Homework 27, Calculus I

Remark: I'm not checking these work, but you should have.
that is $\int f(x)\,dx = F(x)$ then $\frac{dF}{dx} = f(x)$.

§4.9#1

$$\int (x-3)\,dx = \boxed{\tfrac{1}{2}x^2 - 3x + c}.$$

§4.9#3

$$f(x) = \tfrac{1}{2} + \tfrac{3}{4}x^2 - \tfrac{4}{5}x^3$$

$$\boxed{F(x) = \tfrac{1}{2}x + \tfrac{1}{4}x^3 - \tfrac{1}{5}x^4 + C}$$

§4.9#5

$$\int (x+1)(2x-1)\,dx = \int (2x^2 + x - 1)\,dx$$

$$= \boxed{\tfrac{2}{3}x^3 + \tfrac{1}{2}x^2 - x + C}$$

§4.9#7

$$\int \left(5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}\right)dx = 5\left(\tfrac{4}{5}x^{\frac{5}{4}}\right) - 7\left(\tfrac{4}{7}x^{\frac{7}{4}}\right) + C$$

$$= \boxed{4x^{5/4} - 4x^{7/4} + C}$$

Remark: The unusual arithmatic $\frac{1}{\frac{b}{a}} = \frac{a}{b}$ comes up a lot with fractional exponents.

$$\int x^{\frac{1}{4}}\,dx = \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + C \qquad \left(\text{Power Rule for integrals}\right)$$

$$= \frac{x^{5/4}}{\frac{5}{4}}$$

$$= \tfrac{4}{5}x^{5/4}$$

← I just write this an do the rest in my noggin.

§4.9 #9

$$\int\left(6\sqrt{x}-\sqrt[6]{x}\right)dx = \int\left(6x^{1/2}-x^{1/6}\right)dx$$
$$= 6\,\frac{2}{3}\,x^{3/2} - \frac{6}{7}\,x^{7/6} + C$$
$$= \boxed{4x^{3/2} - \frac{6}{7}x^{7/6} + C}$$

§4.9#11

$$\int \frac{10}{x^9}\,dx = 10\int x^{-9}dx$$
$$= 10\,\frac{x^{-8}}{-8} + C$$
$$= \boxed{-\frac{5}{4x^8} + C}$$

§4.9#13

$$\int \frac{u^4+3\sqrt{u}}{u^2}\,du = \int\left(u^2+3u^{-3/2}\right)du$$
$$= \boxed{\frac{1}{3}u^3 - 6u^{-1/2} + C}$$

$-\frac{3}{2}+1 = -\frac{1}{2}$

and

$\frac{1}{-\frac{1}{2}} = -2$

§4.9#15

$$\int(\cos\theta - 5\sin\theta)\,d\theta = \boxed{\sin\theta + 5\cos\theta + C}$$

§4.9#17

$$\int\left(2\sec(t)\tan(t) + \frac{1}{2}t^{-1/2}\right)dt = \boxed{2\sec(t) + \sqrt{t} + C}$$

Remark:
The "antiderivative of $f(x)$" is denoted $F(x)$ and the "indefinite integral of $f(x)$ is denoted $\int f(x)dx$. These both obey:

$$\frac{d}{dx}\left(F(x)\right) = \frac{d}{dx}\int f(x)dx = f(x)$$

We insist however that $\int f(x)dx$ include the "C"

Remark Continued:

The difference between "F" and $\int f(x)dx$ is that F may not include the arbitrary constant C. We insist $\int f(x)dx$ be "the most general antiderivative" this simply means it must allow for different possible values for C. In truth $\int f(x)dx$ is a whole <u>family</u> of functions which differ only by at most a constant. For example,

$$f(x) = x^2 \qquad F(x) = \tfrac{1}{3}x^3 \quad \text{or} \quad F(x) = \tfrac{1}{3}x^3 + 3$$

But, we must have $\int x^2 dx = \frac{1}{3}x^3 + C$

this is largely a matter of <u>conventions</u>.

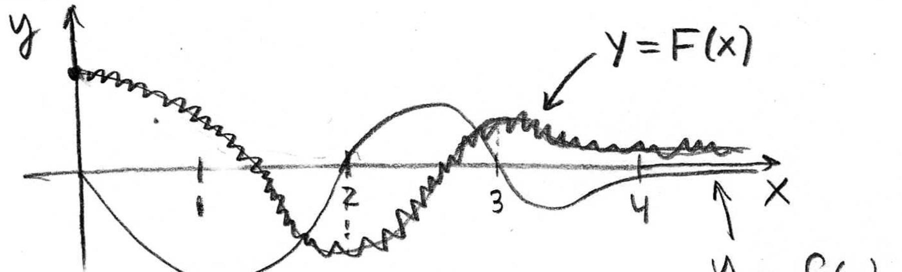
§4.9#41) Given graph of f passes through (1, 6) and that the slope of its tangent line at $(x, f(x))$ is $2x+1$ find $f(2)$.

$$\frac{df}{dx} = 2x+1 \xrightarrow[\text{antidifferentiation}]{} f(x) = x^2 + x + C$$

Now $f(1) = 6$ thus $6 = 1+1+C \rightarrow C = 4$.
Hence $f(x) = x^2 + x + 4$. In particular,

$$f(2) = 4+2+4 = \boxed{10}$$

§4.9#45) Graph $y = F(x)$ given $F(0) = 1$ and the graph of $f(x)$ below.



Need that

$$\frac{dF}{dx} = f(x)$$

$f(x) < 0$ F decreases
$f(x) > 0$ F increases.
this idea does it.

Remark: see solⁿ in Stewart, his is a little better, you can't get to exact w/o a formula in my opinion