Section 6.3

5. Find the arc length of the curve $x = 1+3t^2$, $y = 4+2t^3$ from $t=0$ to $t=1$.

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{36t^2 + 36t^4}\, dt$$

$$= \int_0^1 \sqrt{36t^2(1+t^2)}\, dt$$

$$= \int_0^1 6t\sqrt{1+t^2}\, dt$$

let $u = 1+t^2$; $du = 2t\,dt$

when $t=0$, $u = 1+0^2 = 1$

$t=1$, $u = 1+1^2 = 2$

$$= \int_1^2 3\sqrt{u}\, du$$

$$= 3\int_1^2 u^{\frac{1}{2}}\, du$$

$$= 3 \cdot \frac{u^{3/2}}{3/2}\Big|_1^2 = 2u^{3/2}\Big|_1^2$$

$$= 2\left(2^{3/2} - 1\right) = 4\sqrt{2} - 2.$$