Homework 15, Calculus I, Fall 2008

§3.5#41)

$$\frac{d}{dx}\left(\sqrt{x+\sqrt{x}}\right) = \frac{1}{2\sqrt{x+\sqrt{x}}}\frac{d}{dx}\left[x+\sqrt{x}\right]$$

$$= \boxed{\frac{1}{2\sqrt{x+\sqrt{x}}}\left[1+\frac{1}{2\sqrt{x}}\right]}$$

Remark: Yes, I would like expect you to write the middle step in this course

§3.5#43) Assume r, n, P are constants.

$$\frac{d}{dx}\left[(2r\sin(rx)+n)^P\right] = P(2r\sin(rx)+n)^{P-1}\frac{d}{dx}\left[2r\sin(rx)+n\right]$$

$$= P(2r\sin(rx)+n)^{P-1}\left[2r\cos(rx)\frac{d}{dx}[rx]+0\right]$$

$$= \boxed{2r^2P\cos(rx)\,(2r\sin(rx)+n)^{P-1}}$$

§3.5#48) Find y' and y''

$$y' = \frac{d}{dt}\left[\sin^2(\pi t)\right] = 2\sin(\pi t)\frac{d}{dt}\left[\sin(\pi t)\right]$$

$$= 2\sin(\pi t)\cos(\pi t)\frac{d}{dt}\left[\pi t\right]$$

$$= \boxed{2\pi\sin(\pi t)\cos(\pi t) = y'}$$

$$y'' = \frac{d}{dt}\left[2\pi\sin\pi t\cos\pi t\right] \quad : \text{ use } \underline{\text{product rule}}$$

$$= 2\pi\left[\frac{d}{dt}(\sin\pi t)\cos\pi t + \sin\pi t\frac{d}{dt}[\cos\pi t]\right]$$

$$= 2\pi\left[(\cos(\pi t)\cdot\pi)\cos\pi t + \sin(\pi t)(-\sin(\pi t)\cdot\pi)\right] \quad : \text{(chain rule twice } u=\pi t)$$

$$= \boxed{2\pi^2\left[\cos^2(\pi t)-\sin^2(\pi t)\right] = y''}$$

<u>Alternatively</u> : $2\pi\sin\pi t\cos\pi t = \underline{\pi\sin(2\pi t) = y'}$

$$y'' = \frac{d}{dt}\left[\pi\sin 2\pi t\right] = \pi\cos(2\pi t)\frac{d}{dt}[2\pi t]$$

$$= \underline{2\pi^2\cos(2\pi t) = y''}$$

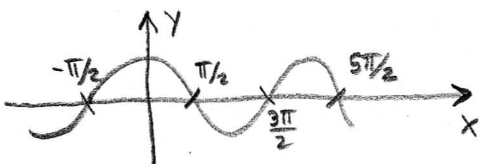
(trig. identities can save calculation)

§3.5 #59) Find **all** points on $y = f(x) = 2\sin(x) + \sin^2(x)$ for which the graph has a horizontal tangent line. We must find where $f'(x)$ can be zero.

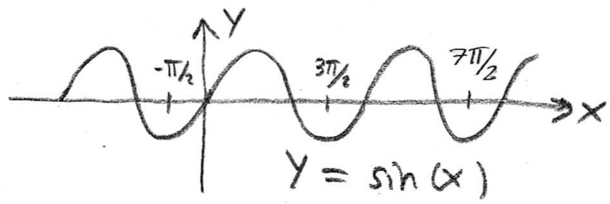
$$
\begin{aligned}
f'(x) &= \frac{d}{dx}\left[2\sin(x) + \sin^2(x)\right] \\
&= 2\cos(x) + 2\sin(x)\frac{d}{dx}(\sin(x)) \\
&= 2\cos(x) + 2\sin(x)\cos(x) \\
&= 2\cos(x)\left[1 + \sin(x)\right] = 0
\end{aligned}
$$

Thus we want points where either

$$\underbrace{\cos(x) = 0}_{x = \pm\frac{\pi}{2},\ \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ \dots} \quad \underline{\text{OR}} \quad \underbrace{1 + \sin(x) = 0}_{x = \frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{7\pi}{2},\ -\frac{5\pi}{2},\ \dots}$$



$Y = \cos(X)$



$Y = \sin(x)$

all of these already included in sol$^{\underline{n}}$ to $\cos(x) = 0$.

