§3.3#61 Suppose $s(t) = t^3 - 3t$ represents the equation of motion find,

(a.) acceleration & velocity,

$$v(t) = \frac{ds}{dt} = \underline{3t^2 - 3}.$$

$$a(t) = \frac{dv}{dt} = \underline{6t}.$$

(b.) $a(2) = 6(2) = \underline{12}.$

(c.) $v(t) = 3t^2 - 3 = 0$

$$3t^2 = 3$$
$$t^2 = 1$$
$$\underline{t = \pm 1} \quad \text{thus} \quad \boxed{a(-1) = -6 \quad \text{and} \quad a(1) = 6}$$

probably the text throws out $t = -1$ since its negative, I don't see the need, time can be negative.

§3.3#71 Find points on $y = f(x) = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is <u>horizontal</u>. Note this means we must search for x such that $f'(x) = 0$,

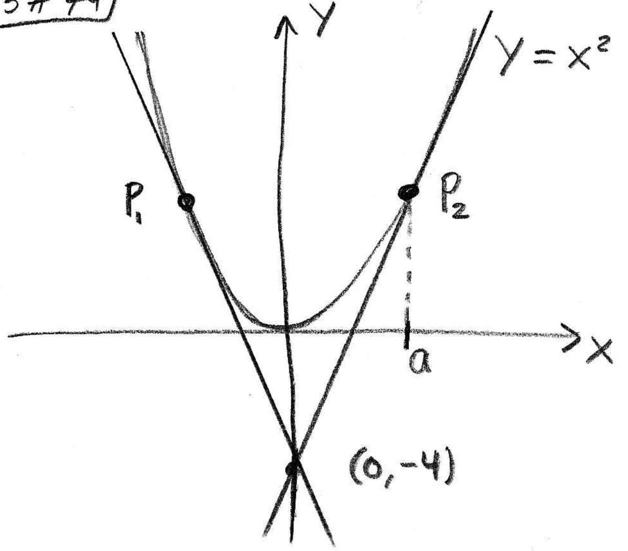
$$f'(x) = 6x^2 + 6x - 12 = 0$$
$$(6x + 12)(x - 1) = 0$$
$$6(x+2)(x-1) = 0 \Rightarrow x = -2 \text{ or } x = 1$$

Then $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 1 = -16 + 12 + 24 + 1 = 21$

$f(1) = 2 + 3 - 12 + 1 = -6$

Thus the points where the tangent is <u>horizontal</u> are $\boxed{(-2, 21), (1, -6)}$.

§3.3 #79)



We need to introduce some variable other than X or else confusion will abound. I'll use "a" as in picture

Let $Y = f(x) = x^2$ then our tangent lines must follow a few simple rules,

1.) they're a line: $Y = mx + b$

2.) slope given by $f'(a) = 2a$: $Y = 2ax + b$

3.) they go through $(0, -4)$: $\underline{b = -4}$.

So we have

$$Y = 2ax - 4$$

Now we still have one more piece of data. We know the tangent line has to pass through $(a, f(a)) = (a, a^2)$ hence

$$a^2 = 2a^2 - 4 \rightarrow a^2 = 4 \rightarrow \underline{a = \pm 2}.$$

Thus the points are $\boxed{(2, 4) \text{ and } (-2, 4)}$
"P_2" "P_1" (my picture)

§3.3#83| Find $P(x) = Ax^2 + Bx + C$ such that $P(2) = 5$
$P'(2) = 3$ and $P''(2) = 2$. Observe,

$$P(x) = Ax^2 + Bx + C$$
$$P'(x) = 2Ax + B$$
$$P''(x) = 2A$$

Thus $P''(2) = 2 = 2A \Rightarrow \underline{A = 1}$
$P'(2) = 3 = 4A + B = 4 + B \Rightarrow \underline{B = -1}$.
$P(2) = 5 = 4A + 2B + C = 4 - 2 + C \Rightarrow \underline{C = 3}$.

Hence $\boxed{P(x) = x^2 - x + 3}$

§3.3#85| Find cubic $y = ax^3 + bx^2 + cx + d$ with horizontal tangent at the points $(-2, 6)$ and $(2, 0)$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

We want $\underbrace{\frac{dy}{dx}(-2, 6) = 0}_{\text{I}}$ and $\underbrace{\frac{dy}{dx}(2, 0) = 0}_{\text{II}}$ thus,

I : $0 = 12a - 4b + c$
II : $0 = 12a + 4b + c$

Subtract equations to obtain $-8b = 0$ thus $\underline{b = 0}$.
Adding equations yields $24a + 2c = 0$ thus $\underline{c = -12a}$.
We find $y = ax^3 - 12ax + d$. We can pin this down further since the points $(-2, 6)$ and $(2, 0)$ are on the cubic,

$$\begin{array}{rl} & \begin{pmatrix} 6 = -8a + 24a + d \\ 0 = 8a - 24a + d \end{pmatrix} \\ + & \\ \hline & 6 = 2d \longrightarrow \underline{d = 3} \end{array}$$

and $16a = d = 3$
$\Rightarrow \underline{a = \frac{3}{16}}$.

