Homework 14, Calculus I, Fall 2008

§3.5 #1

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(4x)\right] = \cos(4x)\frac{d}{dx}(4x) \qquad (\text{here } u = 4x)$$
$$= \boxed{4\cos(4x)}$$

§3.5 #3

$$\frac{d}{dx}\left[(1-x^2)^{10}\right] = \frac{d}{dx}\left(u^{10}\right) \quad : \text{letting } u = 1-x^2$$
$$= 10u^9\frac{du}{dx} \quad : \text{chain-rule}$$
$$= 10(1-x^2)^9(-2x)$$
$$= \boxed{-20x(1-x^2)^9}$$

§3.5 #5

$$\frac{d}{dx}\left[\sqrt{\sin(x)}\right] = \frac{d}{dx}\left[\sqrt{u}\right] \quad : \text{letting } u = \sin(x)$$
$$= \frac{1}{2\sqrt{u}}\frac{du}{dx} \quad : \text{chain-rule}$$
$$= \boxed{\frac{1}{2\sqrt{\sin(x)}}(\cos(x))}$$

§3.5 #7

$$F'(x) = \frac{d}{dx}\left[(x^4+3x^2-2)^5\right] \quad : \text{let us use } u = x^4+3x^2-2$$
$$= \frac{d}{dx}\left[u^5\right]$$
$$= 5u^4\frac{du}{dx}$$
$$= 5(x^4+3x^2-2)^4\frac{d}{dx}\left[x^4+3x^2-2\right]$$
$$= \boxed{5(x^4+3x^2-2)^4(4x^3+6x)}$$

§3.5 #9 let $u = 1+2x+x^3$ in the calculation below, $\frac{du}{dx} = 2+3x^2$ and,

$$\frac{d}{dx}\left(\sqrt[4]{1+2x+x^3}\right) = \frac{d}{dx}\left(u^{\frac{1}{4}}\right)\frac{du}{dx} = \frac{1}{4}u^{-\frac{3}{4}}\frac{du}{dx} = \boxed{\frac{1}{4}(1+2x+x^3)^{-3/4}(2+3x^2)}$$

§ 3.5 # 13) Let a be a constant w.r.t. x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\cos(a^3+x^3)\right]$$

$$= \frac{d}{dx}\left[\cos(u)\right] \quad : \text{ let } u = a^3 + x^3$$

$$= -\sin(u)\frac{du}{dx}$$

$$= \boxed{-3x^2 \sin(a^3+x^3)}$$

§ 3.5 # 15) let k be a constant,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x \sec(kx)\right]$$

$$= \sec(kx) + x\frac{d}{dx}\left[\sec(kx)\right] \quad : \text{ used product rule}$$

$$= \sec(kx) + x\frac{d}{dx}\left[\sec(u)\right] \quad : \text{ let } u = kx$$

$$= \sec(kx) + x\sec(u)\tan(u)\frac{du}{dx} \quad : \text{ chain rule.}$$

$$= \sec(kx) + x\sec(kx)\tan(kx)\,k \quad : \frac{du}{dx} = k$$

$$= \boxed{\sec(kx)\left[1 + kx\tan(kx)\right]} \quad : \text{ simplified answer.}$$

§ 3.5 # 17)

$$g'(x) = \frac{d}{dx}\left[(1+4x)^5(3+x-x^2)^8\right] \quad : \text{ let } \begin{cases} u = 1+4x \\ w = 3+x-x^2 \end{cases}$$

$$= \frac{d}{dx}\left[u^5 w^8\right]$$

$$= \frac{d}{dx}(u^5)\,w^8 + u^5\frac{d}{dx}(w^8) \quad : \text{ product rule}$$

$$= 5u^4\frac{du}{dx}w^8 + u^5 \cdot 8w^7\frac{dw}{dx} \quad : \text{ chain rule } (\times 2)$$

$$= 20u^4w^8 + 8u^5w^7(1-2x) \quad : \frac{du}{dx} = 4,\ \frac{dw}{dx} = 1-2x$$

$$= 4u^4w^7\left[5w + 2u(1-2x)\right] \quad : \text{ simplifying answer}$$

$$= 4u^4w^7\left[15 + 5x - 5x^2 + 2(1+4x)(1-2x)\right] \quad : \text{ still simplifying.}$$

$$= 4u^4w^7\left[15 + 5x - 5x^2 + 2 - 4x + 8x - 16x^2\right] \quad : \text{ again simplifying}$$

$$= \boxed{4(1+4x)^4(3+x-x^2)^7(17 + x - 21x^2)}$$

Remark: try logarithmic diff. on this later

§3.5 # 23

$$\frac{d}{dx}\left[\sin(x\cos(x))\right] = \frac{d}{dx}\left[\sin(u)\right] \qquad : u = x\cos(x)$$

$$= \cos(u)\frac{du}{dx} \qquad : \text{chain rule}$$

$$= \cos(u)\frac{d}{dx}\left[x\cos(x)\right]$$

$$= \cos(u)\left[\cos(x) - x\sin(x)\right] \qquad : [\text{product rule}]$$

$$= \boxed{\cos(x\cos(x))\left[\cos(x) - x\sin(x)\right]}.$$

§3.5 # 25 Warning: your instructor identifies z and z as same.

$$\frac{d}{dz}\left[\sqrt{\frac{z-1}{z+1}}\right] = \frac{d}{dz}\left(\sqrt{u}\right) \qquad : u = \frac{z-1}{z+1}$$

$$= \frac{1}{2\sqrt{u}}\frac{du}{dz} \qquad : \text{chain rule}$$

$$= \frac{1}{2\sqrt{u}}\left[\frac{1(\cancel{z}+1) - (\cancel{z}-1)1}{(z+1)^2}\right] \qquad : \text{quotient rule on } u.$$

$$= \sqrt{\frac{1}{u}}\,\frac{1}{(z+1)^2} \qquad : \text{simplifying answer}$$

$$= \sqrt{\frac{z+1}{(z-1)^2}}\sqrt{\frac{1}{(z+1)^4}} \qquad : \text{simplifying}$$

$$= \boxed{\frac{1}{\sqrt{(z-1)^2(z+1)^3}}} \qquad : \text{simplifying}$$

Remark: I've shown more work than usual here. As the semester progresses I will often omit some of the middle steps like saying what "u" is. When problems are complicated it becomes more important to explain steps. If I ask to "show work including choice of u explicitly" then I mean for you to give sol$^{\underline{n}}$ along the lines given here.