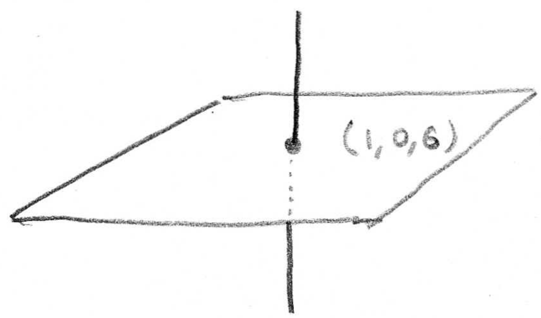
§13.5 #5 Find line through point (1, 0, 6) and perpendicular to the plane $x + 3y + z = 5$. Let's picture this,



the direction vector (or tangent vector) to the line is in the same direction as the normal to the plane.

The equation for a plane with normal $\langle a, b, c\rangle$ and a point (x_0, y_0, z_0) is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$. We can also write $ax+by+cz = ax_0+by_0+cz_0 = $ "d".
So comparing $x+3y+z=5$ to $ax+by+cz=d$ we see the normal is $\langle 1, 3, 1\rangle$. Thus the equation of the line through $\vec{r}_0 = (1, 0, 6)$ is

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \boxed{\langle 1,0,6\rangle + t\langle 1,3,1\rangle = \vec{r}(t)}$$

↑ "vector equation"

Equivalently, $\vec{r}(t) = \langle x, y, z\rangle$ so

$$\boxed{\begin{aligned} x &= 1+t \\ y &= 3t \\ z &= 6+t \end{aligned}}$$

← "parametric equations"

§13.5 #10 Find line through (2, 1, 0) and perpendicular to $\vec{A} = \hat{i}+\hat{j}$ and $\vec{B} = \hat{j}+\hat{k}$. Recall $\vec{A}\times\vec{B}$ is $\perp$ to both $\vec{A}$ & $\vec{B}$.
Choose direction vector $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k} = \langle 1, -1, 1\rangle$.
As a check, notice $\vec{A}\cdot\langle 1,-1,1\rangle = 0$ and $\vec{B}\cdot\langle 1,-1,1\rangle = 0$.
We find

$$\boxed{\vec{r}(t) = \langle 2, 1, 0\rangle + t\langle 1, -1, 1\rangle}$$

No need to bother with symmetric equations.

§13.5 #16

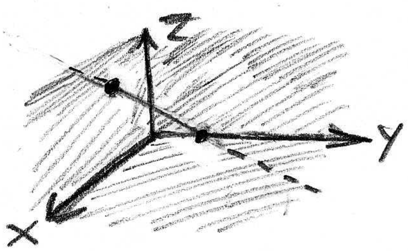
a.) find parametric equations for line through $(2,4,6)$ that is $\perp$ to plane $x-y+3z=7$. Same as #5,

$$\boxed{\begin{array}{l} \vec{r}(t) = \langle 2,4,6\rangle + t\langle 1,-1,3\rangle \\ \hline x = 2+t \\ y = 4-t \\ z = 6+3t \end{array}}$$

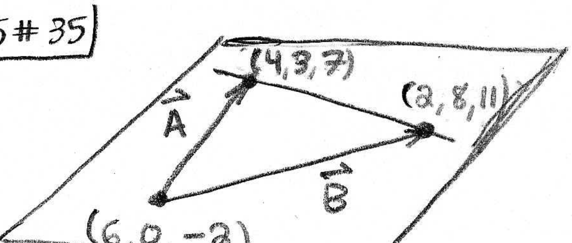
b.) Where does this line intersect xy, yz, zx planes?

xy: $z=0=6+3t \Rightarrow t=-2 \therefore \underbrace{\vec{r}(-2) = \boxed{(0,6,0)}}_{\text{also on } yz\text{-plane}}$

zx: $y=0=4-t \Rightarrow t=4 \therefore \vec{r}(4) = \boxed{(6,0,18)}$



§13.5 #35



Given a point $(6,0,-2)$ and a line $x=4-2t$, $y=3+5t$, $z=7+4t$ contained in the plane find the equation for this plane. Begin with picture. Pick two points on line, lets use the points on the line where $t=0$ and $t=1$, $\vec{r}(0)=(4,3,7)$, $\vec{r}(1)=(2,8,11)$

$$\vec{A} = \langle 4-6,\ 3-0,\ 7-(-2)\rangle = \langle -2,3,9\rangle$$
$$\vec{B} = \langle 2-6,\ 8-0,\ 11-(-2)\rangle = \langle -4,8,13\rangle$$

We want the normal vector $\perp$ to both $\vec{A}$ & $\vec{B}$ thus we

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 9 \\ -4 & 8 & 13 \end{vmatrix} = \langle 39-72,\ -(-26+36),\ -16+12\rangle$$

Thus $\vec{A}\times\vec{B} = \langle -33,-10,-4\rangle$ is a normal for the plane. Thus, the equation for plane is $\boxed{-33(x-6)-10y-4(z+2)=0}$