Section 7.8

\# 3. $y'' - 2y' = \sin(4x)$

The characteristic equation is

$\lambda^2 - 2\lambda = 0 \Rightarrow \lambda(\lambda - 2) = 0$

$\Rightarrow \lambda = 0$ or $\lambda = 2$

Hence $y_c(x) = c_1 e^0 + c_2 e^{2x} = c_1 + c_2 e^{2x}$

Now, let $y_p(x) = A\cos(4x) + B\sin(4x)$

then $y_p'(x) = 4(-A\sin(4x) + B\cos(4x))$

$y_p''(x) = 16(-A\cos(4x) - B\sin(4x))$

hence, we have $16(-A\cos(4x) - B\sin(4x)) - 8(-A\sin(4x) + B\cos(4x)) = \sin(4x)$

$\Rightarrow (-16A - 8B)\cos(4x) + (-16B + 8A)\sin(4x) = \sin(4x)$

$$\Rightarrow \begin{cases} -16A - 8B = 0 \\ +8A - 16B = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{40} \\ B = -\frac{1}{20} \end{cases}$$

$\therefore y(x) = c_1 + c_2 e^{2x} + \frac{1}{40}\cos(4x) - \frac{1}{20}\sin(4x)$

\#9. $y'' - y = xe^{3x}$

The characteristic equation is

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\therefore y_c(x) = c_1 e^x + c_2 e^{-x}$

Now, let $y_p(x) = Axe^{3x} + Be^{3x}$

then $y_p'(x) = Ae^{3x} + 3Axe^{3x} + 3Be^{3x} = (A + 3B)e^{3x} + 3Axe^{3x}$

$y_p''(x) = 3(A + 3B)e^{3x} + 3Ae^{3x} + 9Axe^{3x} = (6A + 9B)e^{3x} + 9Axe^{3x}$

hence we have $(6A + 9B)e^{3x} + 9Axe^{3x} - Axe^{3x} - Be^{3x} = xe^{3x}$

$$\Rightarrow \left.\begin{matrix} 6A + 8B = 0 \\ 8A = 1 \end{matrix}\right\} \Rightarrow \begin{matrix} A = \frac{1}{8} \\ B = -\frac{3}{32} \end{matrix}$$

$\therefore y(x) = c_1 e^x + c_2 e^{-x} + \frac{1}{8}xe^{3x} - \frac{3}{32}e^{3x}$