§4.1 #29) Find critical #'s of function: $f(x) = 5x^2 + 4x$

$f'(x) = 10x + 4 = 0 \Rightarrow x = \frac{-4}{10} = -2/5$

the critical # is $\boxed{c = -2/5}$

§4.1 #31) Find critical #'s of function: $f(x) = x^3 + 3x^2 - 24x$

$f'(x) = 3x^2 + 6x - 24 = 0$

$(3x - 6)(x + 4) = 0$

$\Rightarrow$ $\boxed{x = 2 \text{ and } x = -4 \text{ critical \#'s}}$

§4.1 #33) Find critical #'s of function: $s(t) = 3t^4 + 4t^3 - 6t^2$

$\frac{ds}{dt} = 12t^3 + 12t^2 - 12t = 0$

$12t(t^2 + t - 1) = 0$

$t = 0$ or $t = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$

$\boxed{\text{Critical \#'s are } 0 \text{ and } \frac{-1}{2} \pm \frac{\sqrt{5}}{2}}$

§4.1 #35) Find critical #'s of $g(y) = \frac{y-1}{y^2-y+1}$. Notice to begin that $y^2 - y + 1 = 0$ has no real sol^{n}'s

$\boxed{\begin{array}{l} y^2 - y + 1 = 0 \\ y = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2} \\ \text{no real sol}^{\underline{n}}\text{s.} \end{array}}$

$\frac{dg}{dy} = \frac{d}{dy}\left(\frac{y-1}{y^2-y+1}\right)$

$= \frac{y^2 - y + 1 - (y-1)(2y-1)}{(y^2 - y + 1)^2}$

$= \frac{y^2 - y + 1 - 2y^2 + 3y - 1}{(y^2 - y + 1)^2}$

$= \frac{-y^2 + 2y}{\underbrace{(y^2 - y + 1)^2}_{\text{this is never zero so it doesn't give us places where } g'(y) \text{ d.n.e.}}} = 0 \longrightarrow y(2 - y) = 0$

$\boxed{\begin{array}{c} y = 0 \ \& \ y = 2 \\ \text{critical \#'s} \end{array}}$

§4.1 # 45 Find absolute extrema of $f(x) = 3x^2 - 12x + 5$ on the closed interval $[0, 3]$. Use Closed Int. Method.

1.) $f'(x) = 6x - 12 = 0 \rightarrow c = 2$ only critical #.

$$f(2) = 3(2)^2 - 12(2) + 5 = 12 - 24 + 5 = -7 = f(2)$$

2.) $f(0) = 5$

$$f(3) = 3(9) - 36 + 5 = 27 - 36 + 5 = -4 = f(3)$$

3.) Comparing the possible extrema we find

The absolute max. of $f(x)$ on $[0, 3]$ is $f(0) = 5$.
The absolute min of $f(x)$ on $[0, 3]$ is $f(2) = -7$.

§4.1 # 72 Consider $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$ notice $f'(x) = 3ax^2 + 2bx + C = 0$ can have three types of sol^{n}'s. Think of it this way: $A = 3a$, $B = 2b$, $C = C$ we want to solve $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

The discriminant $B^2 - 4AC$ can be

(i.) $B^2 - 4AC = 0$: double root
(ii) $B^2 - 4AC > 0$: distinct real roots
(iii) $B^2 - 4AC < 0$: no real sol^{n}'s.

Three examples that come to mind are

(i.) $(x - 3)^2 = 0$
(ii) $(x - 1)(x - 2) = 0$
(iii) $x^2 + 1 = 0$

§4.1 # 72 Continuing we had examples

i.) $\frac{dy}{dx} = (x-3)^2 = x^2 - 6x + 9$ (one critical # for f(x))

ii.) $\frac{dy}{dx} = (x-1)(x-2) = x^2 - 3x + 2$ (two critical #'s for f(x))

iii) $\frac{dy}{dx} = x^2 + 1$ (no critical #'s for f(x))

which illustrate the three types of critical numbers possible for a cubic function y. You can check that

i.) $y = \frac{1}{3}x^3 - 3x^2 + 9x$

ii.) $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$

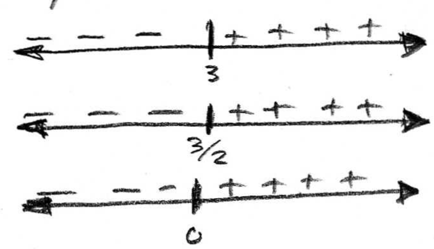
iii) $y = \frac{1}{3}x^3 + x$

are precisely the cubic functions which yield the dy/dx given at top of page. I'll use ideas from later sections to help graph.

