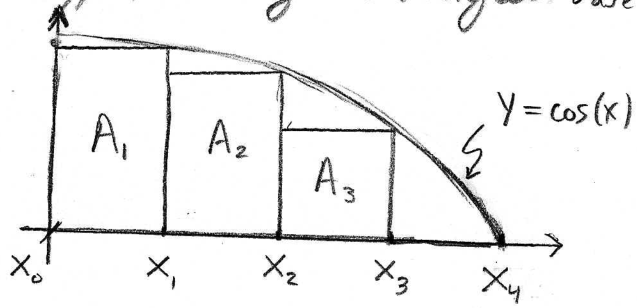
Homework 26, Calculus I

§5.1#3 Estimate area under $Y = f(x) = \cos(x)$ from $x=0$ to $x = \pi/2$ using 4 approximating rectangles based on right endpoints. Illustrate,



$X_0 = 0$

$X_4 = \pi/2$

$\Delta X = \dfrac{\pi/2 - 0}{4} = \dfrac{\pi}{8}$

$X_1 = X_0 + \Delta X = \pi/8$

$X_2 = X_0 + 2\Delta X = 2\pi/8 = \pi/4$

$X_3 = X_0 + 3\Delta X = 3\pi/8$

$$A_R \cong A_1 + A_2 + A_3 + A_4$$

$$= \cos(\pi/8)\Delta X + \cos(\pi/4)\Delta X + \cos(3\pi/8)\Delta X + \underbrace{\cos(\pi/2)}\,\Delta X$$

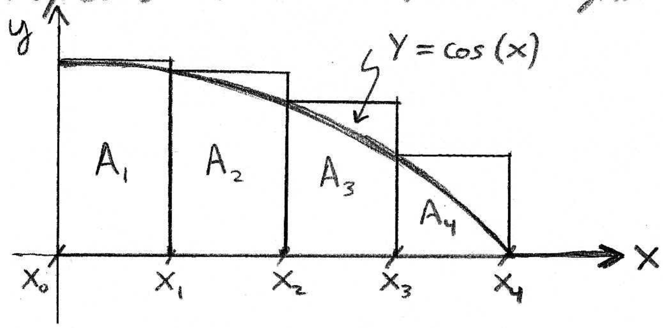
zero. The A_4 is hard to see. ☺

$$= \left(\cos(\pi/8) + \cos(\pi/4) + \cos(3\pi/8)\right)\Delta X$$

$$= (0.9239 + 0.7070 + 0.3825)\,0.3928$$

$$= \boxed{0.791 = \text{area by right endpts using } n=4 \text{ on } [0,\pi/2]}$$

Its clearly an underestimate of the true area.
Now repeat with left endpts.



$$A_L = A_1 + A_2 + A_3 + A_4$$

$$= \cos(0)\Delta X + \cos(\pi/8)\Delta X + \cos(\pi/4)\Delta X + \cos(3\pi/8)\Delta X$$

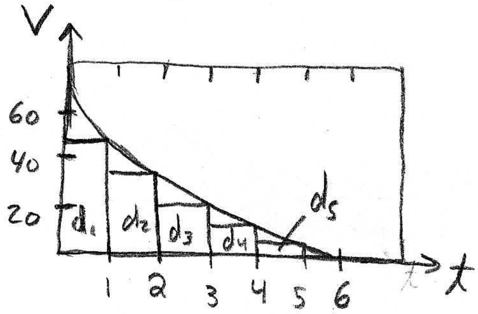
$$= \left(\cos(0) + \cos(\pi/8) + \cos(\pi/4) + \cos(3\pi/8)\right)\Delta X$$

$$= (1 + 0.9239 + 0.7070 + 0.3825)(0.3928)$$

$$= \boxed{1.18 = \text{area by left endpts. using } n=4 \text{ on } [0,\pi/2]}$$

Remark:
This is clearly an overestimate, you can see this from the picture. Soon we'll learn the true area $A = 1$. This follows from taking $n \to \infty$ so $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x \longrightarrow \int_a^b f(x)\,dx$.

§5.1 #15 The velocity graph of a braking car is shown in text. I'll reproduce it here, estimate distance travelled as t goes from 0 to 6.



I'll use 6 approximating rectangles notice $V_{avg}\,\Delta t$ gives the distance travelled during Δt at velocity V_{avg}. Area gives distance in a (vt)-graph. Use right end pts.

$$d_{total} = d_1 + d_2 + d_3 + d_4 + d_5 + d_6$$

$$\cong V_1\Delta t + V_2\Delta t + V_3\Delta t + V_4\Delta t + V_5\Delta t + \underbrace{V_6\Delta t}_{\text{zero.}}$$

$$= (45 + 35 + 20 + 15 + 10)\,1\left(\frac{ft}{s}\cdot s\right)$$

$$= \boxed{125\ ft \approx d_{total}}$$

Clearly mine is an underestimate, the text finds about 20 more ft, to be careful I need a better graph and I'd do better to use the midpoint to base the height of the approximating rectangles.