Note $-12\left(\frac{3}{16}\right) = -9/4$ thus,

$$\boxed{y = \frac{3}{16}x^3 - \frac{9}{4}x + 3}$$

is the cubic with $\frac{dy}{dx} = 0$ at $(-2, 6)$ and $(2, 0)$.

§3.3 #96 Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find values for the constants m & b which make f differentiable everywhere. To begin, notice $\frac{d}{dx}(x^2) = 2x$ and $\frac{d}{dx}(mx+b) = m$ are true for all x so they're certainly true for $x<2$ and $x>2$ respective. We need only verify that $f'(2)$ exists. This means, at $x=2$ we need

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(mx+b) \quad \text{at } x=2$$

$$2x = m \quad \text{at } x=2$$

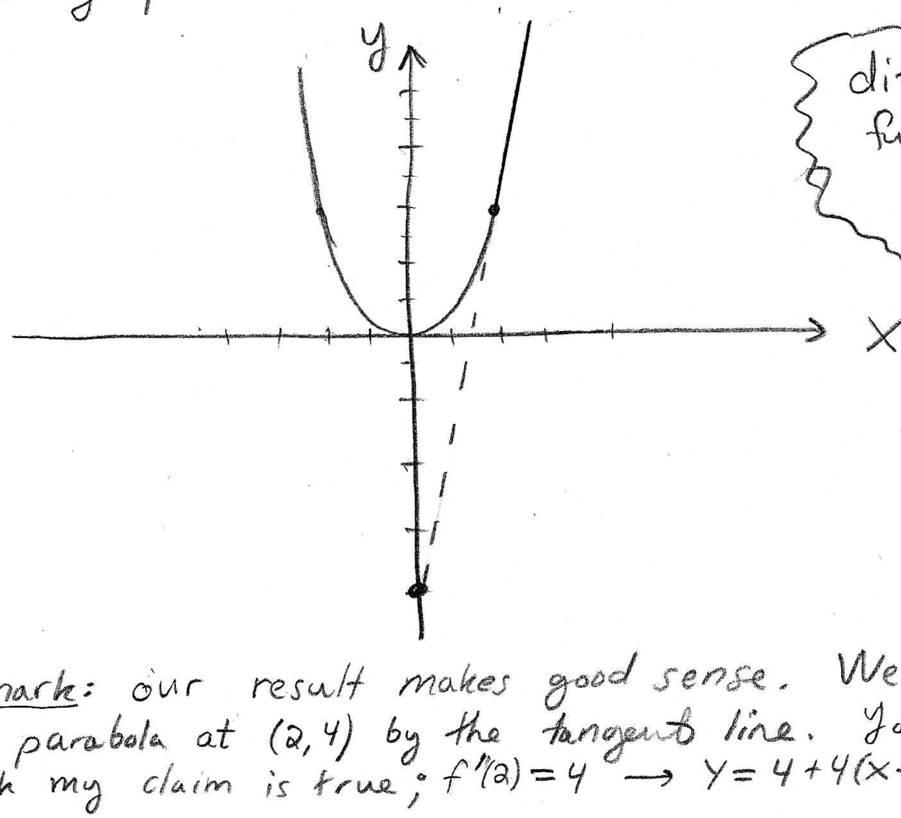
$$\boxed{4 = m.}$$

What about b? Well we also need continuity, so $\lim_{x\to 2} f(x) = f(2) = 4$. That is,

$$\begin{array}{ccc} \lim_{x\to 2^+} f(x) & = & \lim_{x\to 2^-} f(x) \\ \| & & \| \\ 2m+b & = & 4 \end{array}$$

But $m=4$ thus $8+b=4$ so $\boxed{b=-4}$

Let's graph the situation here,



differentiable functions can't have kinks or corners in their graphs.

Remark: our result makes good sense. We continue the parabola at $(2,4)$ by the tangent line. You can check my claim is true; $f'(2)=4 \longrightarrow y = 4+4(x-2) = 4x-4$.