i.) $y'' = 2(x-3)$

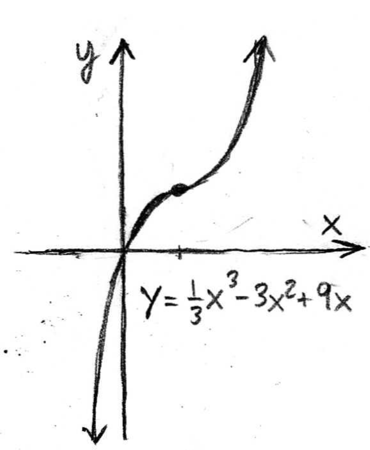
ii.) $y'' = 2x - 3$

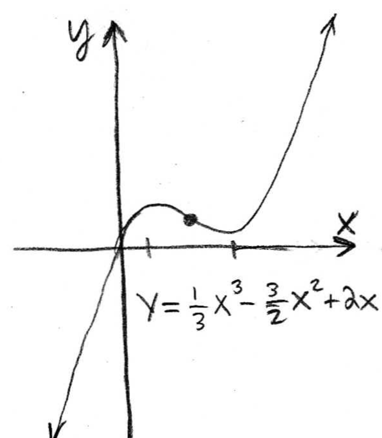
iii) $y'' = 2x$

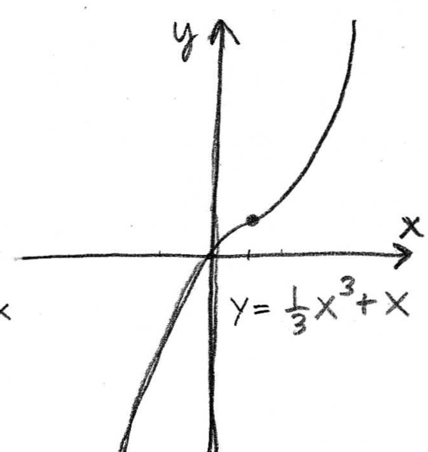


inflection points are at $x = 3, 3/2, 0$ for i, ii, iii respectively.

You can use the 1st derivative test to show only case ii.) has local max/min. Cases i) and iii) have no local extrema. Thus a cubic either has one local max and one local min or it has no local extrema at all.







§4.2 #1 | Verify Rolle's Th^{m} applies and find c such that $f'(c) = 0$.
Let $f(x) = 5 - 12x + 3x^2$ consider the closed interval $[1, 3]$.
Observe that f is continuous on $[1, 3]$. Moreover
$f'(x) = -12 + 6x$ thus f is diff. on $(1,3)$. Finally note

$$f(1) = 5 - 12 + 3 = -4$$
$$f(3) = 5 - 36 + 27 = -4$$

Thus Rolle's Th^{m} applies to f on $[1,3]$. In particular
$f'(c) = -12 + 6c = 0 \therefore$ $c = 2$ has $f'(c) = 0$

§4.2 #5 | Let $f(x) = 1 - x^{2/3}$ show that $f(1) = f(-1)$ but
there is no $c \in (-1,1)$ such that $f'(c) = 0$. Why does
this not contradict Rolle's Th^{m}?

Sol^{n}: It is clear $f(1) = 1 - 1 = 0$ and $f(-1) = 1 - 1 = 0$. Consider

$$f'(x) = -\frac{2}{3} x^{-1/3} = \frac{-2}{3\sqrt[3]{x}}$$

this is not well-defined at $x = 0$, thus f is not differentiable
at $x = 0$. Rolle's Th^{m} requires f be diff on $(-1,1)$ if we
are to apply the Th^{m}. Notice that

$$f'(c) = \frac{-2}{3\sqrt[3]{c}} = 0$$ has no solutions.

§4.2 #11 | Verify $f(x) = 3x^2 + 2x + 5$ satisfies the mean value Th^{m}
with respect to the closed interval $[-1, 1]$.

Sol^{n}: notice f is continuous and $f'(x) = 6x + 2$ is well-defined so
f is differentiable on $(-1, 1)$, thus the mean value Th^{m} should apply.

$$f(-1) = 3 - 2 + 5 = 6$$
$$f(1) = 3 + 2 + 5 = 10$$

$$f_{avg} = \frac{\Delta f}{\Delta x} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{10 - 6}{2} = 2.$$

Let's find $c \in (-1,1)$ such that $f'(c) = 2$,

$$6c + 2 = 2 \longrightarrow \boxed{c = 0, \quad f'(0) = 2}$$