^{-1}\right)$$

$$= \frac{1}{c} + \delta_j\left\{(\omega_j^2 - \omega^2)^{-1} - \omega(\omega_j^2 - \omega^2)^{-2}(-2\omega)\right\} \qquad \text{Let } \beta = (\omega_j^2 - \omega^2)$$

$$= \frac{1}{c} + \delta_j\left\{\beta^{-1} + 2\omega^2 \beta^{-2}\right\}$$

$$= \frac{1}{c} + \delta_j\left\{\frac{\beta + 2\omega^2}{\beta^2}\right\} \qquad \text{note } \beta + 2\omega^2 = \omega_j^2 + \omega^2$$

$$\therefore \; V_g = \frac{1}{\frac{1}{c} + \delta_j\left\{\frac{\beta + 2\omega^2}{\beta^2}\right\}} = \frac{1}{\frac{1}{c} + \underbrace{\frac{Nq^2}{2m\epsilon_0 C}\sum_j \left[\frac{\omega_j^2 + \omega^2}{(\omega_j^2 - \omega^2)^2}\right]}_{Q}}$$

$$V_g = \frac{1}{\frac{1}{c} + Q} \Rightarrow V_g(1 + QC) = C$$

$\therefore \; V_g > C \iff 1 + QC < 1 \Rightarrow V_g > C$ only if $QC < 0$.

Thus all that remains is for me to show that $QC \geq 0$

Then V_g cannot be greater than C!

$$QC = \frac{Nq^2 c}{2m\epsilon_0 C}\sum_j \left[\frac{\omega_j^2 + \omega^2}{(\omega_j^2 - \omega^2)^2}\right]$$, then note, $N, q^2, m, \epsilon_0, \omega_j^2, \omega^2, (\omega_j^2 - \omega^2)^2 > 0$

And as we know the sum of positive numbers is positive, thus $QC > 0$

$\therefore$ V_g is necessarily less than C, Even if $V > C$.