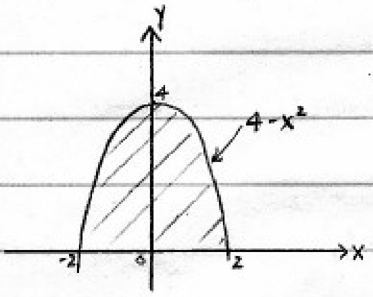
Section 6.5

35. $y = 4 - x^2$, $y = 0$



$A = \int_{-2}^{2} 4 - x^2 dx = (4x - \frac{x^3}{3})\Big|_{-2}^{2} = \frac{32}{3}$

$\bar{x} = \frac{1}{A}\int_{-2}^{2} x(4-x^2)dx$

$= \frac{3}{32}(2x^2 - \frac{x^4}{4})\Big|_{-2}^{2} = 0.$

$\bar{y} = \frac{1}{A}\cdot\frac{1}{2}\int_{-2}^{2}(4-x^2)^2 dx$

$= \frac{3}{64}(16x - \frac{8x^3}{3} + \frac{x^5}{5})\Big|_{-2}^{2}$

$= \frac{3}{64}(32 - \frac{64}{3} + \frac{32}{5})2 = \frac{6}{64}(\frac{256}{15}) = \frac{24}{15} = 1.6$

Section 6.7

3. $f(x) = \frac{3}{64}x\sqrt{16-x^2}$ for $0 \le x \le 4$, $f(x) = 0$ otherwise.

a) $f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x)dx = \frac{3}{64}\int_0^4 x\sqrt{16-x^2}\,dx$

let $u = 16 - x^2$, $du = -2x dx$
when $x=0$, $u=16$
$x=4$, $u=0$

$= \frac{3}{64}\cdot(-\frac{1}{2})\int_{16}^{0}\sqrt{u}\,du$

$= \frac{3}{128}\int_0^{16}\sqrt{u}\,du$

$= \frac{3}{128}\,u^{3/2}\cdot\frac{2}{3}\Big|_0^{16}$

$= \frac{1}{64}(16)^{3/2} = 1$

$\therefore$ f is a probability density function.

b) $P(X<2) = \int_0^2 f(x)dx$

$= \frac{3}{64}\int_0^2 x\sqrt{16-x^2}\,dx$

$u = 16 - x^2$; $du = -2x dx$
when $x=0$, $u=16$
$x=2$, $u=12$

$= \frac{3}{64}\int_{16}^{12}(-\frac{1}{2})\sqrt{u}\,du$

$= \frac{3}{128}\int_{12}^{16}\sqrt{u}\,du$

$= \frac{3}{128}\cdot\frac{2}{3}u^{3/2}\Big|_{12}^{16} = \frac{1}{64}(16^{3/2} - 12^{3/2})$

$= \frac{1}{64}(64 - 24\sqrt{3}) \approx 0.3505$