Homework Quiz on Trig. Subst. 1-20-06

$\boxed{\text{Supp. \#7}}$

$$\int \frac{x^2\,dx}{(x^2-1)^{5/2}} = \int \frac{\sec^2\theta\,\sec\theta\tan\theta\,d\theta}{(\tan^2\theta)^{5/2}}$$

$$\boxed{\begin{aligned} x &= \sec\theta \\ x^2-1 &= \sec^2\theta - 1 = \tan^2\theta \\ dx &= \sec\theta\tan\theta\,d\theta \end{aligned}}$$

$$= \int \frac{\sec^3\theta\,\tan\theta\,d\theta}{\tan^5\theta}$$

$$= \int \sec^3\theta\,\frac{1}{\tan^4\theta}\,d\theta$$

$$= \int \frac{1}{\cos^3\theta}\,\frac{\cos^4\theta}{\sin^4\theta}\,d\theta$$

$$= \int \frac{\cos\theta\,d\theta}{\sin^4\theta}$$

$$= \int \frac{du}{u^4}$$

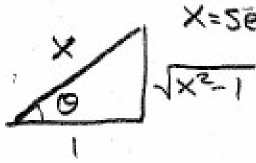
$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\,d\theta \end{aligned}}$$

$$= \frac{-1}{3u^3} + C$$

$$= \frac{-1}{3}\,\frac{1}{\sin^3\theta} + C$$

$$= \boxed{\frac{-1}{3}\left(\frac{x}{\sqrt{x^2-1}}\right)^3 + C}$$

$x = \sec\theta = \frac{hyp}{adj}$



$\boxed{\text{Supp \#1}}$

$$\int \frac{dy}{\sqrt{9+y^2}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9\sec^2\theta}}$$

$$\boxed{\begin{aligned} y &= 3\tan\theta \\ 9+y^2 &= 9 + 9\tan^2\theta = 9\sec^2\theta \\ dy &= 3\sec^2\theta\,d\theta \end{aligned}}$$

$$= \int \sec\theta\,d\theta$$

(see $\boxed{\text{EZO}}$ on 104)

$$\boxed{\begin{aligned} u &= \sec\theta + \tan\theta \\ du &= (\sec\theta\tan\theta + \sec^2\theta)\,d\theta \\ &= \sec\theta\,(\tan\theta + \sec\theta)\,d\theta \\ &= \sec\theta \cdot u\,d\theta \\ \Rightarrow\ \sec\theta\,d\theta &= \frac{du}{u} \end{aligned}}$$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \boxed{\ln\left|\frac{\sqrt{y^2+9}}{3} + \frac{y}{3}\right| + C.}$$

$\tan\theta = \frac{y}{3}$