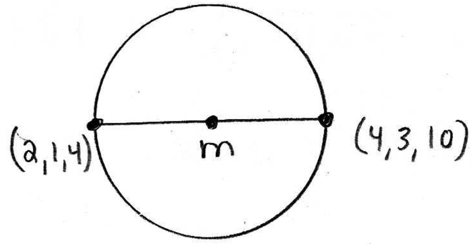
Math 231 – Homework Quiz 1 – Sol^n

§13.1 #20 | Find sphere which has a diameter from the point $(2,1,4)$ to $(4,3,10)$. Look at cross-sectional view,



Notice the center of the sphere is at the midpoint m

$$m = \frac{1}{2}\left[(2,1,4) + (4,3,10)\right]$$

$$m = (3,2,7).$$

The radius of the sphere will be ½ the length of the diameter. $R = \frac{1}{2}\sqrt{(4-2)^2 + (3-1)^2 + (10-4)^2} = \frac{1}{2}\sqrt{4+4+36} = \frac{\sqrt{44}}{2}$

Thus $R = \sqrt{11}$ and the sphere has the equation,

$$\boxed{(x-3)^2 + (y-2)^2 + (z-7)^2 = 11}$$

§13.2 # 24 | Find vector in $\langle -2, 4, 2\rangle$ direction with length 6.

$\vec{v} = k\langle -2,4,2\rangle$ with $|\vec{v}| = 6$

$$\sqrt{4k^2 + 16k^2 + 4k^2} = 6$$

$$\sqrt{24k^2} = 6 \Rightarrow k = \pm\frac{6}{\sqrt{24}} = \pm\frac{6}{2\sqrt{6}}$$

$$\boxed{\vec{v} = \pm\frac{3}{\sqrt{6}}\langle -2, 4, 2\rangle}$$

Or calculate unit vector and multiply by 6. That's an equally good sol^n.

§13.3 #19 Find angle between $\vec{a} = \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$

$$\vec{a} \cdot \vec{b} = \langle 0, 1, 1 \rangle \cdot \langle 1, 2, -3 \rangle = 0 + 2 - 3 = -1$$

$$|\vec{a}| = \sqrt{2}$$

$$|\vec{b}| = \sqrt{1+4+9} = \sqrt{14}$$

$$\therefore \quad \vec{a} \cdot \vec{b} = ab\cos\theta \longrightarrow -1 = \sqrt{14}\cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(-1/\sqrt{14}\right) =}$$

§13.4 #20 Find two unit vectors orthogonal to both $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{k}$. Notice $\underbrace{(\vec{A} \times \vec{B}) \cdot \vec{A} = 0 \text{ and } (\vec{A} \times \vec{B}) \cdot \vec{B} = 0}$.

Likewise for $\vec{B} \times \vec{A}$,

$(\vec{B} \times \vec{A}) \cdot \vec{A} = 0$ & $(\vec{B} \times \vec{A}) \cdot \vec{B} = 0$.

You'll Prove These in general on the Homework Project 4a.

The crossproduct gives us a new vector which is orthogonal to both of its inputs.

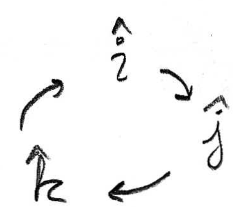
$$\vec{A} \times \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} + \hat{k})$$

$$= \cancel{2\hat{i} \times \hat{i}}^{\,0} + \hat{i} \times \hat{k} + 2\hat{j} \times \hat{i} + \hat{j} \times \hat{k} + 2\hat{k} \times \hat{i} + \cancel{\hat{k} \times \hat{k}}^{\,0}$$

$$= -\hat{j} - 2\hat{k} + \hat{i} + 2\hat{j}$$

$$= \hat{j} - 2\hat{k} + \hat{i}$$

$$= \langle 1, 1, -2 \rangle.$$



$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = \langle -1, -1, 2 \rangle$$

Need to <u>normalize</u> to length one.

$$\vec{u}_1 = \frac{1}{\sqrt{1+1+4}} \langle 1, 1, -2 \rangle \quad \& \quad \vec{u}_2 = \frac{1}{\sqrt{1+1+4}} \langle -1, -1, 2 \rangle$$

$$\boxed{\vec{u}_1 = \frac{1}{\sqrt{6}} \langle 1, 1, -2 \rangle \quad \& \quad \vec{u}_2 = \frac{1}{\sqrt{6}} \langle -1, -1, 2 \rangle}$$