Thus, the tangent line is horizontal at,

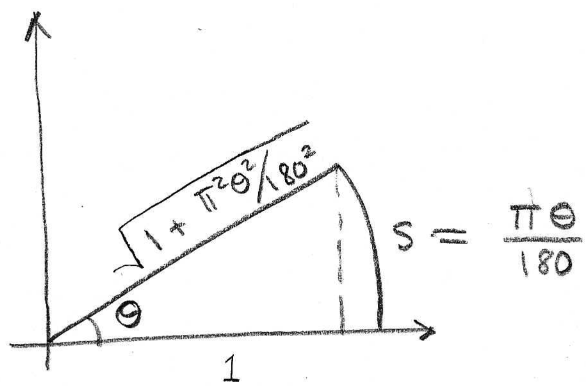
$$\boxed{\begin{aligned} x &= \pm\frac{\pi}{2},\ \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ \dots \\ &= n\pi + \frac{\pi}{2},\ n \in \mathbb{Z}. \\ &= \frac{1}{2}(2n+1)\pi,\ n \in \mathbb{Z}. \end{aligned}}$$

it asked for **all** sol$^{\underline{n}}$'s if we just gave $x = \pi/2$ our answer would be incomplete.

$\mathbb{Z}$ is the set of integers

§3.5 #87 This question is subtle. For me I need to go back to basics to convince my self. If x is in degrees then $\lim_{x \to 0} \left(\frac{\sin_{deg}(x)}{x} \right) \neq 1$. Lets see why. Also lets introduce some notation to emphasize the conceptual distinction,

$$\sin_{deg}(x) = \text{sine calculated from degree measure.}$$



$$\sin_{deg}(\theta) = \frac{\pi\theta/180}{\sqrt{1+\pi^2\theta^2/180^2}}$$

$$\lim_{\theta \to 0^\circ} \left(\frac{\sin_{deg}(\theta)}{\theta} \right) = \lim_{\theta \to 0} \frac{\frac{\pi\theta}{180}}{\theta\sqrt{1+\left(\frac{\pi\theta}{180}\right)^2}}$$

$$= \lim_{\theta \to 0} \frac{\pi}{180\sqrt{1+\underbrace{(\pi\theta/180)^2}_{0}}}$$

$$= \frac{\pi}{180}$$

All the other trig identities are sensible with $\sin \mapsto \sin_{deg}$ and $\cos \mapsto \cos_{deg}$. Thus the arguments for $\frac{d}{d\theta}(\sin\theta) = \cos\theta$ transfer over except $\lim_{x \to 0} \left(\frac{\sin(x)}{x}\right) = 1$ is replaced with $\lim_{\theta \to 0^\circ} \left(\frac{\sin_{deg}\theta}{\theta}\right) = \frac{\pi}{180}$

$$\Rightarrow \boxed{\frac{d}{d\theta}\left[\sin_{deg}(\theta)\right] = \frac{\pi}{180}\cos_{deg}(\theta)}$$

__Most__ texts do not distinguish between sin and $\sin_{deg}$. So this question is hard to make sense of.

§3.5 # 87 Continued

The limit arguments are not necessary really. For me I needed to grasp that sin and $\sin_{deg}$ were truly distinct functions. With that distinction made we can connect them through the following simple formula,

$$\sin_{deg}(x) \equiv \sin\left(\frac{\pi x}{180}\right)$$

$$\cos_{deg}(x) \equiv \cos\left(\frac{\pi x}{180}\right)$$

The $\sin\theta$ and $\cos\theta$ are understood to be the radian-based trig. functions in calculus. We derived (assuming radians) $\frac{d}{d\theta}(\sin\theta) = \cos\theta$ and $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$.

Consider,

$$\begin{aligned}
\frac{d}{dx}\left[\sin_{deg}(x)\right] &= \frac{d}{dx}\left[\sin\left(\frac{\pi x}{180}\right)\right] \\
&= \cos\left(\frac{\pi x}{180}\right)\frac{d}{dx}\left[\frac{\pi x}{180}\right] \\
&= \frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right) \\
&= \frac{\pi}{180}\cos_{deg}(x)
\end{aligned}$$

Thus, I argue $\frac{d}{dx}\left[\sin_{deg}(x)\right] = \frac{\pi}{180}\cos_{deg}(x)$

BUT $\frac{d}{dx}(\sin(x)) = \cos(x)$ (even if x is in degrees because x is a unitless # whether it is measured via radian or degree scaling for angles.)

(Stewart ≈ misleading here, but this issue is debated so I shouldn't complain much here.)