Homework quiz : Appendix G # 19 1/26/06

$$\int \frac{1}{(x+5)^2(x-1)}\,dx$$

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{(x+5)^2} + \frac{B}{(x+5)} + \frac{C}{x-1}$$

$$1 = A(x-1) + B(x-1)(x+5) + C(x+5)^2$$

$x=1$: $1 = C(36) \Rightarrow \boxed{C = 1/36}$

$x=-5$: $1 = A(-6) \Rightarrow \boxed{A = -1/6}$

$x=0$: $1 = -A - 5B + 25C = \frac{1}{6} - 5B + \frac{25}{36} = \frac{36}{36}$

$$-5B = \frac{36}{36} - \frac{6}{36} - \frac{25}{36} = \frac{5}{36}$$

$$\Rightarrow \boxed{B = -1/36}$$

Thus by the algebra above,

$$\int \frac{1}{(x+5)^2(x-1)}\,dx = \int \frac{-1/6}{(x+5)^2}\,dx + \int \frac{-1/36}{(x+5)}\,dx + \int \frac{1/36}{x-1}\,dx$$

$$= -\frac{1}{6}\int \frac{du}{u^2} - \frac{1}{36}\int \frac{du}{u} + \frac{1}{36}\int \frac{dw}{w}$$

$u = x+5$, $w = x-1$, $du = dx$, $dw = dx$

$$= \frac{1}{6}\frac{1}{u} - \frac{1}{36}\ln|u| + \frac{1}{36}\ln|w| + C$$

$$= \boxed{\frac{1}{6}\left(\frac{1}{x+5}\right) - \frac{1}{36}\ln|x+5| + \frac{1}{36}\ln|x-1| + C}$$